Abstract: This paper presents an experimental study on the machining of a cobalt-based refractory material. The machining of such material has been very little studied from the milling standpoint because of its abradability. It is usually machined by grinding. Our aim is to define the optimal cutting conditions for a given tool, a given part, and a given machine. For this, we use a new optimization technique, based on response surface method, using "Kriging interpolation" and two optimization algorithms: the first one is the Broyden-Fletcher-Goldfarb-Shanno algorithm (BFGS) for an unconstrained optimization algorithm, the second one is the sequential quadratic programming algorithm (SQP) for a constrained problem. With these optimizations techniques, a correlation between machining cost, tool cost, surface roughness and cutting parameters has been found for a refractory material, for which cutting conditions are only little studied, with a low experimental and calculation cost.

Key-Words: Cobalt-based refractory material, milling, tool life, cutting conditions, SQP algorithm, Kriging interpolation

1 Introduction
Time and cost constraints make it necessary to reduce the manufacturing costs of each element of a product. Some of these elements being obtained by machining, it will be then necessary to optimize the machining parameters of these elements. For each tool used on each material, it is necessary to be able to define the ideal cutting conditions according to various criteria: cost, surface quality, etc. In this context, we tried to define the ideal cutting conditions in the FSX 414, a cobalt-based refractory material, with a given tool.

This material is frequently used in aeronautics. It is frequently machined by grinding because of its hardness and its abradibility. Its milling has been studied very little: tools have a reduced lifespan and are hardly ever applied in this material. But sometimes, there is no alternative process, and then, the number of worn edges is such that any progress in the tool lifespan has quickly significant effects on the manufacturing costs.

The effects of the various cutting conditions can be studied using Taguchi method [1], to identify the key factors to optimize. For example, Zhang et al. [2] used the “Taguchi method” to identify the significant parameters affecting surface roughness in milling operation; the parameters considered were depth of cut, spindle speed and feed rate. Fuh and Wu. [3] also studied the effect of tool geometry and cutting conditions on the machined surface quality. These authors showed that the feed rate, cutting speed, flank width and tool nose radius have significant effects on the residual stresses. They also have a great effect on the surface roughness. Ghani et al. [4] have used the experimental optimization method in order to optimize the cutting speed, feed rate and depth of cut in hardened steel end milling process, under semi-finishing and finishing conditions with coated carbide tools.

Due to high complexity of the machining optimization problem, Zarei et al. [5], optimize using harmony search algorithm the depth of cut, feed rate and cutting speed for finishing turning operations. For this purpose, the total production cost is used as an objective function. The seam objective function is used by Saravanan et al. [6] and their optimization method was based on a simulated annealing and genetic algorithm. The machining parameters were also depth of cut, cutting speed and feed. Other algorithms are used to optimize the machining performance. Hagiwara et al. [7] use meta-heuristics algorithm to obtain the optimal depth of cut and feed rate. The objective function was based on two measures as weighting factors: the surface roughness (Ra), and chip breakability (CB). However the use of weighting factors can to lead at a local optimum. Oktem et al. [8] minimized an analytical model for surface roughness using genetic algorithm (GA). A factorial design of experiments was used for this purpose. The design variables were the cutting parameters: feed, axial depth of cut, radial depth of cut, cutting speed and machining tolerance.

The majority of the optimization procedures used require a great number of objective evaluations and
constraints functions, increasing by this fact the experimental cost and time test. The optimization algorithm must be carefully chosen. The optimization problem described here is non-linear. To resolve this problem we use an optimization strategy based on the Response Surface Method (RSM) [9] by means of the Kriging interpolation [10]. In the first case, the minimization algorithm is a BFGS method, because only the bounds of the cutting conditions are imposed. In the second case, in which non-linear constraints are imposed, we use the Sequential Quadratic Programming algorithm (SQP) [11], which is applicable to highly nonlinear programming problems and is efficient in reaching global optimum. Two constraints are imposed to each optimization variable. The lower bound is -1 and the upper bound is +1

2. Experiments

2.1. Experimental Procedure

2.1.1. Outputs – parameters optimized
The goal of this experiment is to determine cutting conditions allowing a minimal machining cost, for a given tool, slotting in FSX 414 parts. The machining cost is strongly linked to the tool lifespan, this is why we will focus on this parameter. The machined part will also have to meet some specifications. One of these specifications is the surface quality. This parameter will also be studied.

2.1.2. Inputs
The machine and the machining process are defined by an industrial problem, as well as the tool material and its geometry. Therefore, we studied the influence of the cutting conditions on the tool lifespan and the surface roughness, under dry end milling conditions.
- cutting speed Vc:
  - minimum: machining duration : 15,56 m/min
  - maximum: manufacturers' recommendation and preliminary trials : 40 m/min
- depth of cut ap:
  - minimum: manufacturer’s recommendations: 0.6 mm
  - maximum: preliminary tests and industrial problem to solve: 2.44 mm
- feed per teeth fz:
  - minimum: manufacturers’ recommendations and tool edge acuity: 0.0375 mm/tooth;
  - maximum: preliminary tests and industrial problem to solve: 0.2 mm/tooth

2.1.3. Design of experiment
The number of parameters studied and the number of required levels led us to a composite design.
This design is the combination of a complete factorial design with two variation levels (-1 and +1). To evaluate the repeatability of the phenomena and the error measurement, 6 points from the levels - α and +α, and 1 central point were added. In order to have the almost orthogonal property, α must be equal to 1,414. The tests are described in the table 2.

2.2. Experimental setup

2.2.1. Material Studied
The properties of the FSX 414 are summarized in table 1. Our sample was a parallelepiped cast part measuring 350mm x 85mm x 80mm.

Table 1. Properties of the FSX414

<table>
<thead>
<tr>
<th>Composition (%)</th>
<th>Co</th>
<th>Cr</th>
<th>Ni</th>
<th>W</th>
<th>C</th>
<th>B</th>
<th>Si</th>
<th>Mn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>8.3 g/cm³</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Melting point</td>
<td>1330-1400 °C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanical</td>
<td>440 MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardness</td>
<td>52 HRc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2.2. Tool
The tool was a SANDVIK Coromant R390-A16 016-11L mill, two teeth with standard R390-11 T3 04M-PM carbide plates.

2.2.3. Machine and machining strategy
The machine was a 4-axis (X, Y, Z, A) numerically controlled milling center, model GAMBIN 50C, with a 6.5 kW spindle.

The tests were done by slotting the cast part in its lower width direction. The surface of the sample was milled at the beginning of each test in order to create a homogeneous surface.

2.2.4. Surface roughness measurement
The surface quality was measured using a SURTRONIC 25 (Taylor-Hobson) surface quality measuring device after each slotting. The length of cut was 0.8 mm, the evaluation length was 4 mm, with a Gaussian filter. The measurement was made in the middle of the slot, in parallel of the mill displacement.
2.2.5. Tool life measurement
The flank wear was measured after each slotting (Fig.1). The tool was considered as worn out when the flank wear VB reached 0.3mm.

2.3. Results
In Table 2 are presented the results for each test of the design experiment

<table>
<thead>
<tr>
<th>Trial</th>
<th>( V_c ) [m/min]</th>
<th>( f_z ) [mm/tooth]</th>
<th>( a_p ) [mm]</th>
<th>( R_a ) [µm]</th>
<th>( T ) [min]</th>
<th>( T_c ) [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>17.86</td>
<td>0.0375</td>
<td>0.75</td>
<td>0.68</td>
<td>14.72</td>
<td>205.56</td>
</tr>
<tr>
<td>T2</td>
<td>35.00</td>
<td>0.0375</td>
<td>0.75</td>
<td>0.59</td>
<td>6.8</td>
<td>149.29</td>
</tr>
<tr>
<td>T3</td>
<td>17.86</td>
<td>0.1500</td>
<td>0.75</td>
<td>1.17</td>
<td>2.41</td>
<td>163.35</td>
</tr>
<tr>
<td>T4</td>
<td>35.00</td>
<td>0.1500</td>
<td>0.75</td>
<td>0.91</td>
<td>1.42</td>
<td>128.73</td>
</tr>
<tr>
<td>T5</td>
<td>17.86</td>
<td>0.0375</td>
<td>2.00</td>
<td>0.76</td>
<td>14.01</td>
<td>69.4</td>
</tr>
<tr>
<td>T6</td>
<td>35.00</td>
<td>0.0375</td>
<td>2.00</td>
<td>0.51</td>
<td>6.13</td>
<td>51.13</td>
</tr>
<tr>
<td>T7</td>
<td>17.86</td>
<td>0.1500</td>
<td>2.00</td>
<td>1.09</td>
<td>1.95</td>
<td>65.48</td>
</tr>
<tr>
<td>T8</td>
<td>35.00</td>
<td>0.1500</td>
<td>2.00</td>
<td>0.96</td>
<td>1.04</td>
<td>56.14</td>
</tr>
<tr>
<td>T9</td>
<td>25.00</td>
<td>0.0750</td>
<td>1.23</td>
<td>0.74</td>
<td>5.77</td>
<td>68.39</td>
</tr>
<tr>
<td>T10</td>
<td>15.56</td>
<td>0.0750</td>
<td>1.23</td>
<td>0.86</td>
<td>9.03</td>
<td>88.32</td>
</tr>
<tr>
<td>T11</td>
<td>40.18</td>
<td>0.0750</td>
<td>1.23</td>
<td>0.68</td>
<td>2.6</td>
<td>77</td>
</tr>
<tr>
<td>T12</td>
<td>25.00</td>
<td>0.0300</td>
<td>1.23</td>
<td>0.59</td>
<td>14.6</td>
<td>113.53</td>
</tr>
<tr>
<td>T13</td>
<td>25.00</td>
<td>0.2000</td>
<td>1.23</td>
<td>1.20</td>
<td>1.39</td>
<td>78.76</td>
</tr>
<tr>
<td>T14</td>
<td>25.00</td>
<td>0.0750</td>
<td>0.61</td>
<td>0.69</td>
<td>7.67</td>
<td>126.74</td>
</tr>
<tr>
<td>T15</td>
<td>25.00</td>
<td>0.0750</td>
<td>2.44</td>
<td>0.78</td>
<td>3.49</td>
<td>44.45</td>
</tr>
</tbody>
</table>

3. Optimization problem
In machining process various and sometimes contradictory criteria must be satisfied, so several constraints and objective functions must be introduced in order to obtain low cost and accurate product qualities.

3.1 Objective and constraint functions
The goal of these tests was to define the cutting conditions that allow the lowest machining cost. This optimization problem consists in determining optimal cutting conditions to minimize the total machining cost (F), while avoiding that the value of average surface roughness does not increase more than an imposed value. This condition is integrated by a constraint function (1).

Thus, the optimization problem can be formulated as follows:

\[
\begin{align*}
\text{Min } & \quad F(x) \\
\text{Such that } & \quad g(x) \leq 0 \\
\text{with } & \quad x^a \leq x \leq x^b
\end{align*}
\]

(1)

The normalized objective function to be minimized depends of the volume of material removal and is expressed as:

\[
F(x) = \frac{C_f(x)}{C_0}
\]

(2)

where \( F(x) \) is the optimization variables, \( C_0 \) is the initial total machining cost and \( C_f(x) \) is the total machining cost at iteration k.

This total machining cost is calculated by the integration of all the machining-related costs. These cost needs the machining duration to be calculated. The duration is calculated with the volume V of material to be removed by the tool on the part. This volume is related to the material flow \( Q \) (mm3/min) and the machining duration \( T_u \) (min):

\[
V = QT_u
\]

(3)

In slotting, the material removal flow is given by the product of the milled section by the feed:

\[
Q = D a_p V_f = D a_p f_z z_n = \frac{1000 V_c a_p f_z z}{\pi}
\]

(4)

The machining duration \( T_u \) (min) is then given by the following relation:

\[
T_u = \frac{\pi V}{1000 V_c a_p f_z z}
\]

(5)

The total machining cost is composed of the following costs:

**Machine cost \( C_m \):** it takes into account the machining duration \( T_u \) and unproductive times \( T_i \), as well as the hourly rate of machine use \( C_m \):

\[
C_m = C_m (T_u + T_i)
\]

(6)

**Tool cost \( C_t \):** it is related to the number of cutting edges used to remove the material volume V. It takes into account the tool lifespan \( T \) and the cost of an edge Ca:
Another minimization is carried out by the SQP algorithm. The iterative procedure stops when the successive optimal solutions are superposed with a tolerance.

### 3.3 Kriging interpolation

The Kriging interpolation [12], makes it possible to represent the complex functions effectively. This method is applied in our work to represent the response surface in an explicit form, according to the optimization variables. The approximate relationship of the objective and constraint functions can be expressed as follows:

\[ \tilde{J}(x) = p^T(x)a + Z(x) \]

with, \( p(x) = [p_1(x), \ldots, p_m(x)]^T \), where \( m \) denotes the number of the basis function in regression model, \( a = [a_1, \ldots, a_m]^T \) is the coefficient vector, \( x \) is the design variables, \( \tilde{J}(x) \) is the unknown objective or constraint interpolate function, and \( Z(x) \) is the random fluctuation. The term \( p^T(x)a \) in Eq. (7) indicates a global model of the design space, which is similar to the polynomial model in a Moving Least Squares (MLS) approximation. The second part in Eq. (7) is a correction of the global model. It is used to model the deviation from \( p^T(x)a \) so that the whole model interpolates response data from the function.

The output responses from the function are given as:

\[ F(x) = \{f_1(x), f_2(x), \ldots, f_n(x)\} \]

From these outputs the unknown parameters can be estimated:

\[ a = (P^T R^{-1} P)^{-1} P^T R^{-1} F \]

Where \( P \) is a vector including the value of \( p(x) \) evaluated at each of the design variables and \( R \) is the correlation matrix, which is composed of the correlation function evaluated at each possible combination of the points of design:

\[ R = \begin{bmatrix} R(x_1, x_1) & \cdots & R(x_1, x_n) \\ \vdots & \ddots & \vdots \\ R(x_n, x_1) & \cdots & R(x_n, x_n) \end{bmatrix} + \begin{bmatrix} w(x-x_1) & 0 & \cdots & 0 \\ 0 & w(x-x_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(x-x_n) \end{bmatrix} \]

\[ R_{ij} = \left| x_i - x_j \right| \]

A weight function of Gaussian type with a circular support is adopted for the Kriging interpolation. It takes the Form

\[ w_j(x) = \begin{cases} 1 - e^{-d_j^2/(\sigma^2)} & \text{if } d_j \leq r_w \\ 1 - e^{-r_w^2/(\sigma^2)} & \text{if } d_j \geq r_w \end{cases} \]
where \( d_i = \sqrt{\sum_{j=1}^{2} (x_i^j - x^{*j})^2} \) is the distance from a discrete node \( x_i \) to a sampling point \( x \) in the domain of support with radius \( r_w \).

The second part in Eq. (7) is in fact an interpolation of the residuals of the regression model \( p^T(x) \alpha \). Thus, all response data will be exactly predicted; is given as:

\[
Z(x) = r^T(x) \beta
\]  
(17)

Where \( r^T(x) = \{R(x, x_1), \ldots, R(x, x_n)\} \)

The parameters \( \beta \) are defined as follow:

\[
\beta = R^{-1}(F - Pa)
\]  
(18)

4. Optimization results
The optimization result relates to the total machining cost to be minimized and improve the roughness of machined surface using a constraint function based on experimental data. In the first result an unconstraint optimization problem using BFGS is resolved; i.e. the roughness of machined surface is not taken into account. In the second one, a non linear constraint optimization problem is formulated and resolved using SQP algorithm, to take into account the average roughness of the machined surface.

The \((g)\) constraint is defined according to the average value of surface roughness (Ra). A zero value of \( g \) indicates that the Ra equals the acceptable imposed value.

Table 3: Summary of the optimization results

<table>
<thead>
<tr>
<th>Case 1</th>
<th>( V_c )</th>
<th>( f )</th>
<th>( a_p )</th>
<th>Ra</th>
<th>( T_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconst. problem</td>
<td>Initial values</td>
<td>17.86</td>
<td>0.15</td>
<td>0.75</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>Optimal values</td>
<td>40.18</td>
<td>0.2</td>
<td>2.44</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Improvement</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>---%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>( V_c )</th>
<th>( f )</th>
<th>( a_p )</th>
<th>Ra</th>
<th>( T_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint problem</td>
<td>Initial values</td>
<td>17.86</td>
<td>0.15</td>
<td>0.75</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>Optimal values</td>
<td>40.18</td>
<td>0.12</td>
<td>2.44</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Improvement</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 3 presents a summary of the results obtained with the initial and optimal cutting conditions for the two cases. We note that the cutting speed \( (V_c) \) and depth of cut \( (ap) \) are at their high levels in the two cases.

From the same table, it can be seen that in general, the best compromise between total cost (which represent tool cost, material flow removed, and machining cost) and Ra roughness of machined surface, consists in high level of cutting speed \( (V_c) \), middle level of feed \( (fz) \) and high level of depth of cut \( (ap) \). However, only concerning the total cost, we can see that the best value of the objective functions is given by the high level of cutting speed \( (V_c) \), a middle level of the feed \( (fz) \), and a high level of the depth of cut \( (ap) \).

The response surfaces of the objective and constraint functions are presented respectively in (Figures 2 and 3). The maximum value of the objective function is obtained with the high level of cutting conditions, but with the constraint function, the best value is given by the high level of cutting speed \( (V_c) \) and the low level of feed \( (fz) \) and depth of cut \( (ap) \).
The convergence history during the optimization run is presented in Figure 4 and Figure 5. According to figure 4, we note that for the unconstrained problem, the objective function decrease significantly at the first iteration and converge to the value of 4.28 € but leads to a bad surface roughness. Furthermore, in the case of the constrained problem (Fig. 5), the objective function decreases during the optimization iterations and the constraint function is respected. The optimal values lead to a surface quality improvement and to the decrease of the total cost. The total cost is not significantly different in the two formulations.

The total cost obtained with the optimal cutting conditions is improved respectively for the unconstrained and constrained optimization problem of about 97% and 90% compared to the initial one. However, when the roughness of machined surface is not taken into account, the unconstrained optimization problem is resolved, and leads to a bad surface quality approximately. For the second case a constraint function is imposed to decrease the roughness of the machined surface and. This constraint is respected and the average value of Ra is decreased by 30%.

5. Conclusion
In this paper, we present the application of an optimization method for dry machining process. This method is based on a Surface Response Method and Kriging algorithms, which are interesting when time and cost consuming experiments are involved. The method was applied to determine optimal cutting conditions for milling FSX414, a cobalt based refractory material.

The numerical optimization algorithm presented in this work shows its suitability and robustness as a tool for optimizing a machining process. It proves its capability of predicting optimal cutting conditions to improve the surface roughness (good surface quality) while minimizing the total cost, taking into account the tool life.

In a first unconstrained case, for which minimizing the total cost is the only objective, the minimal cost is attained by maximizing the depth of cut, the middle level of the feed and the low level of the cutting speed. Depth of cut has only little influence on the tool life, therefore it can be easily maximized. Cutting speed has a large influence on the tool life, this is why it is minimized for an optimized cost. Feed only has a limited influence on tool life, therefore the optimal value is at the middle level.

In the constrained case, we can that the optimal situation is different. Depth of cut is still maximized because it has no influence on the roughness, but feed is reduced compared to the preceding case, because it has influence on surface roughness.

References


