Computation of Stability Regions for PID Controllers

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Abstract: - This contribution is focused on computation of stability regions for Proportional-Integral-Derivative (PID) controllers. The area of possible placement of the controller parameters which guarantee feedback stabilization of a controlled plant is obtained via plotting the stability boundary locus. This approach is subsequently combined with the desired model method which is used for final controller design. The applicability of the technique is demonstrated on an example where a third order electronic laboratory model is successfully controlled.

Key-Words: - Linear Control, Stabilization, Stability Regions, PID Controllers, Desired Model Method, Electronic Model

1 Introduction

Over 95% of contemporary practical industrial applications use PID (or PI as a special case) control algorithms [1], [2] and thus the appropriate PI(D) control design is still very topical especially for systems under some nonlinearities, perturbations or time-variant behaviour. Without any doubts, the absolutely primary and essential requirement is the stability of closed control loop.

There is an array of techniques to computation of stabilizing PI(D) controllers in the literature such as calculations presented in [3], stability boundary approach from [4], [5] or Kronecker summation method published in [6]. However, all those tools solve “only” the problem of finding the area of all possible stabilizing variations of PI(D) controller parameters. For the control design itself, potentially with additional safety or performance specifications, another method has to be utilized. For the purpose of this paper, the desired model method, formerly known as inversion dynamics method, has been applied [7].

This paper presents the PID controller design using the combination of preliminary stability regions computation via stability boundary locus approach and consequent suitable parameters tuning with the assistance of the desired model method. The efficiency of the studied technique has been verified through a simulation example where the third order electronic laboratory model has been successfully stabilized and controlled.

The work is organized as follows. In Section 2, the basic ideas and rules for computation of stabilizing regions for PID controllers are described. The Section 3 than presents the stabilization of the electronic laboratory model. Further, the specific controller design along with its simulative verification is provided in Section 4. And finally, Section 5 offers some conclusion remarks.

2 Computation of Stabilizing Regions

The primary and essential step is to determine the parameters of the PID controller which guarantee stabilization of the feedback control loop containing the plant:

$$G(s) = \frac{B(s)}{A(s)} \tag{1}$$

and ideal PID controller:

$$C(s) = k_P + \frac{k_I}{s} + k_D s = \frac{k_P s + k_I + k_D s^2}{s} \tag{2}$$

One of the possible approaches has been published in [4], [5]. It is based on plotting the stability boundary locus. First, the substitution $s = j\omega$ in the transfer function (1) and subsequent decomposition of the numerator and denominator into their even and odd parts lead to:

$$G(j\omega) = \frac{B_E(-\omega^2) + j\omega B_O(-\omega^2)}{A_E(-\omega^2) + j\omega A_O(-\omega^2)} \tag{3}$$

Then, the expression of closed-loop characteristic polynomial and equaling the real and imaginary parts to zero result in the relations for proportional and integral gains:

$$k_P(\omega,k_D)=\frac{P_5(\omega)P_4(\omega)-P_6(\omega)P_2(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)}$$
$$k_I(\omega,k_D)=\frac{P_6(\omega)P_1(\omega)-P_5(\omega)P_3(\omega)}{P_1(\omega)P_4(\omega)-P_2(\omega)P_3(\omega)} \tag{4}$$

where

$$\begin{aligned}
P_1(\omega)&=-\omega^2 B_O(-\omega^2)\\
P_2(\omega)&=B_E(-\omega^2)\\
P_3(\omega)&=\omega B_E(-\omega^2)\\
P_4(\omega)&=\omega B_O(-\omega^2)\\
P_5(\omega)&=\omega^2 A_O(-\omega^2)+\omega^2 B_E(-\omega^2)k_D\\
P_6(\omega)&=-\omega A_E(-\omega^2)+\omega^3 B_O(-\omega^2)k_D
\end{aligned} \tag{5}$$

Simultaneous solution of equations (4) and plotting the obtained values into the (k_P,k_I,k_D) space define the stability boundary locus.

As can be noticed, the last two terms and thus also the parameters k_P and k_I depend on derivative constant k_D, which is practically considered to be chosen and fixed for one set of calculations. In other words, k_D is preset and corresponding set of boundary parameters k_P, k_I is computed. The obtained curve splits the (k_P,k_I) plane into the stable and unstable regions. The selection of the stabilizing/unstabilizing areas can be performed through a test point within each region. The final stability region(s) are consequently plotted via the "(k_P,k_I) slices" into the (k_P,k_I,k_D) space.

The computation process is going to be illustrated in the following example.

3 Stabilization of an Electronic Model

The laboratory plant in the form of electronic model has been considered as a controlled system. Its transfer function adopted from [8], [9] can be written as:

$$G(s)=\frac{2.925}{175.5s^3+137.5s^2+22s+1} \tag{6}$$

It means that the even and odd parts in (3) are:

$$\begin{aligned}
B_E(-\omega^2)&=2.925\\
B_O(-\omega^2)&=0\\
A_E(-\omega^2)&=137.5(-\omega^2)+1\\
A_O(-\omega^2)&=175.5(-\omega^2)+22
\end{aligned} \tag{7}$$

In the first instance, the derivative gain k_D was fixed to 1 and then the relations (5) and (4) were computed for a range of nonnegative frequencies. The corresponding (k_P,k_I) pairs are plotted in fig. 1. The stabilizing area lies inside the depicted shape as can be easily verified using an arbitrary (k_P,k_I) from this region and testing the closed-loop characteristic polynomial stability.

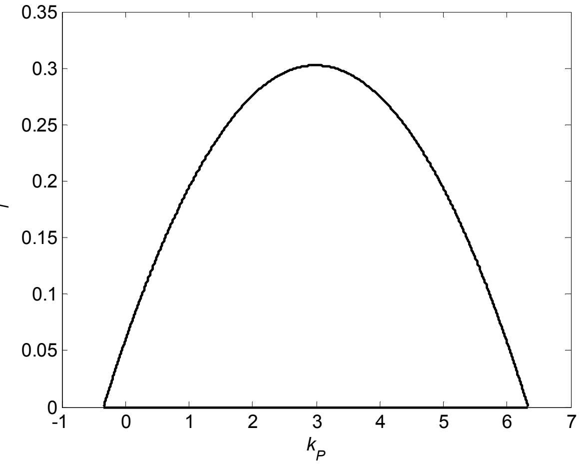

Fig. 1: Stability region for $k_D=1$

Afterward, the stability regions were computed and visualized for 11 equally spaced k_D from 0 to 10. The result is shown in fig. 2.

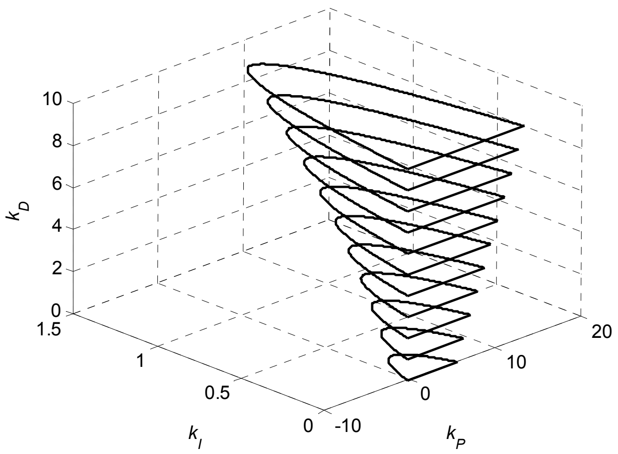

Fig. 2: Stability regions for $k_D \in \langle 0,10\rangle$

Thus, all the variations of PID controller parameters which are located inside the shape defined by stability regions from fig. 2 ensure the feedback stabilization of the plant (6).

4 Control Experiment

Now, the natural question follows. How to choose the controller with desired performance from the pre-calculated stabilizing pool? In fact, the paper does not attempt to bring any novel control design method, but utilizes an existing one and combines it with the previous stabilizing approach. From the number of available techniques, the desired model method (formerly known as inversion dynamics method) [7] was applied.

First of all, the appropriate mathematical model of the controlled plant is requested. For that reason, the third order transfer function (6) can be simply approximated by the second order one as:

$$G_N(s) = \frac{2.925}{137.5s^2 + 22s + 1} \approx \approx \frac{2.925}{175.5s^3 + 137.5s^2 + 22s + 1} \tag{8}$$

which corresponds with the one of desired transfer function forms:

$$G_N(s) = \frac{K}{T^2s^2 + 2\xi Ts + 1} \tag{9}$$

where:

$$\begin{aligned} K &= 2.925\,[-] \\ T &= 11.726\,[\mathrm{sec}] \\ \xi &= 0.9381\,[-] \end{aligned} \tag{10}$$

The controller tuning is handled through the choice of closed control loop time constant T_w. Here, it was adjusted to:

$$T_w = 20\,[\mathrm{sec}] \tag{11}$$

The parameters of the controller:

$$C(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{12}$$

can be calculated according to:

$$\begin{aligned} T_I &= 2\xi T = 22 \\ K_P &= \frac{T_I}{KT_w} = 0.3761 \\ T_D &= \frac{T}{2\xi} = 6.25 \end{aligned} \tag{13}$$

Thus, the final parameters of the controller in the form (2) are:

$$\begin{aligned} k_P &= K_P = 0.3761 \\ k_I &= \frac{K_P}{T_I} = 0.0171 \\ k_D &= K_P T_D = 2.3504 \end{aligned} \tag{14}$$

Thanks to the fact that this variation of parameters lies inside the stability region from fig. 2, the obtained PID controller stabilizes the original plant (6). The actual control behaviour can be found in fig. 3.

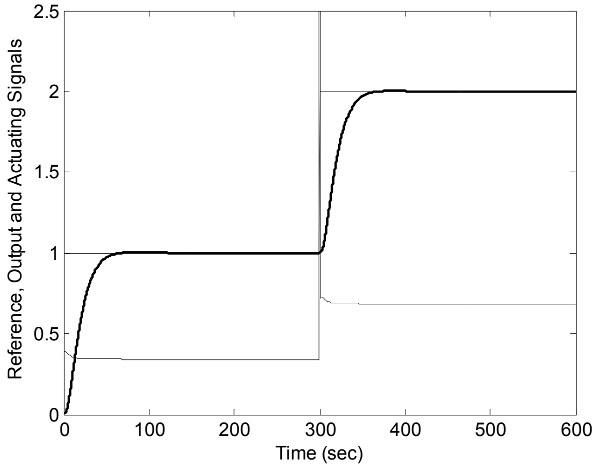

Fig. 3: Control of plant (6) using PID controller (14)

Such control result should be appropriate for most of common industrial applications.

5 Conclusions

The paper has dealt with computation of stability regions for ideal PID controllers. The combination of pre-computing the stabilizing areas and designing the PID controller which lies inside it represents relatively easy but effective way of obtaining the stabilizing PID controller with acceptable performance. The future extension of this research should be focused on robust stabilization under parameter perturbations in controlled plant.

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