Diffusion of Biostimulator in Curing Cultural Plants

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Abstract: In this paper we have presented mathematical model based on the diffusion mechanism which enable us to estimate time necessary for the penetration of biostimulator into the plant body. In reality, the biostimulator surface layer must be in the liquid phase to realize the above diffusion process.

Key-Words: - mathematic modeling, biostimulator, diffusion process.

1 Introduction

Leather

Besides its main products, food processing industry generates significant number of waste which is a valuable raw material in agriculture.

Nowadays, a very perspective raw stock is collagen proteinous waste generated during production of sausage casings, so called collagen paste, used for the production of biostimulators. The collagen paste is a very valuable raw stock because it consists of practically pure collagen with minimum ash content (less than 0.5% on the free moisture base).

Proteinous biostimulators enhance the immune system of cultivated crops and thus reduce the need of chemical plant protection agents. A favourable side effect of proteinous biostimulators is their action as leaf fertilizers. The main requirements for biostimulators are their good solubility in water and fast penetration of the active component into the plant fine inner structure. Both requirements are fulfilled via collagen paste hydrolysis, which can be carried out in both acid and alkaline conditions with the use of proteolytic enzymes in the latter case. The collagen paste is obtained by alkaline processing of beef calcimine (limed collagen splits) with lime wash. After deliming, pure collagen paste is extracted into an "infinite" cylinder. The batch that does not meet the preliminary mechanical tests is our raw stock – waste collagen paste for biostimulator production.

To show a biostimulator`s positive effect, it is necessary that an optimal amount of the biostimulator penetrates into the treated plant inner volume in the form of diluted aqueous spray. Absorption of the optimal amount by the plant body, however, requires some time, which depends on the biostimulator`s concentration gradient on the surface and the value of its effective diffusion coefficient. During this time, the surface film of the biostimulator must be in the liquid state, because if there is quick water evaporation, the transport of the biostimulator ceases due to practically zero value of the effective diffusion coefficient in solid state. Further, if rain occurs, the surface biostimulator film is washed away and its positive effect will be completely eliminated. The speed of evaporation depends on climatic and hydrodynamic conditions such as temperature, relative air humidity and wind speed. To elaborate an estimation of recommended conditions for spraying in field conditions, we have worked out a mathematical model which includes simultaneous diffusion of biostimulator into the plant inner volume and mass transport into the surroundings (air).

2 Theory

Mathematical model

Let us assume a plant surface covered on both sides with a continuous and even layer of liquid containing a dissolved biostimulator.

We limit ourselves to a one-dimension model where the spatial variable will represent the thickness of the plant part. Further, let us assume isotropic properties of the treated plant in relation to the inner mass transport, which enables us to apply axially symmetric boundary condition for our proposed model. Due to the surface water evaporation, the concentration of the biostimulator will rise and consequently, the value of the effective diffusion coefficient will decrease. This decrease however will be eliminated by the increasing concentration gradient on the treated plant surface and therefore we will consider the value of the effective diffusion coefficient constant during the existence of the liquid phase on the plant surface.

$$\frac{\partial c}{\partial \tau}(x,\tau) = D\frac{\partial^2 c}{\partial x^2}(x,\tau) \qquad 0 < x < b \qquad 0 < \tau < \tau_k \tag{1}$$

$$\frac{\partial c}{\partial x}(0,\tau) = 0 \tag{2}$$

$$c(b,\tau) = \varepsilon c_0 \tag{3}$$

$$c(x,0) = 0 \tag{4}$$

$$c_0(0) = c_{0p} \qquad (\qquad 5)$$

$$V_0(\tau)\frac{\partial c_0}{\partial \tau}(\tau) = -SD\frac{\partial c}{\partial x}(b,\tau) \tag{6}$$

$$V_0(\tau) = V_{0p} - \frac{k\, c_n(1-\varphi)\tau S}{\rho} \tag{7}$$

$$c_n = \frac{p_n M}{RT} \tag{8}$$

$$V_{0p}c_{0p} = V_0(\tau)\, c_0(\tau) + \tau DS\frac{\partial c}{\partial x}(b,\tau) \tag{9}$$

Equation (1) describes a non-stationary concentration field of the biostimulator in the treated plant body. The time-limited validity of $0 < \tau < \tau_k$ is given by the critical time when all surface water is evaporated and the biostimulator concentration equals its density. In this case, as said before, the value of the effective diffusion coefficient is zero. Equation (2) is an axially symmetric problem which represents isotropic properties in relation to the biostimulator inner diffusion. Equation (3) expresses the assumption of a perfect transport of the biostimulator into the plant inner volume, ε stands for the plant porosity. Equations (3) and (4) are the initial conditions. Equation (6) represents mass balance, i.e. the speed of the biostimulator accumulation in the plant body is equal to the diffusion flow on the surface. $V_0(\tau)$ is the time dependence of the surface film volume – this dependence is described by equation (7), where φ is the relative air humidity and ρ is the density of water. c_n is the saturation concentration of water vapour which depends on the partial saturation pressure; this is given by equation (8) where p_n is the partial pressure of water vapour at the point of saturation, M stands for the molar mass of water, R is the universal gas constant and T is the absolute air temperature which is equivalent the saturation temperature. The time dependence of the biostimulator concentration in the surface layer is given by the balance equation (9).

The mathematical model represented by equations (1) – (9) is non-linear due to the initial conditions (6) – (9). In the case when air is saturated with water vapours, the second term of the right side of equation (7) equals zero and then the surface film volume is constant. The model becomes linear and the solution for this case is as follows:

$$C(X,Fo)\ \frac{Na}{\varepsilon+Na} + 2Na\cdot\sum_{n=1}^{\infty}\frac{\cos(X\cdot g_n)\cdot e^{-Fo\cdot g_n^2}}{\varepsilon\cdot\cos(g_n) - \frac{\varepsilon\cdot\sin(g_n)}{g_n} - g_n\cdot Na\cdot\sin(g_n)} \tag{10}$$

where X, Fo, Na, ε are dimensional parameters and q_n are roots of equation:

$$tg(g_n) = -\frac{Na\cdot g_n}{\varepsilon} \qquad (g_n > 0) \tag{11}$$

Equation (10) describes a non-stationary concentration field in the treated plant body. From the practical point of view is important the integral mean concentration of the biostimulator in the treated plant inner volume. The integral mean concentration and its dependence on the dimensionless time is given by the following equation (12) with parameter Na, which stands for the ratio of the surface liquid layer volume to the volume of the plant.

$$\bar{C}(Fo) = \frac{Na}{\varepsilon+Na} - 2Na^2\cdot\sum_{n=1}^{\infty}\frac{e^{-Fo\cdot g_n^2}}{\varepsilon^2+Na\cdot\varepsilon+Na^2\cdot g_n^2} \tag{12}$$

Graphical expression of equation (17) is shown in Figure 1.

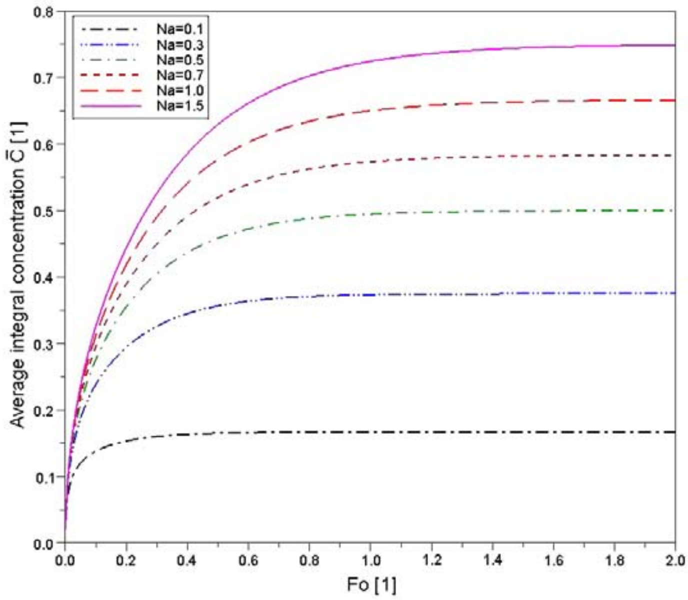

Figure 1. Average integral concentration of biostimulator in the plant body

The graphical interpretation (Figure 1) enable us to estimate critical time which is necessary to ensure that desired amount of biostimulator have penetrated into the plant body.

3 Conclusions

We have presented mathematical model based on the diffusion mechanism which enable us to estimate time necessary for the penetration of biostimulator into the plant body. In reality, the biostimulator surface layer must be in the liquid phase to realize the above diffusion process. In the reality, these conditions are fulfilled when the relative humidity of air is close to saturated state of air – for example, this corresponds to biostimulator curing which is proceeds in the greenhouse or immediately when the rain is stopped in the field conditions. In other cases, it is inevitable to repeat biostimulat application to obtain the desired results.

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List of symbols

c	concentration of biostimulator in the plant body	[kg/m^3]
x	x coordinate	[m]
τ	time	[s]
D	effective diffusion coefficient of biostimulator in the plant body	[m^2/s]
V_0	Volume of liquid biostimulator surface layer	[m^3]
S	Total surface of the plant body	[m^2]
k	mass-transfer coefficient	[m/s]
ρ	density of water	[kg/m^3]
p_n	saturated water vapour pressure	[Pa]
M	molecular weight of water	[kg/kmol]
R	universal gas constant	[J/(kmol.K)]
T	temperature	[K]
b	half thickeness of plant body	[m]
Na	ratio of the surface liquid layer volume to the volume of the plant	[1]
X	$X=x/b$, dimensionless coordinate	[1]
C	$C=c/c_p$, dimensionless concentration of biostimulator in the plant body	[1]
c_p	initial surface concentration of biostimulator	[kg/m^3]
Fo	$Fo=D\,\tau/b^2$, dimensionless time (Fourier diffusion number)	[1]
c_0	concentration of biostimulator in the surface liquid layer	[kg/m^3]
c_{0p}	initial concentration of biostimulator in the surface liquid layer	[kg/m^3]
ε	plant body porosity	[1]