The Value of Demand Postponement under Demand Uncertainty

Rawee Suwandechochai

Abstract— Resource or capacity investment has a high impact on the firm profitability. However, this decision must be made earlier when demand is uncertain and it is very difficult to change later on. Many firms search for other strategies to deal with uncertainty in order to gain more profit and stay in a business. Postponement strategy is one of the strategies that many companies use to hedge against the uncertainty. A firm can increase its profitability when it can postpone some decisions or activities to the time later until the more information is obtained.

In this paper, we consider a firm producing two substitutable products. The firm needs to make two decisions: the capacity investment and production quantity. The objective of this work is to study how the demand postponement affects the firm's capacity investment and its profitability under demand uncertainty and how degrees of substitution impact our findings. Based on this framework, we model the problem as a two-stage stochastic programming. We characterize the optimal investment capacity and production quantity under different postponement strategies. In addition, the necessary and sufficient conditions for investment in flexible capacity are obtained.

***Keywords*—**Capacity investment, Demand postponement, Demand uncertainty, Stochastic programming, Substitutable Products.

I. Introduction

POSTPONEMENT strategy is one of the strategies that companies can use to hedge against uncertainty. Many type of postponement have been studied as stated in [1] and [2] such as time postponement, place postponement, and form postponement, etc. There are applications of postponement in industry and results from [3] indicated that there is a positive relationship between postponement and company performance.

The postpone strategies we consider in this work are the quantity postponement and demand postponement strategies. The production quantity postponement refers to a firm's ability to postpone its production quantity until after demand information is obtained. Considering the quantity postponement along with the flexible resources increases the firm profitability. The firm's flexible resources allow the firm to switch capacity amount of the multiple products and provides a risk pooling effect.

Many researchers have focus on the flexibility provided by the production postponement strategies as shown in [1], [4], and [5]. The impact of quantity postponement on the optimal capacity has been studied by [6] for a single-product firm. On the other hand, there is a few works studying on demand postponement. [7] formulates and solves the capacity planning problem for one product. In this paper, we extend the model to the substitutable products. In addition, we are also interested in the effect of the demand postponement strategy when the firm has implemented the production quantity postponement.

Therefore, the objective of this paper is to consider the impact of the demand postponement on the optimal capacity investment under different postponement strategies. Specifically, we consider the no postponement and the production quantity postponement as base cases. Then, we examine the effect of demand postponement on the optimal capacity investment and the firm's profit under both strategies as well as study how the degrees of substitution between products affect the findings.

II. Notations, Models, and Assumptions

In this work, we consider a firm that produces two substitutable products in a monopolistic setting. This firm needs to determine capacity, pricing, and production quantity.

Let K denote the capacity of the flexible resources that can produce both products,

c_K be a unit investment cost,

c_h be a unit holding cost,

c_q be a unit product cost,

c be a discount price (or reimbursement cost) for the demand in postponement period. The production cost for each item is included here,

d_i be a demand of product i,

p_i be a selling price of product i and, without loss of generality, let ($p_1 \geq p_2$),

s_i be an amount of selling product i,

q_i and q_{pi} be production quantities of product i for regular and postponement period, respectively.

Timeline is divided into two stages shown in Fig 1. Stage 1 is a planning period and Stage 2 includes the regular and postponement periods.

This work was supported in part by the Thailand Research Fund (TRF) under Grant MRG5080200 and Faculty of Graduate Studies, Mahidol Univesity.

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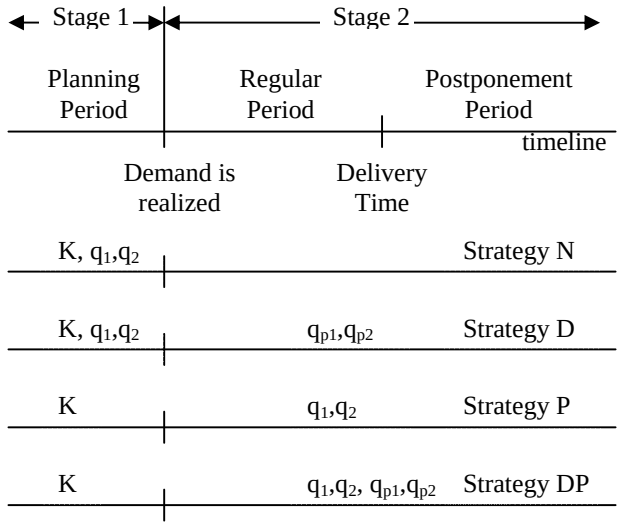

Figure 1: Timeline

In Fig 1, the investment capacity (K) is determined in the planning period when demand is highly uncertain. There are four cases that are considered, No postponement (N), Demand postponement (D), Production postponement (P), and production and demand postponement (PD). All the decision variables are determined in the planning period under strategy (N). For strategy (P), the production quantity, q_i, is determined after the demand curves are realized. When the demand postponement strategy is implemented, the postponed production quantity, q_{pi}, is determined after the demand curves are realized and this products will be delivered in the postponement period.

We assume that the fraction of unsatisfied demand, 1-γ, is retained in the postponement period. That is, when the demand exceeds the production quantity, there is only $(1-\gamma)(d_i-q_i)$ remaining demand in the postponement period.

Aggregate linear demand function for two substitutable products is considered. It is a function of the prices of both products:

$$d_i = \xi_i - \alpha p_i + \beta p_{3-i},\ i = 1,2$$

where ξ_i is an intercept of product i, β is the degree of substitution ($0 \le \beta \le \alpha$).

Let E[.] be an expectation operator with respect to random variables, ξ_i, and Pr(.) be a probability. Let $\pi(.)$ be the profit function in Stage 2.

III. Mathematical formulations

From the above, we can divide the problem into two stages. Stage 1 corresponds to the time when the demand is uncertain and Stage 2 is the time after the demand information is obtained.

The mathematical model for the problem under demand strategy (D) can be formulated as

$$\text{(stage 1)}\ \max_{K,\vec{q},\vec{p}} V \equiv E[\pi(K,\vec{q})] - c_K K - \sum_{i=1}^{2}(c_q + c_h)q_i$$

$$\text{s.t.}\quad \sum_{i=1}^{2} q_i \le K \qquad (1)$$

$$K, q_i \ge 0 \qquad \text{for } i = 1,2 \qquad (2)$$

$$\text{(stage2)}\ \max_{\vec{s},\vec{q}_p} \pi(K,\vec{q}) = \sum_{i=1}^{2} p_i s_i + \sum_{i=1}^{2}(p_i - c)q_{pi}$$

$$\text{s.t.}\quad s_i \le q_i \qquad \text{for } i = 1,2 \qquad (3)$$

$$s_i \le (\varepsilon_i - \alpha p_i + \beta p_{3-i})^+ \quad \text{for } i = 1,2 \qquad (4)$$

$$q_{pi} \le q_i \qquad \text{for } i = 1,2 \qquad (5)$$

$$q_{pi} \le (1-\gamma)(\varepsilon_i - \alpha p_i + \beta p_{3-i} - s_i)^+ \text{for } i = 1,2 \qquad (6)$$

$$s_i, q_{pi} \ge 0 \qquad \text{for } i = 1,2 \qquad (7)$$

The objective of the model in Stage 1 is to maximize the expected profit function. The objective function in Stage 2 is to maximize the firm profit by determining the amount of sell, s_i, and the postponed production quantity q_{pi}. Constraint (1) states that the total production quantity should not exceed the firm's capacity. Constraints (3) and (4) imply that the amount of sell cannot exceed production quantity and nonnegative demand of each product. In addition, (5) and (6) ensures that the postponed production quantity, q_{pi}, does not exceed the production quantity and it does not exceed retained demand. Constraints (2) and (7) are non-negativity constraints for investment capacity, production quantity, selling amount, and postponed production quantity, respectively.

Note that the model for the Problem N is similar except that q_{pi} is set to zero.

The mathematical model for the problem under production quantity and demand postponement strategy (PD) can be formulated as

$$\text{(stage 1)}\ \max_{K} V \equiv E[\pi(K)] - c_K K$$

$$\text{s.t.}\quad K \ge 0 \qquad (8)$$

$$\text{(stage2)}\ \max_{\vec{q},\vec{q}_p} \pi(K) = \sum_{i=1}^{2}(p_i - c_q)q_i + \sum_{i=1}^{2}(p_i - c)q_{pi}$$

$$\text{s.t.}\quad q_1 + q_2 \le K \qquad (9)$$

$$q_i \le (\varepsilon_i - \alpha p_i + \beta p_{3-i})^+ \qquad \text{for } i = 1,2 \ (10)$$

$$q_{p1} + q_{p2} \le K \qquad (11)$$

$$q_{pi} \le (1-\gamma)(\varepsilon_i - \alpha p_i + \beta p_{3-i} - s_i)^+ \text{for } i = 1,2 \ (12)$$

$$q_i, q_{pi} \ge 0 \qquad \text{for } i = 1,2 \ (13)$$

The objective of the model in Stage 1 is to maximize the expected profit function. In this case, the firm determines the production quantity after the demand is realized. Therefore, the production and the holding costs do not occur in the first

stage. In Stage 2, the objective function is to maximize the firm profit by determining the production quantity and the postponed production quantity given that the investment capacity and prices. Constraints (9) and (11) state that the total production quantity and the total postponed production quantity should not exceed the firm's capacity, respectively. Constraint (10) implies that the production quantity should not exceed the nonnegative demand of each product. In addition, (12) the postponed production quantity (q_{pi}) does not exceed the retained demand. (8) and (13) are non-negativity constraints for investment capacity, production quantity, and postponed production quantity.

The model for the production quantity postponement (P) is similar to Problem (PD) except that q_{pi} is set to zero.

IV. Results

The optimal solution to Problems N, D, and P can be characterized in the similar procedures. Hence, in what follows, we focus on Problem PD. To analyze the problem, we solve the problem backward. First, for a given set of p_1 and p_2, we derive the optimal solution for the problem in Stage 2 and show that the expected profit function in Stage 1 is strictly jointly concave in q_1, and q_2. Consequently, the optimal solution is unique and the Karush-Kuhn-Tucker (KKT) first-order conditions are necessary and sufficient for optimality to the problem. This leads to the following results.

Theorem 1 The production quantities q_1 and q_2 are the unique optimal solution to Problem PD if and only if there exists $v \geq 0$ that satisfies the following conditions:

$$[c - c_p + \gamma(p_1 - c)]\Pr(\Omega_5^{QD} \cup \Omega_{11}^{QD} \cup \Omega_{12}^{QD}) +$$
$$[c - c_p + \gamma(p_2 - c)]\Pr(\Omega_6^{QD} \cup \Omega_9^{QD}) +$$
$$[p_2 - c_p + (1-\gamma)(p_2 - c)]\Pr(\Omega_{12}^{QD}) +$$
$$[2p_1 - c_p - c]\Pr(\Omega_7^{QD}) + [2p_2 - c_p - c]\Pr(\Omega_8^{QD} \cup \Omega_{10}^{QD}) = c_K - v$$
$$K \cdot v = 0$$

where

$$\Omega_5^{QD} = \{K + \alpha p_1 - \beta p_2 < \xi_1 < \frac{2-\gamma}{1-\gamma}K + \alpha p_1 - \beta p_2,$$
$$\xi_2 < \alpha p_2 - \beta p_1\}$$
$$\Omega_6^{QD} = \{\xi_1 < \alpha p_1 - \beta p_2, \alpha p_2 - \beta p_1 < \xi_2 < K + \alpha p_2 - \beta p_1\},$$
$$\Omega_7^{QD} = \{\xi_1 > \frac{2-\gamma}{1-\gamma}K + \alpha p_1 - \beta p_2\},$$
$$\Omega_8^{QD} = \{\xi_1 < \alpha p_1 - \beta p_2, \xi_2 > K + \alpha p_2 - \beta p_1\},$$
$$\Omega_9^{QD} = \{\alpha p_1 - \beta p_2 < \xi_1 < K + \alpha p_1 - \beta p_2,$$
$$K + (\alpha - \beta)(p_1 + p_2) < \xi_1 + \xi_2 < \frac{2-\gamma}{1-\gamma}K + (\alpha - \beta)(p_1 + p_2)\}$$
$$\Omega_{10}^{QD} = \{\alpha p_1 - \beta p_2 < \xi_1 < K + \alpha p_1 - \beta p_2,$$
$$\xi_1 + \xi_2 > \frac{2-\gamma}{1-\gamma}K + (\alpha - \beta)(p_1 + p_2)\}$$
$$\Omega_{11}^{QD} = \{\xi_1 > K + \alpha p_1 - \beta p_2, \xi_2 > \alpha p_2 - \beta p_1,$$
$$\xi_1 + \xi_2 < \frac{2-\gamma}{1-\gamma}K + (\alpha - \beta)(p_1 + p_2)\}$$
$$\Omega_{12}^{QD} = K + \alpha p_1 - \beta p_2 < \xi_1 < \frac{2-\gamma}{1-\gamma}K + \alpha p_1 - \beta p_2,$$
$$\xi_1 + \xi_2 > \frac{2-\gamma}{1-\gamma}K + (\alpha - \beta)(p_1 + p_2)$$

Proof: See Appendix.

Theorem 2 The production quantities q_1 and q_2 are the unique optimal solution to Problem N if and only if there exists v_1 and $v_2 \geq 0$ that satisfies the following conditions:

$$p_1 \Pr(\xi_1 > q_1 + \alpha p_1 - \beta p_2) = c_K + c_h + c_q - v_1$$
$$p_2 \Pr(\xi_2 > q_2 + \alpha p_2 - \beta p_1) = c_K + c_h + c_q - v_2$$
$$q_i \cdot v_i = 0, \quad \text{for } i = 1,2$$

Theorem 3 The production quantities q_1 and q_2 are the unique optimal solution to Problem D if and only if there exists v_1 and $v_2 \geq 0$ that satisfies the following conditions:

$$(2p_1 - c)\Pr(\xi_1 > \frac{2-\gamma}{1-\gamma}q_1 + \alpha p_1 - \beta p_2) +$$
$$(\gamma p_1 + (1-\gamma)c)\Pr(\Omega_1^D) = c_K + c_h + c_q - v_1$$
$$(2p_2 - c)\Pr(\xi_2 > \frac{2-\gamma}{1-\gamma}q_2 + \alpha p_2 - \beta p_1) +$$
$$(\gamma p_2 + (1-\gamma)c)\Pr(\Omega_2^D) = c_K + c_h + c_q - v_2$$
$$q_i \cdot v_i = 0, \quad \text{for } i = 1,2$$

where

$$\Omega_1^D = \{q_1 + \alpha p_1 - \beta p_2 < \xi_1 < \frac{2-\gamma}{1-\gamma}q_1 + \alpha p_1 - \beta p_2\},$$
$$\Omega_2^D = \{q_2 + \alpha p_2 - \beta p_1 < \xi_2 < \frac{2-\gamma}{1-\gamma}q_2 + \alpha p_2 - \beta p_1\}.$$

Theorem 4 The production quantities q_1 and q_2 are the unique optimal solution to Problem P if and only if there exists $v \geq 0$ that satisfies the following conditions:

$$p_2 \Pr(\xi_1 > \alpha p_1 - \beta p_2, \xi_2 > K + \alpha p_2 - \beta p_1)$$
$$+ p_1 \Pr(\xi_1 > K + \alpha p_1 - \beta p_2) + p_2 \Pr(\Omega^Q) = c_K - v$$
$$K \cdot v = 0$$

where

$$\Omega^Q = \{\alpha p_1 - \beta p_2 < \xi_1 < K + \alpha p_1 - \beta p_2, \xi_1 + \xi_2 > K + (\alpha - \beta)(p_1 + p_2)\}$$

The optimal capacity investment under each postponement strategy follows a threshold policy.

Corollary 1 The optimal investment policy under postponement strategy is one of the following cost threshold policies:

1) If $c_K + c_h + c_q < \bar{c}^N$, then $K^* > 0$. Otherwise, $K^* = 0$.

where $K^* = q_1^* + q_2^*$ the threshold values is given as

$$\bar{c}^N = \max\{ p_1 \Pr(\xi_1 > \alpha p_1 + \beta p_2), p_2 \Pr(\xi_2 > \alpha p_2 + \beta p_1)\}$$

2) If $c_K + c_h + c_q < \bar{c}^D$, then $K^* > 0$. Otherwise, $K^* = 0$.
where $K^* = q_1^* + q_2^*$ the threshold values is given as

$$\bar{c}^D = \max\{ (2p_1 - c)\Pr(\xi_1 > \alpha p_1 - \beta p_2), (2p_2 - c)\Pr(\xi_2 > \alpha p_2 - \beta p_1)\}$$

3) If $c_K < \bar{c}^Q$, then $K^* > 0$. Otherwise, $K^* = 0$.
where the threshold values is given as

$$\bar{c}^Q = p_1 \Pr(\xi_1 > \alpha p_1 - \beta p_2) + p_2 \Pr(0 < \xi_1 < \alpha p_1 - \beta p_2, \xi_2 > \alpha p_2 - \beta p_1)$$

4) If $c_K < \bar{c}^{QD}$, then $K^* > 0$. Otherwise, $K^* = 0$.
where the threshold values is given as

$$\bar{c}^{QD} = (2p_1 - c - c_p)\Pr(\xi_1 > \alpha p_1 - \beta p_2) + (2p_2 - c - c_p)\Pr(0 < \xi_1 < \alpha p_1 - \beta p_2, \xi_2 > \alpha p_2 - \beta p_1)$$

Proof: It follows directly from Theorems 1-4.

The above corollary suggests that if the total unit cost $c_K + c_h + c_p$ is less than the cost threshold, the firm always invests in the capacity under Strategies N and D. On the other hand, if the total unit cost is higher than the cost threshold, it is better for the firm not to invest. Similarly, under the Strategies P and PD, if the unit investment cost is less than the cost threshold value indicated in Corollary 1, then the firm should invest in the capacity.

Corollary 2

$$\bar{c}^N \le \bar{c}^D \text{ and } \bar{c}^Q \le \bar{c}^{QD}$$

Proof: It follows directly from Proposition 1.

Corollary 2 indicates that there are some regions that the firm should invest in its capacity to gain more profit when the demand postponement is considered.

Next, we perform a numerical study to investigate the impact of the optimal investment capacity changes in demand substitution parameter β.

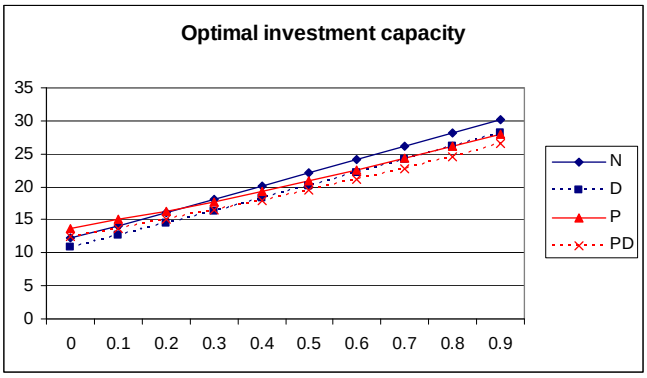

Figure 2: Optimal investment capacity K^* versus β/ν.

Results from the numerical studies suggest that the optimal investment capacity K^* is increasing in β for all strategies (N, D, P, PD) as shown in Fig. 2. The results show that the firm will invest more when the products are more substitutable. This is because the firm can take advantage from the substitution. However, the rate of change of the optimal investment capacity is less when production postponement is implemented. This is because of the flexible resources which is also used to hedge against demand variability. It is not surprising that when the products are independent or not substitutable the firm invests more under strategy P. This is because the firm has ability to deal with the demand variability by using the production quantity postponement from the flexible resource. The results also show that the demand postponement strategy leads to invest less in the firm's capacity and it leads to lower investment when addition production quantity is implemented along with.

Moreover, our finding indicates that the firm invests less when the demand postponement is additionally implemented. Due to the ability of the firm to postpone part of the demand to time later, it may cause the firm invests less.

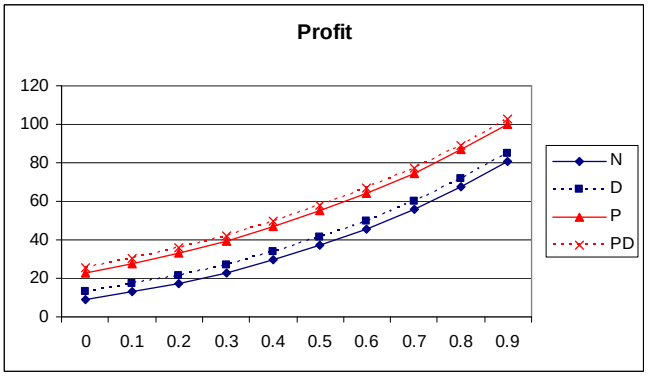

Figure 3: The firm's profit versus β/ν.

Similar results are obtained for the firm's profit. Fig 3 shows that the profit is increasing in substitution parameter for all strategies. The demand postponement generates the higher profit for the firm.

V. Conclusion

In this paper, we study the impact of the demand postponement on the optimal capacity under demand uncertainty for a two-product firm in a monopolistic setting. The firm makes decisions 1) investment capacity and 2) product quantity to maximize the profit function. We formulate mathematical models for different postponement strategies. The necessary and sufficient condition for the optimal capacity investment is obtained for each strategy. Our findings show that implementing the demand postponement leads to invest less in optimal capacity and increase in the firm's profitability.

Our results come with limitations. We consider the linear demand function which depends on the price of the products.

It would be interesting to consider other realistic demand function. We also assume a firm that is in a monopolistic setting. Considering the competition would be another interesting extension to our models.

APPENDIX

Consider problem QD, we first solve the Stage 2 Problem. The demand space can be decomposed into different disjoint sets; see Fig. 4.

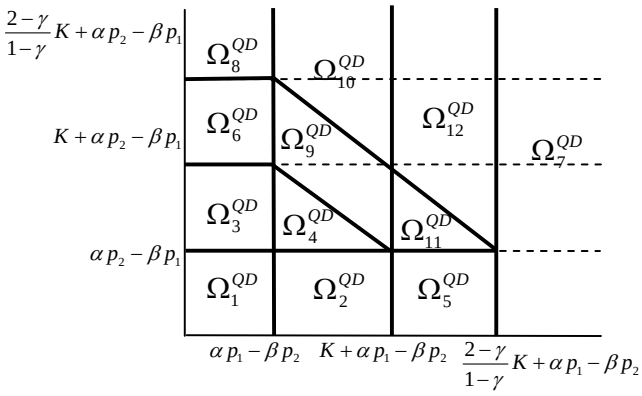

Figure 4: The demand space for Problem PD

We can obtain the closed form expressions for the production quantities and postponed production quantities for each region. It is easy to verify that the optimal solution to the Stage 2 Problem is unique and the optimal Stage 2 profit can be obtained directly. Before we solve the Stage 1 problem, let ε_i be a realization of ξ_i and $f(\cdot,\cdot)$ be a joint probability density function of ξ_1 and ξ_2, respectively.

To solve the Stage 1 Problem, the Stage 2 profit is considered. We derive

$$\begin{aligned}\frac{\partial E[\Pi(K)]}{\partial K} &= [c-c_p+\gamma(p_1-c)]\Pr(\Omega_5^{QD}\cup\Omega_{11}^{QD}\cup\Omega_{12}^{QD})\\ &+[c-c_p+\gamma(p_2-c)]\Pr(\Omega_6^{QD}\cup\Omega_9^{QD})\\ &+\ [p_2-c_p+(1-\gamma)(p_2-c)]\Pr(\Omega_{12}^{QD})\\ &+\ [2p_1-c_p-c]\Pr(\Omega_7^{QD})\\ &+[2p_2-c_p-c]\Pr(\Omega_8^{QD}\cup\Omega_{10}^{QD})\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 E[\Pi(K)]}{\partial K^2} &= -\int_0^{\alpha p_2-\beta p_1}[(p_1-c_p)-(1-\gamma)(p_1-c)]\\ &\qquad f(K+\alpha p_1-\beta p_2,\varepsilon_2)\,d\varepsilon_2\\ &-\int_{\alpha p_2-\beta p_1}^{\infty}\gamma(p_1-p_2)f(K+\alpha p_1-\beta p_2,\varepsilon_2)\,d\varepsilon_2\\ &-\int_0^{\alpha p_1-\beta p_2}\left(\frac{2-\gamma}{1-\gamma}\right)[\gamma p_2+(1-\gamma)c-c_p]\\ &\qquad f(\varepsilon_1,K+(\alpha-\beta)(p_1-p_2))\,d\varepsilon_1\\ &-\int_{K+\alpha p_1-\beta p_2}^{\frac{2-\gamma}{1-\gamma}K+\alpha p_1-\beta p_2}\frac{(2-\gamma)^2}{1-\gamma}(p_2-c)\\ &\qquad f(\varepsilon_1,\frac{2-\gamma}{1-\gamma}K+(\alpha-\beta)(p_1-p_2)-\varepsilon_1)\,d\varepsilon_1\\ &<0.\end{aligned}$$

It can be shown that $V=E[\pi(K)]-c_K K$ is strictly concave in K. Therefore, the optimal solution to the Stage 1 Problem is unique, and since the constraints are linear, the KKT first-order optimality conditions, given in Theorem 1, are necessary and sufficient for optimality.

Similar method can be used to obtain the results in Theorems 2-4.

ACKNOWLEDGMENT

I would like to thank Thailand Research Fund (TRF) for the financial support and Dr. Kanchit Malaivongs for all his comments and support for this research.

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