Expansion of the basic EOQ model with inclusion of trim-loss costs

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Abstract—We suggest an expansion of the basic economic order quantity model by including costs of trim-loss into the model. The expansion is based on the findings about trim loss costs from our previous work. The ratio between the stock size and order size significantly affects the trim loss costs thus it is important to take that ratio into consideration when proposing the expanded model. In the first part of the contribution we conducted a literature research of current model expansions and have not found any expansions that would include trim loss costs. In the second part the proposal for expansion is presented.

Keywords—costs optimization, cutting stock problem, economic order quantity, inventory management.

I. INTRODUCTION

In relationship to some costs related to stock, a model for optimal order size (economic order quantity – EOQ model) was developed at the beginning of the last century [1], [2]. This model is still used today in many varieties and expansions. The model is suitable for solving the problem of the optimal order for one product [3], and particularly adequate for solving a one-dimensional cutting stock optimization problem in which companies order one type of item. There is a limitation for companies that order the same type of items but different lengths. It is possible to address the problem by averaging order lengths and transforming them into a unified product so that the EOQ model can be applied.

This paper first presents the basic model of optimal order size with some of the most common expansions. This is followed by a proposal to expand the model of optimal order size with trim-loss costs.

II. MODEL OF OPTIMAL ORDER SIZE

The model of optimal order size focuses on the question of when and what amount should be ordered to keep the total stock costs as low as possible. It was developed at the beginning of the twentieth century and it is still used in many versions to solve problems today. The description of the basic model of optimal order size is recapitulated from [4] – [6].

The following costs are included in the basic model:

- Stock costs;
- Order costs;
- Costs per unit in stock.

In the ideal case, the company could have infinite stock at stock costs equal to zero. If the order costs were equal to zero, the company could order the minimum amount at minimal intervals. However, such situations are not possible in practice and it is therefore necessary to determine what the order size is at which the total costs are minimized. This problem can be solved by the model of optimal order size under certain limitations. The presumptions of the basic model of optimal order size are as follows:

- Demand is known and constant in time;
- Costs are known and constant in time;
- Supply time is equal to zero;
- Stock shortages are not allowed;
- Each order is delivered from a single supply;
- Purchase price is constant regardless of the amount ordered.

The costs of stock per unit are constant in the basic model because the company pays an equal price for all units purchased. The viability of including the cost of stock per unit, which does not affect the final result in the basic model, is presented later in the explanation of expanded models. Stock costs increase with the growth of individual order size. When orders are larger, the intervals between individual orders are also larger. This means that companies will have higher average stocks, which consequently lead to higher stock costs. In the basic model of optimal order size, constant growth of stock cost is presupposed.

Stock cost falls with larger order size. The larger the order is, the fewer times the company will have to order new deliveries. Because certain costs do not depend on order size (e.g., administrative costs of the order) and some costs do not grow linearly with the order size (e.g., equipment-use costs, transport cost, quality-control cost, etc.), the total cost of ordering decreases. Due to these components of ordering costs, these costs do not fall linearly but are dependent on each individual instance.

All three types of costs are shown in Fig. 1. From the figure it is evident that the costs per stock unit do not affect the optimal order size, which is found when the total costs of stock, total cost of ordering, and total cost of stock units are minimal.
The basic model of optimal order size can offer an answer to optimal order size, either in the number of units that need to be ordered or in the order value. The basic model of optimal order size therefore provides a result that presents the optimal order. However, the result may not always be an integer, which can lead to a useless result because certain types of items are only able to be delivered in discrete amounts. It is possible to find many cases in practice. For example, a company cannot order 3.71 computers or 18.9 trucks. Thus, rounding up or down and the sensitivity analysis are necessary due to the optimal result [7]. The second limitation of the result arises from packing or transport. Assume, for instance, that it is possible to load a container with 13 tons of loose cargo. Based on computation, it is determined that the optimal order size is equal to 14 tons. This means that the second container would be almost empty and the cost of transport would rise. In this case it is also necessary to take other costs into account.

Due to these limitations and because of the limitations on the basic EOQ model that arise from basic assumptions that indicate the narrowness of use of the model in practice, many expansions of the model have been developed. The most important and recent models are introduced below.

