

Lattices and Patterns

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Abstract: This work is aimed at finding interesting patterns in knowledge discovery applications. We first apply formal concept analysis methods for deriving concepts from particular data sets. We also investigate concepts stabilities by removing attributes from the same data set. Another way to search for interesting patterns is to build nested lattices and compare the obtained concepts with the ones resulting from applying the first two approaches.

Key Words: Clustering, formal concept analysis, patterns, knowledge discovery

1 Introduction

A pattern is a concise and semantically rich representation of raw data. Pattern management is widely discussed in [1] and [8]. An example of a pattern could be a cluster that represents a set of transactions with the common bought items or an association rule which states that whenever we buy cheese we tend to buy crackers too, [7].

Conceptual clustering is a machine learning paradigm for unsupervised classification developed mainly during the 1980s, [2]. Sometimes conceptual clustering is referred to as 'learning by observation'.

The stability index of a concept indicates how much the concept intent depends on particular objects of the extent, [6]. Concept stability is applied in this work in an attempt to find a reasonably small set of objects and attributes that represent the most interesting tendencies in a particular study.

This work is aimed at finding interesting patterns in knowledge discovery applications. We first apply formal concept analysis methods for deriving concepts from particular data sets. We also investigate concepts stabilities by removing attributes from the same data set. Another way to search for interesting patterns is to build nested lattices and compare the obtained concepts with the ones resulting from applying the first two approaches.

Postgraduate students are enrolled in a research methods course. They are regularly invited to take web based tests in order to receive feedback on their current knowledge during the course. We are investigating changes in concepts after some restrictions on particular attributes or merging some attributes. Joint efforts of the research community may develop an approach for knowledge evaluation that will overcome the problem of over testing.

2 Concepts

Let \( P \) be a non-empty ordered set. If \( \sup\{x,y\} \) and \( \inf\{x,y\} \) exist for all \( x, y \in P \), then \( P \) is called a lattice [3]. In a lattice illustrating partial ordering of knowledge values, the logical conjunction is identified with the meet operation and the logical disjunction with the join operation.

A context is a triple \((G, M, I)\) where \( G \) and \( M \) are sets and \( I \subseteq G \times M \). The elements of \( G \) and \( M \) are called objects and attributes respectively [3], [11].

For \( A \subseteq G \) and \( B \subseteq M \), define

\[
A' = \{ m \in M \mid (\forall g \in A) \ g Im \},
\]

\[
B' = \{ g \in G \mid (\forall m \in B) \ g Im \}
\]

where \( A' \) is the set of attributes common to all the objects in \( A \) and \( B' \) is the set of objects possessing the attributes in \( B \).

A concept of the context \((G, M, I)\) is defined to be a pair \((A, B)\) where \( A \subseteq G \), \( B \subseteq M \), \( A' = B \) and \( B' = A \). The extent of the concept \((A, B)\) is \( A \) while its intent is \( B \). A subset \( A \) of \( G \) is the extent of some concept if and only if \( A'' = A \) in which case the unique concept of the which \( A \) is an extent is \((A, A')\).

The corresponding statement applies to those subsets \( B \in M \) which is the intent of some concepts.

The set of all concepts of the context \((G, M, I)\) is denoted by \( \mathcal{B}(G, M, I) \). \( (\mathcal{B}(G, M, I); \leq) \) is a complete lattice and it is known as the concept lattice of the context \((G, M, I)\).
Incremental techniques are discussed in [9] and [10]. The presented framework is adapted to the maintenance of concept lattices upon the insertion of a set of objects into the context. We apply the 'Bottom-up Search' which is a dual of the Incremental techniques.

A concept lattice summarizing the outcome of tests related to Chapter 1 and Chapter 2 is presented in Fig. 1.

The outcome of tests related to Chapter 1, Chapter 2 and a summary test is presented in Fig. 2. The summary tests confirmed the tendency indicated by tests related to Chapter 1 and Chapter 2.

The outcome of tests related to Chapter 3, Chapter 4 and Chapter 5 is presented in Fig. 3.

The outcome of tests related to a summary test Chapter 3, Chapter 4 and Chapter 5 is presented in Fig. 4. This confirmed the tendency indicated by the one in Fig. 3.

Outcome of tests related to Chapter 1, Chapter 2, a summary test, Chapter 3, Chapter 4 and Chapter 5 is presented in Fig. 5.

Intensity depends on object counting.

### Table 1: Part 1

<table>
<thead>
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<th>Ch 2</th>
<th>Summary test</th>
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### 3 Formal Concepts Stabilities

Stability of a formal concept is discussed in [5] and [6].

**Definition 1** Let \((A, B)\) a formal concept of \(\mathfrak{B}(G, M, I)\). Stability of \((A, B)\) is

\[
\gamma(A, B) = \frac{|\{C \subseteq A | C' = A' = B\}|}{2|A|}
\]

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Figure 4: Outcome of tests related to a summary test, Chapter 3, Chapter 4 and Chapter 5

<table>
<thead>
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<th>Ch 4</th>
<th>Ch 5</th>
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Table 2: Part 2

The stability index of a concept indicates how much the concept intent depends on particular objects of the extent. Given a concept \((A, B)\), the stability index measures the number of elements of \(G\) that are in the same equivalence class of \(A\), where an equivalence class is defined as follows.

**Definition 2** Let \(X \subseteq G\), we denote by \(X\) the equivalence class of \(X\) where:

\[
\langle X \rangle = \{ Y \subseteq G | Y' = X' \}
\]

Note that when \(X\) is closed, any \(Y\) in \(X\) is a subset of \(X\). Thus, considering a formal concept \((A, B)\), definition 3 can be rewritten as:

\[
\gamma(A, B) = \frac{|\langle A \rangle|}{2|A|}
\]

Then, the larger the equivalence class of an extent is (with respect to extent size), the more stable the concept is. The idea behind stability is that a stable concept is likely to have a real world interpretation even if the description of some its objects (i.e. elements in the extent) is "noisy".

Figure 5: Outcome of tests related to Chapter 1, Chapter 2, a midterm test, Chapter 3, Chapter 4 and Chapter 5

The lattice in Fig. 6 corresponds to the lattice in Fig. 1. The amount of concepts is reduced by 50%. This can considerably improve the decision process since the amount of cases to work on is decreased by 50%. The lattice in Fig. 7 corresponds to the lattice in Fig. 2. The lattice in Fig. 8 corresponds to the lattice in Fig. 3. The lattice in Fig. 9 corresponds to the lattice in Fig. 4. The lattice in Fig. 10 corresponds to the...
lattice in Fig. 5.

Figure 7: Reduced context related to Chapter 1, Chapter 2 and a summary test

Figure 8: Reduced context related to Chapter 3, Chapter 4 and Chapter 5

4 Nested lattices

A nested lattice is the product of two concept lattices, graphically represented by a nested line diagram, sometimes referred to as inner and outer lattice.

A nested line diagram consists of an outer diagram that contains inner diagrams in each node. Inner diagrams are not necessarily congruent but only substructures of congruent diagrams. Congruent diagrams are shown as structures possessing some unrealized concepts. Nested line diagrams, similar to the tree structure, make the focus concepts the roots and associate each sequence of concepts below the focus with a path like the trunk of a tree, [4]. The formal concept analysis nested line diagrams are based on combining multiple partial views of the data represented in the context, [4].

The overall effect of building nested line diagrams is mapping concepts into a clear identifiable picture of reality. That means having several complete lattices of a partial context nested into one another rather than a partial lattice of a context, [4].

The nested lattices in Fig. 11, Fig. 12, and Fig. 13 illustrate dependencies between different tests results.
Figure 11: Nested lattice relating outcomes from Chapter 1 and Chapter 2

Figure 12: Nested lattice relating outcomes from Chapter 1 and Chapter 5

5 Conclusion

Considering the relatively small sample of initial data we would say that early feedback presented to students has positive effect on their final grades. Appropriate removal of objects can preserve the most important concepts and present a clearer picture of current tendencies.

Future work is needed to determine the amount of objects and attributes who’s removal does not significantly change important concepts.

References:


