A simple automatic classifier of PSK and FSK signals using characteristic cyclic spectrum

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Abstract: - This article deals with current issues of automatic modulation recognition. As a suitable method, classification investigating characteristic shape of the signal cyclic spectrum was chosen. First, a theoretical analysis of the issue of calculating of cyclic spectrum estimation is made using Strip Spectral Correlation Algorithm (SSCA). In another part, important findings learned during experiments are presented and conditions for obtaining the characteristic cyclic spectrum are determined. After that, a simple automatic classifier of modulations is designed and optimized by using a series of experiments. The classifier designed was tested by using a series of random modulated signal realizations with AWGN noise.

Key-Words: - automatic modulation recognition, modulation classifier, PSK, FSK, cyclic spectrum, cyclostationarity, Matlab

1 Introduction
Partial, but very complex problem of radio signal analysis is recognition of the modulation type used. Difficulty of a task is based on the variety of possible modulation types used and their parameters, and also on the specifics of radio channels, such as fading or multipath propagation. Contemporary trend is automatization of modulation recognition.

The issue of automatic recognition of modulated signals recorded considerable progress in the last two decades. This is due to both technological development and digitization of range of branches and particularly dynamic growth in the field of pattern recognition.

Originally, the area of modulation recognition belonged primarily to the domain of specialized civilian institutions dealing with management and use of frequency spectrum and selected security forces (police, army, intelligence services). Nowadays, it is also used in commercial communications systems, such as cognitive radio [1].

In this article, we describe a simple classifier of digitally modulated PSK and FSK signals based on examining the cyclic spectrum shape. The second chapter of the article deals with theoretical analysis of cyclostationary signals. In the third chapter, a simple classifier will be designed and a set of experimental results will be proposed. Chapter 4 contains a summary of the results achieved.

2 Estimation of signal cyclic spectrum
A random process \( x(t) \) is said to be \( N \)th order cyclostationary in the strict sense if its \( N \)th order distribution function exhibits periodicity in time with period \( T \) [2]

\[
F(x_1,x_2,\ldots,x_n; t_1,t_2,\ldots,t_n) = F(x_1,x_2,\ldots,x_n; t_1 + mT, t_2 + mT,\ldots,t_n + mT).
\] (1)

In practice it is often sufficient to use only second order statistics, which lead to the definition of second-order cyclostationarity in wide sense. The key second order statistical characteristic is instantaneous autocorrelation function \( R_x(t,\tau) \). So, the process \( x(t) \) is said to be wide sense cyclostationary if its autocorrelation function is periodic in time with period \( T \). The instantaneous autocorrelation function is defined as

\[
R_x(t,\tau) = x(t + \tau/2)x^*(t - \tau/2),
\] (2)
where \( \tau \) is the time lag and \(*\) represents the complex conjugate of the signal \( x(t) \). Because the instantaneous autocorrelation function is periodical
in time (for all \( \tau \)) it can be expanded as Fourier series (the convergence of series is assumption)
\[
R_{xx}(t, \tau) = \sum_{n=-\infty}^{\infty} R_{xx}^\alpha(\tau)e^{2\pi in\alpha t},
\]
where \( \alpha = n/T \) are called cyclic frequencies, and \( R_{xx}^\alpha(\tau) \) are the Fourier coefficients of the instantaneous autocorrelation function that are also referred to as cyclic autocorrelation function
\[
R_{xx}^\alpha(\tau) = \frac{1}{T} \int R_{xx}(t, \tau)e^{-i2\pi n\alpha t} dt.
\]

The cyclic autocorrelation function presents the statistical descriptor of cyclostationary signals in two dimension time-frequency domain. When we reexpress the equation (4) by substitution (2), we obtain
\[
R_{xx}^\alpha(\tau) = \frac{1}{T} \int \left[ \int x(t + \tau/2)e^{i2\pi n\alpha(t+\tau/2)} \right] \left[ \int x(t - \tau/2)e^{-i2\pi n\alpha(t-\tau/2)} \right] dt = E\{u(t)v(t)\}
\]

Now it is possible to bring out two other, but equivalent, definitions of cyclostationary signals [3]. Firstly, the signal \( x(t) \) contains second order periodicity (is cyclostationary signal) if and only if the power of spectral density of the delay-product signal (2) for some delays \( \tau \) contains spectral lines at some nonzero frequencies \( \alpha \neq 0 \), that is if and only if \( R_{xx}^\alpha(\tau) \neq 0 \) is satisfied.

Secondly, the multiplication with \( e^{\pm i2\pi n\alpha t} \) shifts the signal \( x(t) \) in the frequency domain of \( \pm \alpha/2 \). So the cyclic autocorrelation function of a signal \( x(t) \) is cross-correlation function of the frequency shifted versions \( u(t) \) and \( v(t) \) of the same signal \( x(t) \) which leads to third definition. The signal \( x(t) \) exhibits second order cyclostationary if and only if frequency shifted versions of \( x(t) \) are correlated with each other, that is if \( E\{u(t)v(t)\} = R_{xx}^\alpha(\tau) \) is not identically zero as a function of \( \tau \) for some \( \alpha \neq 0 \).