An important extension of the basic model of optimal order size is represented by taking quantity discounts from suppliers into account [8] – [10]. Suppliers offer quantity discounts for larger orders, which results in a decrease of average cost per stock unit. Hence the cost per stock unit curve, which is horizontal in Fig. 1, acquires a downward character. The most common situation is stepped-curve drops, which also lead to stepped drops in the total-cost curve. The reasons for stepped drops are stepped quantity discounts, in which a discount is applied to all items at a certain order quantity. The topicality of expansion is also reflected in its recent discussion by various researchers [11].

In practice, payments are not carried out immediately but are shifted for various reasons. Extending the basic model of optimal order size with regard to payment shift was first introduced by [12]. The opportunity costs of interest were included later [13]. This extension is also present in current expansions of the model [14] – [17].

By removing the assumption about pre-known supply times and by introducing uncertainty, running out of stock became an important factor of the model. Costs that arise from inability to meet demand due to insufficient stocks were introduced to the model by [18]. They were followed by many other researchers [19] – [21].

There are many other expansions of the model, but these are not mentioned separately in this paper. What is important is that in the literature it is not possible to find an extension of a model of optimal order size that takes trim-loss costs into consideration. The proposal for expanding the basic model of optimal order size with costs of trim loss that arise from cutting is presented in the following section.

III. PROPOSITION OF EXPANSION

The proposed expansion of the basic model of optimal order size is presented by introducing trim-loss cost with the assumption that the trim-loss costs are similar to the costs of holding the unit in stock. The costs of the order should be about the same size as well. If the costs are smaller, the trim-loss costs will not significantly affect the total stock cost. It is therefore reasonable to include them in the model only for theoretical purposes because it is not useful in practice.

The ratio between the stock size and order size significantly affects the trim-loss costs. It is therefore important to take this ratio into consideration when proposing an expanded model [22]. With increasing individual order size to suppliers in relation to individual order size from customers, the average stock also grows in relation to the average order. Based on findings in [22], it is assumed that trim-loss costs will be lower by increasing the average stock (if the order size remains unchanged). This phenomenon occurs due to the greater variety of stock items from which the company can cut orders received from its customers. Hence it is necessary to appropriately adjust the horizontal axis on the graph of the basic model of optimal order size.

Therefore it is not possible to find the absolute order size for suppliers on the horizontal axis of the expanded model, but the relative size of average stock, which represents half of the size of individual order to suppliers due to size of individual customer orders. By converting the values on the horizontal axis, the initial shapes of the curves of costs do not change in the basic model.

This explanation of the uniformity of values on the horizontal axis of the costs presented can also be expressed with equations. The order size for suppliers from Fig. 1 can be reformulated as:

\[ \text{order size for suppliers} = 2 \times \text{average stock size} \]

(1)

The ratio defined in [22] can be reformulated as:

\[ \frac{\text{average stock size}}{\text{average customer order size}} \]

(2)
For the purposes of expanding the basic model of optimal order size, it is possible to presuppose that the average order size for customers is constant, and so the following relation can be established:

\[
\text{order size for suppliers} = \frac{\text{average stock size}}{\text{const.}} \tag{3}
\]

The graph of the proposed extended basic model of optimal order size is presented in Fig. 2.

Fig. 2: Graphic presentation of the proposed expanded EOQ model

The basic shape of trim-loss costs curve is assumed based on simulations carried out in [22]. The position of the trim-loss costs curve in relation to the other cost curves in Fig. 2 is presented symbolically. The position depends on the relative value of material in a ratio with other costs. The higher the value of cut material is (if other costs remain unchanged), the higher the curve of trim-loss cost will be positioned. The influence on the total-cost curve will also rise. The closer the global maximum of the trim-loss costs curve to optimal order size in the basic model (which means that optimal order size in the basic model will be close to the value of the ratio between stock size and order size), the larger the influence trim-loss costs have on the minimum of total cost in the proposed expanded model.

In proposing an expansion of the model, we relied on findings about the optimal ratio between stock size and order size [22]. Because the cutting problem is NP hard and including the cutting problem into the problem of optimal order size makes the problem mathematically more demanding, we suggest using simulations for searching for the optimal order size in the proposed expansion of the model. Further studies could therefore involve the use of simulations for the proposed model.

REFERENCES