Analysis of environment - DFB-FL sensors interaction by using coupled-mode equations

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Abstract—This paper is pointing to an investigation of various aspects of distributed feedback fiber laser sensors and their interaction with environment and of possible applications by numerically solving coupled-mode equations system describing the laser field propagation. The developed numerical analysis has the aim of a better understanding of DFB-FL itself and of its interaction with environment in order to be operated as a sensor. The main idea of the paper consists into investigating how various environment parameters modify the coefficients of coupled-mode equations describing laser field propagation through the DFB-FL structure. Basically, this is the key of understanding operation of DFB-FL sensors.

Keywords— DFB-FL, sensor, coupled-mode equations, environment, laser optimization.

I. INTRODUCTION

DISTRIBUTED feedback fiber lasers (DFB-FL) are optoelectronic devices with an increasing number of applications in various fields, being the subject of much research over the past ten years. They proved high reliability, fiber compatibility, high output power, narrow bandwidth, good beam quality, low phase noise, and low relative intensity noise (RIN) [1]–[4]. Distributed-Feed-Back fiber lasers (denoted as DFB-FL) can be designed with a grating structure to provide high output power (up to 60 mW), single frequency [5], single polarization [6] and high optical signal-to-noise ratio [6]. DFB-FLs have been widely used in sensing [7], communication systems [8]–[11], and high precision spectroscopy [12] all of which require single-mode, single-frequency lasers.

Single polarization and narrow line bandwidth of DFB-FL lasers are very desirable for a large number of applications among which sensor systems are an important one [5]–[7]. Alternatively, DFB lasers can be made to operate in stable dual polarization regime, defined with respect to the induced spatial modulation of the core index of refraction (Bragg grating), so that simultaneous measurements can be carried out [8]–[10]. In addition to the sensing and telecommunication applications, DFB fiber lasers suitable for high-power applications have been demonstrated [11]. In order to obtain increased output power DFB-FL, doping levels have been increased to allow more pump light to be absorbed with the doping densities of commercial Erbium (Er3+) and Ytterbium (Yb3+) ions co-doped fibers. With the commercial availability of over 500 mW-pump single emissive structure diode lasers, the absorption transition inside doped optical fiber relative easy becomes saturated. The phenomenon of un-pumped lengths of optical fiber appearance is immediately implied. Because the majority of the pump light is absorbed and/or converted into lasing photons over limited, even relatively long at the Bragg grating scale, portions of optical fiber, the remainder of the fiber remains essentially un-pumped. The transition length between the pumped and un-pumped regions is given by the small-signal absorption, which is of the order of a millimeter or less, a phenomenon which is defined as gain apodization. This gain apodization effect with pumped and un-pumped sections of the DFB laser is a phenomenon which has so far been neglected.

In this article, a first stage results obtained by numerical simulation of effects of gain apodization in DFB-FL are presented. At a first glance, the impact on threshold behavior of DFB-FL structure is explored along with its effect on output power and mode discrimination, pointing to the implications as sensor use. In the following sections several items of DFB-FL subject are investigated. A “conventional” model based on coupled-wave equations is sketched and applied to several cases of possible DFB-FL structures. The physics of gain apodization in DFB lasers is necessarily studied and a comparison with conventional configurations is performed. The impact of gain apodization on phase-shifted DFB fiber lasers is investigated; lasing thresholds and the output power ratio from both ends of the fiber lasers are analyzed; and, finally, techniques for using gain apodization as an optimization tool are discussed.

II. THEORY

Traditionally, defined by use or non-use of a π phase-shift and its location inside the diffraction Bragg grating, there have
been three main DFB-FL cavity designs which were intensively investigated because of ease of manufacturing and because they offer different performance and distinctive operational characteristics [13]-[14]. Although a DFB-FL structure is widely used for single-mode operation, in general, its mode spectrum is more complicated. In a uniform index-coupled DFB-FL without π phase shift or end mirrors, DFB-FL can operate in one of two degenerate longitudinal modes symmetrically located along \( \lambda_0 \), the Bragg wavelength of the grating.

As a “classical” approach, coupled-mode theory [15]-[16] can be used to analyze the threshold behavior in simple DFB-FLs. Fig. 1 is a schematic representation of the well-known coupled-mode theory, consisting, essentially, into the coupling between forward and backward electric field waves propagating inside the waveguides created by the periodic modulation of the optical fiber core refractive index \( n \). For a uniform optical fiber Bragg grating with uniform gain, using a widely performed notation, the coupled-wave equations can be written as [15]-[17]:

\[
\frac{dE_A}{dz} = \kappa \cdot e^{i[2\Delta\beta z]} E_A + g E_A
\]

\[
\frac{dE_B}{dz} = \kappa \cdot e^{-i[2\Delta\beta z]} E_A - g E_B
\]

In the system of Eq. (1) and (2), \( E_A \) and \( E_B \) are the complex amplitudes of the forward and backward propagating waves. In Eqs. (1) and (2), \( \Delta\beta \) is the propagation constant difference between the waves in the \( z \) direction along the optical fiber axis and the \( m \)-th Bragg frequency of the grating (\( m = 1 \) for first-order gratings). \( \Delta\beta \) is defined as:

\[
\Delta\beta = \beta - \frac{m \pi}{\Lambda}
\]

\( \phi \) is the wave phase at the position \( z = 0 \), \( \kappa \) is the coupling coefficient between the forward and backward waves propagating inside the Bragg grating, \( g \) is the gain coefficient of the active medium collocated with the Bragg grating, existing inside it, \( \Lambda \) is the wavelength of the optical fiber Bragg grating spatial modulation. In the absence of reflections from either sides of the structures the system of Eq. (1) and (2) can be solved analytically, as a first glimpse.

\[
\begin{bmatrix}
E_A(z_{j+1}) \\
E_A(z_{j+1})
\end{bmatrix} =
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
E_A(z_{j}) \\
E_A(z_{j})
\end{bmatrix}
\]

(4)

In Eq. (4) the matrix elements are the solutions to system of Eq. (1) and (2). The matrix elements of Eq. (4) are defined as:

\[
F_{11} = \cosh(\gamma_j L_j) + \frac{\Delta\beta_j}{\gamma_j L_j} \sinh(\gamma_j L_j) e^{i \phi_k j}
\]

(5)

\[
F_{12} = -\kappa_j L_j \sinh(\gamma_j L_j) \frac{e^{-i \phi_k j}}{\gamma_j L_j}
\]

(6)

\[
F_{21} = -\kappa_j L_j \sinh(\gamma_j L_j) \frac{e^{i \phi_k j}}{\gamma_j L_j}
\]

(7)

\[
F_{22} = \cosh(\gamma_j L_j) - \frac{\Delta\beta_j}{\gamma_j L_j} \sinh(\gamma_j L_j) e^{-i \phi_k j}
\]

(8)

Into the matrix Eq. (4), more precisely, into the \( F \) matrix elements the following parameters are introduced as:

\[
\Delta\beta_j = \Delta\beta + ig_j
\]

(9)

\[
\gamma_j = \gamma_j - (\Delta\beta_j)^2
\]

(10)

\[
\beta_j = \frac{\pi}{\Lambda j}
\]

(11)

The basic idea of this formalism consists in the splitting of the active gratings into \( N \) sections where the total matrix will be defined as:

\[
F_i = F_{i,N} F_{i,N-1} \ldots F_{2} F_1
\]

For a non-uniform DFB fiber laser, the coupling coefficient \( \kappa \) and gain coefficient \( g \) can change with the position \( z \). For DFB fiber lasers without a phase shift, the phase terms in the definitions of matrix \( F \) elements can be written as

\[
\phi_k = \phi_k^{(1)} + \Delta\beta_k^{(1)}
\]

(13)

where \( \kappa = 1, 2, \ldots N \). For phase-shifted DFB fiber lasers, the phase terms in the definitions of matrix \( F \) elements can be written as

\[
\phi_k = \phi_k^{(1)} + 2\beta_k^{(1)} L_k^{(1)} + \Delta\phi_k
\]

(14)

The boundary conditions are defined as:

\[
E_A(0) = E_B(L) = 0
\]

(15)

By using the boundary conditions, the gain threshold condition can be obtained from the relation [17]:

\[
F_{12} = 0
\]

(16)

Nominally, this relation will produce a mode spectrum with different modes appearing at different frequencies \( \Delta\beta \). For high-power operation, it is desirable not only to have a low threshold but also to have most of the light coming out of only one side of the cavity. By using the total matrix \( F_T \), the output power ratio from both ends of the fiber can be written as

\[
\frac{P_1}{P_2} = \frac{\left| E_A(0) \right|^2}{\left| E_A(L) \right|^2} = |F_{21}|^2
\]

(17)

In Eq. (17) \( P_1/P_2 \) represents the ratio of the power output coupling at \( z = 0 \), as compared to \( z = L \). For an improved
understanding of the physics introduced by gain apodization, the previously defined formalism is applied to several significant cases. In all cases, the grating strength $\kappa$ and period $\Lambda$ are kept constant and no phase shift will be included. The peak reflectivity of the grating is determined, in each investigated case, by

$$R = \tanh^2(kd)$$  \hspace{1cm} (18)

As can be noticed the desired generality is not lost, as typical values for $\kappa$ and $L$ being chosen. In the followings, the coupling coefficient of the fiber grating is $\kappa \sim 1 \text{ cm}^{-1}$ and the grating lengths are 3 - 5 cm in most cases. Since the length under which the gain will drop from its maximum value to zero is less than 1 mm, the gain apodization along the $z$ axis will be approximated by using a defined step function. The investigated gain-apodized DFB-FL is schematically presented in Fig. 2, 3 and 4. In these figures, the $L_1$ section is highly doped with the uniform gain coefficient $g$ and $L_2$ has no gain. This case will be compared to two other cases: The first, a DFB fiber laser of length $L_1$ and uniform gain but no un-pumped section, is shown in Fig. 2. The second case, shown in Fig. 3, is the same laser as shown in Fig. 2, but with a reflector at the end of the cavity where the grating would be in the apodized case. The reflectivity value is chosen to be the peak reflectivity of the unpumped fiber grating of the case shown in Fig. 3, namely,

$$R_2 = \tanh^2(kd_2)$$ \hspace{1cm} (19)

This value was chosen to directly compare to the apodized case shown in Fig. 3.

III. NUMERICAL SIMULATION RESULTS

The numerical simulations were performed using self made programs developed on the basis of commercial usual software packages. The coupled-mode equations describing the back-ward and forward field propagation are numerically solved using a DAE algorithm based on a Runge-Kutta-Felhberg procedure. This procedure is further developed by considering the algebraic equations derived from matrix Eq. (4), and introducing the results obtained by numerically solving these equations into the border conditions of coupled-mode differential equations and/or using them for estimation of DFB-FL output parameters.

The gain thresholds for these cases where $L_1 = 2.5$ cm and $L_2 = 0.5$ cm are shown in Fig. 5. The horizontal axis is the normalized frequency $\Delta\beta L$ (where $L=L_1+L_2$), while the vertical axis is the normalized gain threshold $g_{th}L_1$. The gain is normalized with $L_1$ since the value of $gL_1$ relates to the pump power. The mode spectra of the three different lasers are nearly identical since the lasing cavities are of nearly equal length. When compared to the short DFB-FL structure, the gain-apodized DFB-FLs show a nearly 30 % reduction in lasing threshold due to its passive grating section. The DFB with the reflector similarly shows a reduction in the lasing threshold for its first-order mode. However, the threshold reduction applies significantly to all modes since the reflector is spectrally uniform. For the gain-apodized DFB laser, whose passive section has spectral dependence, the additional reflector also aids in modal discrimination with higher-order modes.

It is also important to note that although the passive grating system introduces system asymmetry, the zeroth-order mode cannot reach the lasing threshold since the phase of the transition between the two sections is maintained. Nevertheless, Fig. 6 demonstrates the advantage of using gain apodization for reduced lasing threshold without the penalty of decreased spectral purity.
In Eq. (20), the parameter \( \delta(\omega) \) is defined as:

\[
\delta(\omega) = \left( \frac{\pi}{c} \right) (\omega - \omega_b) = \beta(\omega) - \beta_b
\]

In Eq. (20), \( \delta(\omega) \) represents the dispersion effect relative to Bragg grating characteristic wavelength, implicitly, frequency. \( \kappa \) is the coupling coefficient of the grating, \( L \) is the grating length, and parameter \( q \) is defined as:

\[
q = \pm \sqrt{\delta^2 - \kappa^2}
\]

Fig. 6 Modal frequencies of a gain-apodized DFB fiber laser with \( L_1 = 0.5 \) cm, \( L_2 = 2.5 \) cm, and a reflection spectrum of a 3-cm fiber Bragg grating.

Fig. 7 Modal frequencies of a 0.5-cm uniform-gain DFB fiber laser and a reflection spectrum of a 0.5-cm fiber Bragg grating.

In order to develop the analysis, the active portion of the gain-apodized DFB fiber laser is chosen to be \( L_1 = 0.5 \) cm with the passive portion longer, \( L_2 = 2.5 \) cm or \( L_2 = 4.5 \) cm. The mode spectrum of this laser and the corresponding reflectivity of a 3-cm fiber Bragg grating (FBG) are shown in Fig. 7. No significant differences between 2.5 cm and 4.5 cm cases were observed. For comparison, Fig. 8 shows the mode spectrum of a conventional 0.5-cm-long DFB laser with the reflectivity spectrum of a 0.5-cm FBG. It is clear from these figures that the mode spectrum of the gain-apodized laser is determined by the entire grating rather than only by the active portion.

Fig. 8 The gain thresholds of the lowest-order mode as a function of gain-apodization profile.

Fig. 9 shows the lowest modal gain threshold versus gain length \( L_1 \) for the gain-apodized DFB laser. From this figure, it is clear that the minimum threshold for \( L_1/L \) is close to 0.7; the gain threshold is 17.9 % less compared to the uniform DFB fiber laser (\( L_1/L = 1 \)). For gain lengths \( L_1/L \) less than unity, the longitudinal distribution of light extends into the un-pumped region, creating an effectively higher reflectivity. Since no gain is extracted from this region, the effective grating strength is increased, thus creating a lower gain threshold. For values of \( L_1/L \) that are too small (less than 0.7 in this case), the grating-length product becomes too small to produce sufficient reflection, effectively increasing the laser threshold via reduced feedback. Fig. 10 demonstrates that gain apodization can decrease the laser threshold if properly tailored.

Fig. 10 The lowest-mode gain threshold versus \( L_1/L \).

In Fig. 10, for the same DFB-FL configuration, the variation of difference between the first two modes gain threshold versus \( L_1/L \) is presented.

Since the gain apodization has introduced system asymmetry, the output power ratio from both ends of the laser will also be modified. To investigate these characteristics, the output power ratio of is plotted against the apodized gain length \( L_1/L \) in Fig. 12.
The power ratio from both ends of the fiber changes monotonically with the apodization gain length $L$. Higher output power from the pumped end of the cavity can be obtained at the optimum pumped length $L_p/L$ for the minimum threshold shown in Fig. 12; the power ratio can be increased by 12.4%. This asymmetry, combined with the 21.2% threshold reduction, can lead to a substantial increase in output power solely because of gain apodization.

IV. CONCLUSIONS

It was shown in the previous section that gain apodization can have a beneficial impact on phase-shifted DFB lasers. It has been previously shown that DFB laser performance can be improved by changing the location of the phase shift and varying $\lambda$ along the laser axis [21]-[22]. To obtain the highest single-frequency output from DFB-FL structures, the gain-apodization length, phase-shift location, and coupling coefficient profile must all be optimized. While this presents a challenging numerical problem, genetic algorithms have proven useful in optimizing laser and amplifier designs [23]–[25].

While the lasing threshold itself will determine the gain-apodization profile for a given DFB laser, this effect can be intentionally introduced. Two separate sections of photosensitive fiber, only one of which is doped with active ions to provide gain, can be spliced together before a grating is written into the fiber. In this way, the independent control of the gain profile, grating strength, and phase-shift location can be used to optimize of DFB laser performance.

In conclusion, the effects of gain apodization in highly doped DFB fiber lasers were investigated. In particular, apodization of the longitudinal gain profile resulted in a lower lasing threshold than a laser with uniform gain without the penalty of modal discrimination. For the case studied, the lasing threshold was reduced by almost 18% for a conventional DFB laser and over 21% for a DFB laser with a $\pi$-phase shift. Furthermore, the longitudinal asymmetry introduced by gain apodization yielded a significantly higher ratio of output power from opposite ends of the laser. Methods of engineering and optimizing such a gain-apodized DFB fiber laser were also discussed.

REFERENCES