Model Predictive Control over Unreliable Communication Links

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Abstract: - In this paper an issue of data dropout in networked control systems is solved. The model predictive control approach is used where the future control inputs up to a given horizon are evaluated at each sampling instant. In case the current output value is lost and the control input can not be computed, the future control input calculated at previous sampling instant can be used instead. The proposed control strategy can be implemented by switching between two or more control laws. The closed loop stability is analyzed provided the switching occurs arbitrarily often. In order to demonstrate the effectiveness of the proposed strategy, the real-time control of simple laboratory plant is represented.

Key-Words: networked control system, model predictive control, data loss, quadratic stability, LMI

1 Introduction
Networked control systems (NCS) gained increasing attention in recent years due to its cost effective and flexible applications. The use of a data network in a control loop enables remote data transfers and data exchanges among users, reduces the complexity in wiring connections and the costs of medias, provides ease in maintenance and offers modularity and flexibility in control system design. Several network protocols for real-time remote industrial control purposes have been developed during last decades, for example Area Network (CAN) or Profinet. The computer networking technologies especially Ethernet have also progressed rapidly and are also appealing for use in control applications. Recently the wireless technologies, such as Bluetooth or Zigbee, received significant attention and can play an important role in networked control systems [1].

In the networked control system, several components of the system may communicate over the common network that connects them together, as illustrated in Fig. 1. This brings many specific control issues giving rise to important research topics. The networked control system design has to deal with the dynamics introduced by the communication network, which may include communication disruptions such as communication channel noise, data losses, bandwidth limitations, time-varying delays, and data quantization. The variable transmission delays can arise due to various reasons, and are of various characteristics and magnitudes - measurement delays, operator delays, computational delay from control or optimization algorithms and communication delays. All these phenomena in the control loop can lead to performance degradation and eventual instability in control systems. Recent research efforts have led to important results on the design of networked control systems [2 – 4].

Fig. 1 Networked control scheme

In this paper, we focus on the design and analysis of network control systems subject to data losses at the sensor-controller link. In this case feedback is lost and the actuator must operate on its own, usually setting the control input to zero or to the last implemented value. The data packets may be lost due to the network congestion or due to the link failures caused by the unreliable nature of the links, such as in the case of wireless networks. The similar problem arises in the control systems with asynchronous measurement, where the samplings are not received at fixed time instants due to the
difficulties of measuring. Although an intensive research activity is devoted to the design and analysis of networked control systems, only few papers deal with the issue of data losses. The stability and disturbance attenuation issues for a class of networked control systems under bounded uncertain access delay and packet dropout effects were investigated in [5]. The optimal control of linear time-invariant systems over unreliable communication links is studied and sufficient conditions for the existence of stabilizing control laws were derived in [6].

If the controller does not receive new feedback data, the plant is regulated in an open-loop system. The intuitive idea of using the plant model at the controller/actuator side to approximate the plant behavior during time periods when sensor data are not available was used in (Montestruque). In [7] a novel timeout scheme and an autoregressive prediction model for delayed/lost sensor was used. In [8] the predictive control for nonlinear systems with guaranteed stability in the presence of data losses was designed.

In the present paper, we also use the model predictive control (MPC) approach to dealing with data losses in the control system. MPC represents a family of advanced control methods which make explicit use of the model to predict the future plant behavior and to calculate the future control sequence minimizing an objective function [9]. The objective function is formulated as a combination of the set-point tracking performances and control effort. MPC belongs to the category of open-loop optimization techniques and its implementation is based on the receding horizon strategy, i.e. only the first term of the future control sequence is used at each sampling instant and the calculation is repeated in the next sampling time. This allows to incorporate a feedback into the control loop and to improve the control performances in the presence of disturbances and unmodelled dynamics.

First predictive control algorithms have been proposed at the end of the 1970s; they quickly became popular and developed considerably over the last three decades both within the research control community and in industry [10]. The popularity of MPC is mainly due to the fact, that it can be used to control a great variety of processes including time-delayed systems, the nonminimum phase systems or the unstable ones. The multivariable case can easily be dealt with as well. Another important feature of this control design approach is that constraints on the input/output variables can be systematically incorporated into the design procedure, which might improve the resulting control system performances and the process operation safety.

MPC has proved its effectiveness in the networked control systems especially in the context of distributed and hierarchical control. The review of decentralized, distributed and hierarchical control architectures based on MPC is in [11]. The problem of variable time delays in control loop has been addressed in [12] where the time-stamped MPC algorithm that uses a communication delay model along with time-stamping and buffering has been proposed. In our paper the MPC approach is employed to treat the issue of data losses. As the sequence of the future control actions up to a given horizon is calculated at each sampling time, the natural idea is to use not only the first control action, but also the next terms of the control sequence in case the sensor data at next sampling times is not available. From the implementation point of view there are two or more control laws (depending on the number of lost output samples) which are switched arbitrarily fast.

The paper is organized as follows. First the standard model predictive control design procedure is briefly described. Then the control strategy for the case of the data loss in the sensor-controller link is proposed. The effectiveness of the proposed control scheme is evaluated by the real-time control of a simple laboratory plant. Finally, some conclusions are given.

2 Model Predictive Control

2.1 Plant model

The plant model is the cornerstone of the predictive control design. It should be accurate enough to fully capture the plant dynamics and allow predictions to be calculated. We consider the plant model in the form of the ARMAX model

\[
A(z^{-1})y(t) = B(z^{-1})u(t - d - 1) + \frac{C(z^{-1})}{D(z^{-1})} \xi(t)
\]

(1)

with

\[
A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{ma} z^{-ma}
\]

\[
B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_{nb} z^{-nb}
\]

\[
C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_{nc} z^{-nc}
\]

\[
D(z^{-1}) = 1 - z^{-1}
\]

(2)

where \(u(t)\) is the control variable, \(y(t)\) the measured plant output, \(d\) denotes the minimum plant model time-delay in sampling periods, \(\xi(t)\) represents the external random disturbance and \(\xi(t)\) is the stationary random process with zero mean value and finite
variance. For simplicity in the following the \( C(z^{-1}) \) polynomial is chosen to be 1.

2.2 Control design
Generalized predictive control (GPC) developed in [13] belongs to the most popular predictive algorithms based on the parametric plant model. The control objective is to compute the future control sequence in such a way that the future plant output is driven close to the prescribed set point value; this is accomplished by minimizing the following cost function

\[
J = E \left\{ \sum_{j=0}^{n_h} \left( \hat{y}(t+j)/t - w(t+j) \right)^2 + \right. \\
\left. + \rho \left( D(z^{-1})u(t+j-sh) \right)^2 \right\}
\]

subject to

\[
D(z^{-1})u(t+i) = 0 \quad \text{for} \quad ch \leq i \leq ph
\]

where \( sh, ph \) and \( ch \) are positive scalars defining the starting horizon, prediction horizon and control horizon, \( \rho \) is a nonnegative control weighting scalar. \( \hat{y}(t+j/t) \) denotes the j-step ahead prediction of \( y(t) \) based on the data available up to the time \( t \) and \( w(t+j) \) is the future set point value at time \( (t+j) \).

The cost function (3)-(4) may be rewritten in the suitable vector form

\[
J = \left( G_1 U(t+ch-1) + Y_0(t) - W(t+ph) \right)^T \\
\left( G_1 U(t+ch-1) + Y_0(t) - W(t+ph) \right) + \\
+ \rho U(t+ch-1)^T U(t+ch-1)
\]

where

\[
G_1 = \begin{bmatrix}
g_{sh-d-1} & \cdots & g_0 & 0 & 0 & 0 \\
g_{sh-d} & \cdots & g_0 & 0 & 0 & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
g_{ch-1} & \cdots & \cdots & \cdots & g_0 \\
g_{ph-d-1} & \cdots & \cdots & \cdots & \cdots & \cdots & g_{ph-ch-d}
\end{bmatrix}
\]

\[
U(t+ch-1) = \left[ D(z^{-1})u(t), \ldots, D(z^{-1})u(t+ch-1) \right]^T
\]

\[
W(t+ph) = \left[ w(t+d+1), \ldots, w(t+ch-1) \right]^T
\]

\[
Y_0(t) = \left[ y_0(t+d+1/t), \ldots, y_0(t+ph/t) \right]^T
\]

\[
y_0(t+j/t) \text{ denotes j-step ahead prediction of the plant free response calculated as follows}
\]

\[
y_0(t+j/t) = H_{j-d}(z^{-1})D(z^{-1})u(t-1) + F_j(z^{-1})y(t)
\]

The polynomials \( F_j(z^{-1}) \) and \( H_{j-d}(z^{-1}) \) as well as the coefficients of matrix \( G_1 \) can be obtained by solving the following Diophantine equations

\[
l = A(z^{-1})D(z^{-1})E_j(z^{-1}) + z^{-1}F_j(z^{-1})
\]

\[
E_j(z^{-1})B(z^{-1}) = G_{j-d}(z^{-1}) + z^{-j+d}H_{j-d}(z^{-1})
\]

The future control sequence minimizing the cost function (5) is given by

\[
U(t+ch-1) = -K(Y_0(t) - W(t+ph))
\]

where \( K = [G_1^T G_1 + \rho I_{ch}]^{-1} G_1^T \)

In standard GPC implementation only the first term of the calculated future control sequence

\[
D(z^{-1})u(t) = -\sum_{j=sh}^{ph} \gamma_j y_0(t+j/t) - y^*(t+j)
\]

is used at each sampling time and the optimization process is repeated at the next sampling time. However, the further control increments can also be calculated and stored for potential use at next sampling times

\[
D(z^{-1})u(t+i) = -\sum_{j=sh}^{ph} \gamma_{i+j} y_0(t+j/t) - y^*(t+j)
\]

for \( i = 1, \ldots, ch - 1 \)

In (15) and (16) the coefficients \( \gamma_j \) for \( i = 1, \ldots, ch - 1, \ j = sh, \ldots, ph \) are the coefficients of i-th line of matrix \( K \).

The control laws (15)-(16) may also be implemented using the standard pole-placement control structure

\[
S_i(z^{-1})D(z^{-1})u(t+i-1) + R_i(z^{-1})y(t) = T_i(z^{-1})w(t)
\]

for \( i = 1, \ldots, ch - 1 \)

The \( R_i(z^{-1}) \), \( S_i(z^{-1}) \) and \( T_i(z^{-1}) \) polynomials depend on the plant model as well as on the choice of the tuning parameters \( sh, ph, ch, \rho \) and can be calculated as follows

\[
R_i(z^{-1}) = \sum_{j=sh}^{ph} \gamma_{ij} F_j(z^{-1})
\]

\[
S_i(z^{-1}) = 1 + \sum_{j=sh}^{ph} \gamma_{ij} z^{-j-i} H_{j-d}(z^{-1})
\]

\[
T_i(z^{-1}) = \sum_{j=sh}^{ph} \gamma_{ij} z^{-ph+j}
\]

3. Model Predictive Control Subject to Sensor Data Loss
In standard implementation of GPC the control
input is calculated at each sampling instant according to the control law (shown in Fig. 2)
\[ S_i(z^{-1})D(z^{-1})u(t) + R_i(z^{-1})y(t) = T_i(z^{-1})w(t) \] (21)
using the measured value of the plant output.

In case the current output value is not available due to the sensor data loss, the control law (21) cannot be evaluated. In this situation the control input is usually set to zero or to the last implemented value.

The model predictive control approach offers another possibility. As it has been stated above, at each sampling instant the sequence of future control inputs, \( u(t+1), i=1,...,ch-1 \) is calculated which can be stored and employed at next sampling instants. Thus in case of data dropout at time \( t \) the control input calculated at previous sampling instant can be used instead, i.e. the control law takes the following form
\[ S_2(z^{-1})D(z^{-1})u(t) + R_2(z^{-1})y(t-1) = T_2(z^{-1})w(t-1) \] (22)
If the data dropout continues at further sampling instants, another terms of future control input sequence can be used.

3.1 Stability analysis
The above described control strategy can be implemented by switching between the control laws (21) and (22) (or the other ones if needed). Even if each applied control law ensures the closed loop stability, it is necessary to prove the closed loop stability in case of switching between these control laws which can occur arbitrarily often.

To guarantee the robust stability of linear systems the concept of quadratic stability of a polytopic system is frequently used [14].

Consider the linear discrete-time uncertain closed loop system in the form
\[ x(t+1) = \sum_{i=1}^{N} \alpha_i A_{ci} x(t), \quad \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \in (0,1), \quad i=1,2,3,...,N \] (23)

**Lemma 1**
The polytopic system (23) is quadratically stable if and only if there exists a positive definite matrix \( P = P' > 0 \) such that
\[ A_{ci}^T P A_{ci} - P \leq 0 \quad \text{for} \quad i=1,2,3,...,N \] (24)

For \( N=1 \) the quadratic stability means the satisfaction of necessary and sufficient conditions, while for \( N>1 \) it implies only the satisfaction of sufficient conditions.

The Ljapunov equation (24) can be rewritten to the LMI (linear matrix inequality) form
\[ \begin{bmatrix} -P & A_{ci}^T P \\ PA_{ci} & -P \end{bmatrix} < 0 \quad \text{for} \quad i=1,2,3,...,N \] (25)

According to the Lemma 1 the stability analysis of the control system with arbitrary switching of controllers necessitates solving the system of \( N \) linear matrix inequalities (25) where \( N \) is the number of lost output samples and \( A_{ci} \) are the discrete state matrices of the closed loop system with the corresponding control law.

4 Experimental Evaluation
The effectiveness of the proposed control scheme has been evaluated by the real-time control of a cylindrical laboratory tank depicted in Fig. 3.

The plant output is the water height measured by a pressure sensor and the plant control input is the inflow servo valve opening. The tank has also the outflow servo valve which has been used to generate a disturbance. The servo valves are governed by voltage within the range 0 – 10 V. The pressure sensor range is 0 – 10 V, too.

The control signal has been calculated in PC and implemented using the programmable logical controller Simatic S7-200. For the communication between PC and PLC the OPC (OLE for Process
Control) communication standard has been used.

The second order model of the water height dynamics has been identified with the sampling period $T_s = 1$ s. Based on this model the GPC controller has been designed using the following control design parameters

$$sh = 1, \ ph = 30, \ ch = 2, \ \rho = 10$$  \hspace{1cm} (26)

As the control horizon $ch$ has been set to 2, in addition to $u(t)$ also the future value of control input $u(t+1)$ has been calculated at each sampling instant, which would be used at the next sampling instant only in case of the output data dropout. The closed loop stability has been verified using the sufficient condition for robust stability in the form of linear matrix inequalities (25).

Figures 4 to 6 show the real-time control results of two experiments. In the first experiment the model predictive control with no data dropouts has been performed. In the second one, 1.5% data dropouts have been artificially generated and the proposed control strategy has been implemented. The measured water height together with its reference value is shown in Fig. 4. Fig. 5 shows the time plots of control input (inflow valve opening). The disturbance signal (outflow valve opening) and the data dropouts are in Fig. 6.

As it can be seen from Fig. 4, the control performances obtained in both experiments are comparables, i.e. the issue of data dropout has been successfully solved using the proposed control strategy.

5 Conclusion

The paper has dealt with the data loss issue in networked control systems. The proposed control strategy is based on the model predictive control design where the future values of control inputs are calculated and used at next sampling instants in case of data losses at the sensor-controller link. To analyze the closed loop stability with the proposed control strategy, the concept of quadratic stability has been used.

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