Cyclic autocorrelation function is a time domain descriptor. For many applications it is a more useful and convenient frequency domain descriptor. It is well known that the Wiener-Khinchin theorem that determines the power spectral density by applying Fourier transforms on autocorrelation function. So when we apply the Fourier transform on the cyclic autocorrelation function, we obtain so called spectral correlation density function \( S_{xx}^\alpha(f) \) (SCD), which represents the key descriptor of cyclostationary signals in frequency domain
\[
S_{xx}^\alpha(f) = F\{R_{xx}(\tau)\} = \int_{-\infty}^{\infty} R_{xx}^\alpha(\tau)e^{-i2\pi \alpha f} d\tau.
\]

The form (7) is conventionally used due to relation to practice calculation of SCD
\[
S_{xx}^\alpha(f) = \lim_{\Delta \to 0} \frac{1}{T\Delta} \int \left| \int x(t, f + \alpha/2)X^\alpha(t, f - \alpha/2) dt \right|^2, \quad (7)
\]
where
\[
X^\alpha(t, f) = \int x(u)e^{-i2\pi \alpha f} du, \quad (8)
\]
is a spectral component of \( x(t) \) at frequency \( f \) with bandwidth \( 1/T \).

2.1 SCD Estimation using SSCA Algorithm
A number of algorithms which are able to calculate the signal cyclic spectrum estimation were published [4]. With regard to the computational efficiency the Strip Spectral Correlation Algorithm (SSCA) was chosen. A block diagram of the SSCA calculation is shown in Figure 1.
The SSCA algorithm can be mathematically expressed as formula
\[
S_{f_i+q\Delta f}(m, f_i - q\Delta f) = \frac{1}{N} \sum_{m=0}^{N-1} X_T(m + n, f_i) x^*(m + n) g(n) e^{-j2\pi qn/N},
\]
where \(k\) is a multiple of frequency resolution \(\Delta f\), \(q\) is a multiple of cyclic resolution \(\Delta \alpha\), \(g(n)\) is a rectangular window of length \(\Delta t\), \(x^*(m)\) is a complex conjugate unfiltered input signal and \(X_T(m, f)\) is a complex demodulation of the discrete signal \(x[m]\) described by the relationship
\[
X_T(m, f) = \sum_{k=-N_c/2}^{N_c/2} a(k) x(m - k) e^{-j2\pi fT_i} e^{-j2\pi f_c kT_i}.
\]
where \(a(k)\) is the Hamming window of length \(N_c\) and \(T_i\) is a sampling period. Cyclic resolution depends on the number of samples of signal \(\Delta \alpha = 1/\Delta t = 1/N\) and frequency resolution is determined by the length of examined sections of the signal \(\Delta f = 1/N_c\) [4].

### 3 Problem solving

In the literary sources, it is possible to find how ideal SCD of various signals looks like (including the derived analytical relationships) [2], [3], [5]. Although, it is often stated that SCD is suitable for automatic signal classification, publications focusing on this topic were issued just a few. As an example we mention [6]. Therefore, we decided to test the simple automatic classifier. The digital modulation types that can be classified are: PSK2, PSK4, and FSK2 FSK4. As programming environment, the tool Matlab was used.

In experiments with PSK and FSK modulations, it was found that the shape of the obtained cyclic spectrum strongly depends on the modulation parameters, namely on the ratio of modulation speed \(v_m\) and frequency of carrier wave \(f_c\). Optimal spectrum - easy to use for classification - can be obtained only when the frequency of the carrier wave is much higher than the modulation speed of transfer data. This fact is illustrated in the Figures 2 and 3.

![Normalized SCD](image)

**Fig. 2.:** SCD PSK4 with parameters \(N=1024, N_c=16, f_c=11\ kHz, v_m=500\ Bd\), \(f_i=44\ kHz\)

![Normalized SCD](image)

**Fig. 3.:** SCD PSK4 with parameters \(N=1024, N_c=16, f_c=11\ kHz, v_m=5500\ Bd\), \(f_i=44\ kHz\)

On this basis and with regard to the frequent subsequent processing on standard PC we have selected parameters listed in Table 1 to simulate the chosen modulation.

<table>
<thead>
<tr>
<th>Table 1: Parameters of simulated modulation</th>
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<tbody>
<tr>
<td>Modulation</td>
</tr>
<tr>
<td>PSK2</td>
</tr>
<tr>
<td>PSK4</td>
</tr>
<tr>
<td>FSK2</td>
</tr>
<tr>
<td>FSK4</td>
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On the basis of examination of the shape of SCD representative realization of PSK and FSK modulation the simple classifier, the block diagram of which is shown in Figure 4, has been designed.

For optimal settings of decision levels of classifier, a series of hundreds experiments was carried out. The values were determined experimentally from the measured curves referred to the graphs in Figures 5 and 6.

The digital modulation classifier designed was tested on simulated signals. For each type of modulation the level of noise was set in the range of the SNR = 30 to -5 dB in steps of 1 dB. For each value of noise 200 random signal realizations with AWGN noise with length of 4416 samples were generated. The success rate of the classifier measured is plotted in the graph in Figure 7.

**4 Conclusion**

In this article, the simple automatic classifier of PSK and FSK digital modulation was designed. Based on a series of experiments its optimization was conducted. It is known that the method of classification based on cyclic spectrum is very resistant against AWGN noise. The classifier designed has confirmed this hypothesis, as it has
proved correct recognition of the modulation up to level of noise SNR = 2 dB.

The experiments demonstrated dependence of the spectrum shape on signal modulation parameters. Of great importance is the size of the carrier frequency and modulation rate of data. The results show, that during the classification of signals that are close to the parameters of the signals on which the classifier has been debugged, the success rate of the classification reaches 100%.

Modulation classification method based on cyclostationarity of signals is very promising. Upon further research we will focus on testing of the classifier on real signal samples and on extension of the set of classified signals.

References: