Implementation of a Transmission Line Model with the PEEC Method for Lightning Surge Analysis

PEERAWUT YUTTHAGOWITH
Department of Electrical Engineering, Faculty of Engineering
King Mongkut’s Institute of Technology Ladkrabang
1, Chalongkrung Road, Ladkrabang, Bangkok
THAILAND
kypeeraw@kmitl.ac.th

Abstract: - This paper proposes a simple transmission line model adopted with the partial element equivalent circuit (PEEC) method for lightning surge analysis. The proposed approach is employed for calculating tower surge responses and voltages across insulators of an actual transmission tower. The proposed approach increases the efficiency of the method in terms of computation time because of reducing a number of elements in the PEEC method. Comparisons of the simulation results by the proposed approach in the time domain, and by the PEEC method without a transmission line model and available experimental data shows satisfactory agreement not only for amplitudes but also for waveshapes.

Key-Words: - Insulator voltage, Lightning surge analysis, Partial element equivalent circuit, Tower surge response, Transmission line model.

1 Introduction

For an economical insulation design of the transmission and distribution systems, predictive calculations of overvoltages due to lightning in the transmission and distribution systems is most essential. Traditionally, circuit-based models and transmission line approaches assuming transverse electromagnetic (TEM) propagation mode were employed for the calculation of overvoltages due to a lightning strike to a transmission system [1]-[4]. The assumption of the TEM propagation mode might not be correct for a tall tower in EHV and UHV transmission systems, because of the nonuniform behavior of the tower, and reflections and scattering from the discontinuities and tower ends [5]-[8]. This behavior cannot be expressed by a time-independent model unless the rise time of the current is much longer than the round-trip time of the traveling wave. To obtain more accurate results, full-wave approaches such as the method of moment (MoM) [5]-[8], the finite-difference time-domain (FDTD) technique [9]-[11], or a hybrid electromagnetic-circuit method such as the PEEC method [12]-[15] are more appropriate than a circuit and transmission line approach.

The PEEC method is derived from a mixed potential integral equation (MPIE) and provides a full wave solution to Maxwell’s equations. The method can be applied to both time and frequency domains. A major difference from the MoM is that the problem under consideration is transformed into the circuit domain where circuit analysis techniques can be applied. A detailed description of the approach can be found in [15], [16]. The calculated results by the full-wave PEEC method have shown good agreement in comparison with the results calculated by the MoM and the FDTD methods, as well as with experimental data [12]-[16]. Moreover, in some specific cases, the computational efficiency of the method was found to be better than that of the MoM and the FDTD method, because post processing for calculating voltages and currents along the studied system is not required.

For the formulation of the full-wave PEEC method in the frequency domain, time responses are obtained by using the inverse Fourier's transform. As well known, a frequency domain analysis for transient studies might be time consuming because of the requirement of inversion of impedance matrix at every frequency step. A full-wave time-domain PEEC formulation can therefore be interesting, especially when dealing with nonlinearities. Voltages and currents along the structure should be memorized at every time step. To consider the presence of conducting ground, the image method can be employed but the solution matrix size is increased due to retardation effects. Therefore, the computational efficiency of the full-wave time-domain PEEC method is not always higher than that of a frequency domain PEEC implementation.

Neglecting retardation effects of electromagnetic waves will increase the computational time efficiency of the PEEC method in the time domain because the past time of voltages and currents are memorized only at one previous time step. The inversion of impedance matrix is required only once and also the model of reduction (MOR) [17], [18] can be employed to reduce the equation size. However, in a typical configuration
A vector of potentials on the horizontal wires. Therefore, a combination of the transmission line theory and the PEEC method would be most of elements (overhead lines) can be represented as horizontal wires. Therefore, a combination of the transmission line theory and the PEEC method would be an effective way to increase the efficiency of the method in terms of computation time and memory requirement. In this paper, the PEEC method neglecting the retardation effect, and quasi-static approximation) in the time domain combined with the transmission line theory is proposed for lightning surge analysis.

### 2 The PEEC Method Combined with a Transmission Line Model

Procedures for obtaining solution of the PEEC method start from discretizing geometry structure into small cells or elements which are composed of current cells and charge or potential cells. The current cells and potential cells are interleaved each other. The rectangular pulse is employed both charge and current basis functions. Then, Galerkin’s method is applied to enforce the mixed potential integral equation which is interpreted as Kirchhoff's voltage law applied to a current cell, and the continuity equation or the charge conservation equation is applied via Kirchhoff's current law to a potential cell. Whole system equations in the frequency domain can be written in a matrix form corresponding to a modified nodal analysis (MNA) formulation as shown in (1).

\[
\begin{bmatrix}
  j\omega P^{-1} + Y & A^T \\
  -A & R + j\omega L
\end{bmatrix}
\begin{bmatrix}
  \Phi \\
  I
\end{bmatrix}
= \begin{bmatrix}
  I_S \\
  U_S
\end{bmatrix}
\tag{1}
\]

where \( A \) is an incidence matrix which expresses the cell connectivity, \( R \) is a matrix of series resistances of current cells, \( L \) is a matrix of partial inductances of current cells including the retardation effect, \( P \) is a matrix of partial potential coefficients of potential cells including the retardation effect, \( \Phi \) is a vector of potentials on potential cells, \( I \) is a vector of currents along current cells, \( U_S \) is a vector of voltage sources, \( I_S \) is a vector of external current sources, and \( Y \) is an additional admittance matrix of linear and non-linear elements.

The equivalent circuit is extracted from three-dimensional geometries of a considered structure. An appropriate solver is employed to obtain solution either in the time domain or in the frequency domain. Fig. 1 shows the procedures of the PEEC simulation. The detail of derivation and formulation of a PEEC for a thin wire structure is found in Reference [15].

![Fig. 1. Procedures in the simulation of PEEC models.](image)

### 2.1 Formulation of the quasi-static PEEC method in the Time domain

The formulation of the quasi-static PEEC method in the time domain can be derived by using (1) and is given in equation (2);

\[
\begin{bmatrix}
  P^{-1} \frac{d}{dt} + Y & A^T \\
  -A & R + L \frac{d}{dt}
\end{bmatrix}
\begin{bmatrix}
  \Phi \\
  I
\end{bmatrix}
= \begin{bmatrix}
  I_S \\
  U_S
\end{bmatrix}
\tag{2}
\]

where \( L \) is a matrix of partial inductances of current cells neglecting the retardation effect, and \( P \) is a matrix of partial potential coefficients of potential cells neglecting the retardation effect.

The matrices, \( P \) and \( L \), express electromagnetic coupling among the cells, and are dense. The solution of equation (2) based on quasi-static assumption can be obtained by employing an appropriate integration scheme. For simplicity, Backward Euler scheme is applied to equation (2) by discretizing in time. The following equation is obtained.

\[
\begin{bmatrix}
  \frac{1}{\Delta t} P^{-1} + Y & A^T \\
  -A & R + \frac{1}{\Delta t} L
\end{bmatrix}
\begin{bmatrix}
  \Phi^n \\
  I^n
\end{bmatrix}
= \begin{bmatrix}
  U_S^n - L \frac{1}{\Delta t} I^{n-1} \\
  I_S^n + P^{-1} \frac{1}{\Delta t} \Phi^{n-1}
\end{bmatrix}
\tag{3}
\]

### 2.2 Transmission Line Model

Formulation of a multi-transmission line in the frequency domain as multi-port network as illustrated in Fig. 2 is given as follow;
To measure the insulator voltages, two current pulses characterized by different rise times were applied. The first pulse (Fig. 3b) has about 0.2 μs rise time (fast rise time current) and the second one (Fig. 3c) has a time to crest of 3 μs. The peak amplitude of the two pulses is 3.4 A. To measure voltages across insulator strings a 10-kΩ resistive voltage dividers were employed.

The full-wave PEEC simulation in the frequency domain involves 256 frequencies up to 5 MHz and 5-m element length. The tower is composed of four main poles of which elements have 0.2 m radius; slant and horizontal elements have 0.1 m radius. The cross arms are composed of 0.2-m radius elements. The same conditions are employed in the quasi-static PEEC simulation in the time domain with a 16 ns time step. The overhead conductors are modeled using a lossless 8-

Recent Researches in Energy & Environment


125
conductor transmission line (6 phase wires and 2 ground wires). The towers and parts of the transmission lines close to the tower are modeled by the PEEC method for taking into account the retardation of electromagnetic fields around the tower by self and mutual impedances of the elements. The total number of elements without the use of the transmission line model is 2488 while the total number of elements when using the transmission line model is reduced to 604.

In the simulation, the tower-footing resistance is represented by a 17-\( \Omega \) resistance by connecting four 68 \( \Omega \) resistors at the bottom of four main poles of the tower.

Table 1 shows computation times relative to the time corresponding to the full-wave PEEC method in the frequency domain. From the results in Table 1, it can be seen that the computation efficiency of the PEEC

![Fig. 4. Comparison between simulated and measured waveforms, fast rise time current injected.](image)

![Fig. 5. Comparison between simulated and measured waveforms, slow rise time current injected.](image)
method in the time domain is much better than that of the PEEC method in the frequency domain. Furthermore, a combination with the transmission line theory for the representation of overhead conductors is very effective to increase the computational efficiency of the method without losing accuracy.

Table 1

Comparison of Computation Time Relative to the Computation Time of the Full-wave PEEC Method

<table>
<thead>
<tr>
<th>PEEC in the time domain</th>
<th>Formulation of an impedance matrix and inversion (p.u.)</th>
<th>Calculation of result by multiplication (p.u.)</th>
<th>Total (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>without adopting transmission line model</td>
<td>5.56 x 10⁻³</td>
<td>5.00 x 10⁻⁴</td>
<td>6.06 x 10⁻³</td>
</tr>
<tr>
<td>with adopting transmission line model</td>
<td>3.33 x 10⁻⁴</td>
<td>6.66 x 10⁻⁵</td>
<td>4.00 x 10⁻⁴</td>
</tr>
</tbody>
</table>

4 Conclusion

The analysis and the results presented in this paper have shown that for applications involving long horizontal structures such as overhead power lines, a combination of the PEEC method with the transmission line theory is computationally very efficient while keeping the accuracy of a full PEEC approach.

References:


