Kinematic analysis of three bar mechanism linked with rotation joints

IOANA PETRE¹, TUDOR DEACONESCU¹, ANDREA DEACONESCU¹, DAN PETRE²
¹Department of Economic Engineering and Production Systems
₂Department of Materials Engineering and Welding
“Transilvania” University of Brasov-Romania
Str. Mihai Viteazu nr. 5, Brașov
ROMANIA
babes_ioana@yahoo.com

Abstract: - The aim of this article is to present the kinematic analysis of a constructive variant of rehabilitation equipment designed for helping the patients with affections of the ankle articulations. For studying its kinematics is has been considered as three flexible bar mechanism linked with rotation joints. The variation limits of the rotation angles of the hip joint and the ankle joint respectively are calculated, and also the evolution of these angles is represented for the entire duration of the stroke.

Key-Words: Isokinetic equipment, Kinematic analysis, Three bar mechanism, Rotation joints

1 Introduction

This paper presents the research of an isokinetic equipment construction variant, for the rehabilitation of the patients with ankle articulations affections. The ill articulation is moved forward and backward in order to obtain the recovery.

It is a kinematic study, in which, the equipment is considered as a three bar equipment linked with three rotation joints and one translation joint.

The results of the paper is the variation limits of the rotation angles the hip joint and the ankle joint, the evolution of these angles and the sliding block travel.

A future direction in this research is to conceive other constructive variants, choose one of all and optimize it.

2. Proposed Isokinetic Rehabilitation Equipment

Isokinetic equipments are instruments used in rehabilitation medicine. Those equipments scope is to allow recovering of the affected functions of the patients by doing equal motion or maintaining the same speed throughout the entire range of motion.

On the market, there are many types of isokinetic equipments, for rehabilitation the shoulder, the arm, or different articulations of the leg (hip, shoulder or ankle articulations). The prices of such equipments are high, exceeding the possibilities of potential users. So, because of the high incidence of those patients with leg affections, and because of the high price of them, it is necessarily to conceive performing rehabilitation equipments at prices below the ones existing on the market [1].

The rehabilitation equipment proposed (fig. 1) uses continuous passive motion, for recovering the patients with affections of the ankle joint of the leg.

Continuous passive motion means that the patient does not make effort, his leg is mechanical driven in order to move the ill articulation and so, to rehabilitate it.

The equipment allows recovery exercises of the ankle joint, the lower limb being immobilized in the device.

Fig. 1 Proposed rehabilitation equipment

The function principle is presented in the figure below (fig.2).

Fig. 2 Working principle
The characteristics of the equipment are: symmetrical, simple and robust construction, has low weight, due to the use of lightweight materials, aluminum, which resists well to use by patients. The efficiency of the utilization of this kind of equipment results from the following: rapid recovery of patients; safety in use; high resistance to daily use; easy of use by computer control; ergonomic; looks good;

3. Kinematic analysis of three bar mechanism linked with rotation joints

Flexible bar mechanism to be analyzed (fig.1) is composed by three bars of different dimensions which realize the movement of the ankle and the hip, in order to rehabilitate it. For this mechanism it will be determined:
- variation limits of the sliding block travel;
- limits of the rotation angle of the hip joint $\phi_1$;
- limits of the rotation angle of the ankle joint $\phi_2$;
- the evolution of these angles on the sliding block travel.

Fig. 1 Kinematic diagram of the flexible bar mechanism

To calculate the above variables, dates of construction are the following: length of bars: $OA = 1000$ mm, $AB = 168$ mm, limits of variation of the angle of rotation of the ankle: $+20$ and $-40$ from the vertical axis (Fig. 2). Maximum possible range of motion of the ankle made for a healthy foot is 60 °, within the limits presented above [2].

Fig. 2 Rotation limits of the ankle joint

Starting from the scheme shown in Figure 1, variation limits of the sliding block travel can be determined. The maximum reached limit is:

$$OB' = OC' - CB'$$

where

$$OC' = \sqrt{OA^2 - AC^2}$$

and

$$CB = AB \cdot \sin 20^\circ = 57.459 \text{ mm}$$

It results:

The minimum limit reached by the sliding block will be determined by relation:

$$OB' = 599,684 \text{ mm}$$

$$OC' = 833,696 \text{ mm}$$

In conclusion, for obtaining the desired rotation angles of the ankle joint, the sliding block will make a translation motion of length:

For the extreme limits of the sliding block travel (points B and B’), $\phi_1$ angle will have values of:
- for point B: $\phi_1 = 9.08^\circ$
- for point B’: $\phi_1 = 7.39^\circ$.

For an intermediate position of the sliding block travel when the bar of length $l_2$ is perpendicular to $x$ axis, we get the maximum value of $\phi_1$ angle:

$$\phi_1 = 9.67^\circ$$

Figure 3 shows the dependence of $\phi_2$ angle against the sliding block travel (B joint).

Fig. 3 $\phi_1 = f(\text{sliding block displacement})$

It can be observed that, in complete contracted state of the sliding block ($l_3 = 883,696$ mm), $\phi_1$ angle has the
value of 7, 39°, it increases until 9,67° in intermediate position, and after that it decreases until 9,08° in complete relaxed state, when l₃ = 1044,919 mm.

To calculate the values of φ₂ angle, from Fig. 1 scheme results:

\[ \phi_2 = \arccos \left( \frac{\sqrt{A^2 + B^2 - L^2}}{2 \cdot AB} \right) \]  \hspace{1cm} (8)

For the extreme limits of the sliding block travel (points B and B’), the φ₂ angle will have the values:
- for point B: \( \phi_2 = 100,916° \)
- for point B’: \( \phi_2 = 42,605° \).

Figure 4 shows the dependence of φ₂ angle against the sliding block travel (B joint).

In maximum contracted state of the sliding block, l₃ = 883,696 mm, φ₂ angle has value of 42,605°, while in maximum relaxed state of it (l₃ = 1044,919 mm), its value increases to 100,916°.

The angle between the foot and the tibia is defined as:

\[ \phi_{2\text{complementar}} = 180° - \phi_2 \]  \hspace{1cm} (9)

The value of the angle φ₂ complementar (fig.5) is of:

\[ \phi_{2\text{complementar}} = 137,395° ... 79,084° \]

Figure 6 shows a sequence of positions which the components of the rehabilitation mechanism make in time, with the visualization of the hip and ankle joints angles variations and also the length in contracted and relaxed state of the sliding block.

The graphs below (Fig. 7, Fig. 8 and Fig.9) show the time related evolution of the angles φ₁, φ₂ and ankle joint angle, considering the stroke length of 161,223 mm being achieved in 20 seconds.

Figure 7 presents the variation of the φ₁ angle against time. φ₁ angle evolution in 20 seconds is situated between 9,08° in complete relaxed state and the value of 7, 39° in contracted state, with a maximum of 9,67° in intermediate position.

In figure 8 it is presented the evolution of φ₂ angle against time.

In 20 seconds, φ₂ angle varies between 79,08° in relaxed state to 137,39° in contracted state of the sliding block.

Figure 9 shows the amplitude of the ankle joint angle, which is by 60°, and the evolution of this angle against the time.
4. Conclusion
The kinematic analysis of isokinetic equipment conceived for rehabilitation of the patients with affections of the ankle articulations had as result the extreme limits of the sliding block travel, the variation limits of the rotation angles of the hip joint and the ankle joint and also the evolution of the angles for the entire duration of the sliding block stroke. This research will be continued with a more in depth analysis of the mechanism and different other constructive variants.

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The stroke realized by the sliding block is by 161,223 mm, between 883,696 mm and 1044,919 mm. Time related evolution of the sliding block travel is presented in figure 10.