A NEW APPROACH TO FUZZY-CONTROL LARGE SCALE SYSTEMS

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Abstract- This paper develops a new approach to the control of interconnected system using fuzzy system theory. The approach is based on incorporating a group of local estimators on the system level to generate the input-output database. An array of feedback fuzzy controllers is then designed to ensure the asymptotic stability of the closed loop system. The developed technique is applied to unstable large-scale system and extensive simulation studies are carried out to illustrate the potential of the new approach.

Keywords—Large scale system Control, Fuzzy Control.

1 Introduction
In control engineering research, problems of decentralized control and stabilization of interconnected systems are receiving considerable interest in recent years [1,2] where most of the effort is focused on dealing with the interaction patterns. It is concluded that a systematic approach to deal with the problems of interconnected systems is twofold: first is to base the analysis and design effort on the subsystem level using conventional control methods and second is to deal with interactions effectively. These methods are facilitated, in general, by virtue of several mathematical tools including linearization, delay approximation, decomposition and model reduction. This constitutes the so-called model-based control system approach for which we have seen numerous techniques [3]. Most of the available results have so far overlooked the operational knowledge of the interconnected system under consideration. In [4], a knowledge-based control system approach has been suggested to deal with the analysis and design problems of interconnected systems by incorporating both the simplest available model as well as the best available knowledge about the system. For single physical systems, one of the earlier efforts along this direction has been on the development of an expert learning system [5-6]. An alternative approach has been on integrating elements of discrete event systems with differential equations [7]. A practically-supported third approach has been through the use of fuzzy logic control by successfully applying fuzzy sets and systems theory [9].

For interconnected systems, the foregoing approach motivates the research into intelligent control by combining techniques of control and systems theory with those from artificial intelligence. The main focus should be on integrating a knowledge base, an approximate (humanlike) reasoning and/or a learning process within a hierarchical structure.

Fuzzy logic controllers [10, 11, 12] are generally considered applicable to plants that are mathematically poorly understood (there is no acceptable mathematical model for the plant) and where experienced human operators are available for satisfactorily controlling the plant and providing qualitative “rules of thumb” (qualitative control rules in terms of vague and fuzzy sentences).

1) Hierarchical ordering of fuzzy rules is used to reduce the size of the inference engine.
2) Real-time implementation, or on-line simulation, of fuzzy controllers can help reduce the burden of large-sized rule sets by fusing sensory data before imputing the system’s output to the inference engine.

A concerted effort has been made to formally reduce the size of the fuzzy rule base to make fuzzy control attractive to interconnected systems. Two of the difficulties with the design of any fuzzy control system are:

- The shape of the membership functions.
- The choice of fuzzy rules.

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The properties that a fuzzy membership function is used to characterize are usually fuzzy. Therefore, we may use different membership functions to characterize the same description.

Conceptually, there are two approaches to determine a membership function. The first approach is to use the knowledge of human experts. Usually this approach can only give a rough formula of the membership function; fine-tuning is required. In the second approach, data are collected from various sensors to determine the membership functions. Specifically, the structures of the membership functions are specified first, then fine-tuning of the membership function parameters should be implemented based on the collected data [8].

In this paper, we contribute to the further development of intelligent control techniques of interconnected systems. It provides a new approach to fuzzy control design for interconnected systems. The approach consists of two stages: In the first stage, a group of local state estimator is constructed to generate the data base of input-output pairs. In the second stage, an array of feedback fuzzy controllers is designed and implemented to ensure the asymptotic stability of the interconnected system. Simulation studies on a large-scale system with unstable eigenvalues are carried out to illustrate the features and capability of the new approach.

2 State Estimation of Interconnected Systems

In the sequel, the terms large-scale and interconnected are used interchangeably. The term large-scale system (LSS) does not have a unique established meaning, but it covers systems that possess several particular features, such as multiple subsystems, multiple control objectives, decentralized and/or hierarchical information structures. Any LSS includes many variables but their control is faced by a well-known fact [3] that the states are not always available for measurement and state must be estimated.

Many authors have considered the state estimation of large-scale systems in input decentralized fashion. Here we summarize one convenient algorithm [2]. Let the state model of the $i^{th}$ subsystem described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{i \neq j}^N G_{ij} x_j \quad (1)$$

$$y_i(t) = C_i x_i(t), \quad i,j = 1, 2, \ldots N \quad (2)$$

Where all vectors and matrices are appropriately defined and $g(.)$ is the interaction function between the $i^{th}$ subsystem and the rest of the system. It is considered that $(C_i, A_i)$ is completely observable for $i = 1, 2, \ldots N$.

The following algorithm finds the optimal states of a large-scale system based on decentralized estimation and control [4]:

**Algorithm 1**

**Step 1**
Read the matrices $A_i, B_i$ and select $Q_i \geq 0$ and $R_i > 0$ as weighted matrices.

**Step 2:**
Solve the following $2N$ algebraic Riccati equations for $H_i, K_i$

$$H_i(A_i^T + \alpha I_i) + (A_i + \alpha I_i)H_i - H_iD_iH_i + Q_i = 0 \quad (3)$$

$$K_i(A_i^T + \alpha I_i) + (A_i + \alpha I_i)K_i - K_iS_iK_i + Q_i = 0 \quad (4)$$

Where $D_i = C_i^T C_i$, $S_i = B_i R_i^{-1} B_i^T$

**Step 3:** Integrate the following set of $N$ simultaneous equation for $e_i(t)$, $i = 1, 2, \ldots, N$, using the initial condition $e_i(0) = x_i(0)$

$$\begin{align*}
\dot{e}_1 &= (A_1 - H_1D_1) e_1 + B_1v_1 \\
\ddots
\end{align*}$$

$$\begin{align*}
\dot{e}_N &= (A_N - H_ND_N) e_N + B_Nv_N
\end{align*}$$

$$e_1 = G_{1N} e_N$$

$$e_2 = G_{2N} e_N$$

$$\vdots$$

$$e_N = G_{NN} e_N$$

$$\begin{align*}
\dot{e}_i &= A_i - H_iD_i \quad (5)
\end{align*}$$
Step 4:
Integrate the following set of $n$ simultaneous equations for $x_i(t)$, $i = 1, 2, \ldots, N$

\[
\begin{align*}
\dot{x}_i &= A_i - S_i K_i + G_i N \quad S_i K_i \quad 0 \\
\dot{x}_N &= G_N N - A_N - S_N N \quad 0 \quad S_N K_N \\
\end{align*}
\]

Step 5:
Generate the input-output pairs $\{v_i, \hat{y}_i = c_i \hat{x}_i\}$.

3 Interconnected system:
Assume the following interconnected system of order 10 [4]:

\[
A = \begin{bmatrix}
-1.5 & -0.3 & -0.25 & 0.1 & 0.5 & r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\
0.1 & 0 & 0 & -0.2 & 0 & r_{21} & r_{22} & r_{23} & r_{24} & r_{25} \\
0 & 0.2 & -1 & 0 & 0.4 & r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\
0.6 & -0.1 & -0.25 & -2 & 0 & r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \\
0.4 & 0.2 & 1 & 0.5 & 0.1 & r_{51} & r_{52} & r_{53} & r_{54} & r_{55} \\
r_{212} & r_{213} & r_{214} & r_{215} & -1.5 & -0.3 & -0.25 & 0.1 & 0.5 & \ldots \\
r_{221} & r_{222} & r_{223} & r_{224} & r_{225} & 0.1 & 0 & 0 & -0.2 & 0 \\
r_{231} & r_{232} & r_{233} & r_{234} & r_{235} & 0 & 0.2 & -1 & 0 & 0.4 \\
r_{241} & r_{242} & r_{243} & r_{244} & r_{245} & 0.6 & -0.1 & -0.25 & -2 & 0 \\
r_{251} & r_{252} & r_{253} & r_{254} & r_{255} & 0.4 & 0.2 & 1 & 0.5 & 0.1 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
\end{bmatrix}
\]
which is considered to be composed of two-coupled subsystems; each of order 5. The coupling parameters are $r_{1jk}$ and $r_{2jk}$ where $j$ and $k$ take values of 1,2,3,4 and 5. In the sequel, we refer to the structure of the interconnected system model as:

\[
\begin{aligned}
\dot{x} &= \begin{bmatrix}
A_{11} & G_{12}(r_1) \\
G_{21}(r_2) & A_{22}
\end{bmatrix} x + \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} v \\
\end{aligned}
\]  

(10)

Where $G_{12}(r_1)$ and $G_{21}(r_2)$ are the coupling matrices.

For a typical values [4] of $r_{15}=-0.1$, $r_{14}=0.1$, $r_{12}=0.2$, $r_{23}=0.1$, $r_{24}=0.15$, $r_{25}=0.11$ and all other values of coupling parameters are zeros, we examined the stability of the system by computing the eigenvalues of matrix $A$. They are {-1.0915, -1.0641, 0.477 + j0.0206, 0.477 – j0.0206, 0.022 + j0.0544, 0.022 – j0.0544, -1.8709 + j0.1713, -1.8709 – j0.1713, -1.9306 + j0.1413, -1.9306 – j0.1413}, and it is quite clear that there are four eigenvalues lie in the open right half of the complex plane and thus the interconnected system is unstable. Further, it is easy to check that the interconnected system is both controllable and observable.

### 3.1 Estimation of the System State Variables and Outputs:

A Matlab program is written to implement the computational algorithm (1) of section 2 on the interconnected system. Different positive and negative step input are applied to estimate the outputs. The results of two cases are illustrated in Fig. 1 and Fig. 2. It is observed that the outputs tend to track conveniently the input signals.

#### 3.2 Design of an Array of Fuzzy Controller

We are going to treat the interconnected system at hand as being composed of two identical and coupled subsystems. The control system to be designed is such that each subsystem has its own fuzzy negative feedback controller which its input being the output of the respective subsystem (Fig.3). Each subsystem fuzzy controller is constructed using two fuzzy systems.
In order to build each fuzzy controller, the following steps are implemented:

**Step 1:** The range of the inputs to each fuzzy controller \([\alpha_i, \beta_i]\) are driven from the estimated value of the respective subsystem outputs, where \(i = 1, 2, 3, 4\).

**Step 2:** \(2N_i+1\) fuzzy set \(M_L^{i}\) in \([\alpha_i, \beta_i]\) that are normal, consistent and complete with triangular membership functions [11], are defined for each controller, where \(L = 1, 2, \ldots, 2N_i+1\). We use \(N_i\) fuzzy set \(M_1^{i}, \ldots, M_{N_i}^{i}\) to cover the negative internal \([\alpha_i, 0]\), the other \(N_i\) fuzzy sets \(M_{N_i+2}^{i}, \ldots, M_{2N_i+1}^{i}\) to cover the positive internal \((0, \beta_i]\), and the center of fuzzy set \(M_{N_i+1}^{i}\) at zero.

**Step 3:** The following \(2N_i+1\) rules are considered

\[
\text{IF } y_{ai} \text{ is } M_L^{i} \text{ or } y_{bi} \text{ is } M_L^{i} \text{ then } u \text{ is } K_L^{i}
\]

Where \(L = 1, 2, \ldots, 2N_i+1\), and \(a_i, b_i\) are the input to the fuzzy controller \(i\), and the center \(y_{ai}^{L}\) and \(y_{bi}^{L}\) of the fuzzy set \(K_L^{i}\) are chosen such that

\[
\begin{align*}
y_{ai}^{L} \text{ and } y_{bi}^{L} & \leq -0 \quad \text{for } l = 1, \ldots, N_i \\
& = 0 \quad \text{for } l = N_i+1 \\
& \geq 0 \quad \text{for } l = N_i+2, \ldots, 2N_i+1
\end{align*}
\]

(11)

**Step 4:** Product inference engine, singleton fuzzyfier, and center average defuzzifier are selected to design the fuzzy controller.

#### 3.3 Simulation Results

**Case 1:**

Fig. 4: Outputs \(y_1\) against \(y_2\)

Fig. 5: Outputs \(y_3\) against \(y_4\)
3.4 Performance of the proposed fuzzy feedback controller array

Now, we examine the effect of coupling matrices on the performance of fuzzy controlled interconnected system. Five additional cases with different coupling ranks are implemented. Fine tuning of membership functions was required to adjust their ranges. The following table summarizes the test cases:

<table>
<thead>
<tr>
<th>Case No.</th>
<th>A11,A22 Norm</th>
<th>G12 Sparsty</th>
<th>G12 Norm</th>
<th>G21 Sparsty</th>
<th>G21 Norm</th>
<th>System Stability without controller</th>
<th>System Stability with controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Fig. 4,5)</td>
<td>2.2529</td>
<td>3/25</td>
<td>0.2</td>
<td>3/25</td>
<td>1.8028</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
<tr>
<td>2 (Fig. 6,7)</td>
<td>2.2529</td>
<td>12/25</td>
<td>0.4712</td>
<td>3/25</td>
<td>0.1803</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
<tr>
<td>3 (Fig. 8, 9)</td>
<td>2.2529</td>
<td>3/25</td>
<td>.2</td>
<td>12/25</td>
<td>0.5341</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
<tr>
<td>4(Fig.10, 11)</td>
<td>2.2529</td>
<td>1</td>
<td>3.0361</td>
<td>3/25</td>
<td>0.1803</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
<tr>
<td>5 (Fig.12,13)</td>
<td>2.2529</td>
<td>3/25</td>
<td>.2</td>
<td>1</td>
<td>3.0364</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
<tr>
<td>6 (Fig. 14,15)</td>
<td>2.2529</td>
<td>1</td>
<td>3.0361</td>
<td>1</td>
<td>3.0417</td>
<td>Unstable</td>
<td>Stable</td>
</tr>
</tbody>
</table>

Table 1

The following figures illustrate the above test cases:

Fig. 6: Case 2 Outputs y1 against y2  
Fig. 7: Case 2 Outputs y3 against y4  

Fig. 8: Case 3 Outputs y1 against y2  
Fig. 9: Case 3 Outputs y3 against y4  

Fig.10 Case 4 Outputs y1 against y2  
Fig. 11Case 4 Outputs y3 against y4  

...
4 Conclusions:
This Paper has developed a new fuzzy control design approach to interconnected system. It has been shown the approach consists of two stages: In stage 1, a group of local state estimator has been constructed to generate the input-output database. Then an array of feedback controllers has been designed and implemented to guarantee the overall asymptotical system stability. Extensive simulation studies have been performed to support the developed design approach.

References

Mohamed MAS Mahmoud Member of IEEE in 1999 and Senior Member (SM) in 2001. Received the B.S. degree in Electrical Engineering from Cairo University and the M.Sc. degree from Kuwait University. He is currently PH.D student in Transilvania University of Brasov, Romania. He occupies a position of Senior Engineer at Al Hosn Gas Co. His current research interests include power delivery, reliability, and protection & control using Fuzzy and Artificial Neural Network Techniques.

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