Searching for Universal Laws and Rules in Music

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Abstract: - Since several decades scientists from different area of investigations are looking for some universal laws and rules. The science synergetics (Haken, 1973) was founded with the aim to discover such laws valid and compatible in physical, biological, sociological, psychological, different human creative etc. processes. In this study we outline some starting points of possible discovery of such laws and rules in music. On several examples concerning musical thinking and musical tone systems we try to show the advisability of such scientific approach in musical processes which underlie – similar to other human creative processes – logical principles. In the sense of synergetics they are known in self-organising and self-regulating systems for which we consider also the musical systems.

Key-Words: - Tone Systems, Modal Thinking, Synergetics, Fractals, Musical Laws, Regularities.

1 Introduction

Searching for universal laws and rules in music we can postulate some questions: What is common to all kind of music over the whole world? Are they some laws or rules out of intention of the artificer? Are they some laws or rules common to self-organising and self-regulating systems?

To answer these questions a long research process from several scientific observational points is needed. Until now several investigations were oriented toward searching of certain fixed points and regularities in musical texts, or setting up statistical tests for their comparison and observing their development. Some interesting results are gained by the analysis based on rank-frequency distributions of pitches and other parameters. Some investigation using mathematical methods try to find regularities in musical texts and to set up statistical tests for their comparison. Also the musicological analysis which do not use the mathematical or statistical methods but which respect the logical development of music show interesting results which may lead to discovering of common rules and laws in music.

In general we can say that certain kind of analysis a musical text is prolific if it allows us to discover regularities some of which may be laws.

2 Regularities in Music

Observing musical systems in different cultures the same or similar principles are evident: dividing of the tone continuum, the preference of special tones, principle of repetition, contrast, reprise, golden section and others. These regularities could be tested by using different interdisciplinary methods: analysis, synthesis, systems theoretical approach, application of proceedings and methods of diverse disciplines (mathematics, statistics, physics, biology, psychology, aesthetics etc.).

2.1 Tone Scales and Modes

In all musical cultures we can find the same dividing of the sound continuum into the octave system given by the natural principles (e.g. the half of the string length is the higher octave) and by overtones and harmonics (fundamental tone = first harmonic; 1st overtone = 2nd harmonic which is the octave).

This octave is basically divided into seven constitutional steps – heptatonic scales/modes, or into five constitutional steps – pentatonic scales/modes. However the sound result of these heptatonic and pentatonic scales or modes is very different because it depends on the combination of intervals or tetrachords and tuning (different dividing of octave into the micro-intervals and summation of them and different distances of intervals given by different number of cents).

In diverse musical cultures and historical periods we can find also other modes, given by using less than octave space for musical text connected with its function (e.g. hexachordal system in ancient European music) or by different dividing of the octave: six (e.g. whole tone scale), eight, nine or ten tone modes in the music of 20th century, for
example, in modes created by Messiaen, Roslavets, Suchoň and other composers.

Musical texts based on these scales or modes in all musical cultures emphasize two most important tones: in European music already in middle age it was called finalis (the beginning and ending tone or the tonic) and repercussa (also reciting tone, tenor tuba or the dominant). The dominant is not always a fifth above the tonic, but very often (later in tonal music is this fifth relation between the tonic and dominant a rule). In fact this principle of the two most important tones is present in the music of all cultures. The attempt of Arnold Schoenberg in his twelve-tone method of musical composition (also dodecaphony or twelve-note composition) in which all 12 notes of the chromatic scale are sounded as often as one another in a piece of music while preventing the emphasis of any through the use of tone rows, is in practice impossible. To put all 12 notes of the equal importance is psychologically in the process of perceiving impossible. More or less we search for more important tones, which could replace the tonic and the dominant.

2.2 Modal System
Modal system is a way of musical thinking after specific rules of tone organization into the scales – modes.

Examination of musical thinking in diverse cultures and historical periods, predomination of modal system is evident. The rules of tone organisation after specific mode determine only choose of tones, which could be repeated and some of them preferred, not the way of combination as in the serial or dodecaphonic thinking. Modal thinking in this sense is typical for many musical texts, not only for that in the European culture.

2.2.2 Modal System of Renaissance Music
On this space it not possible to treat with all types of modal thinking. We only briefly point at the importance of modal thinking without which we can not understand the development of the European music. Modal thinking in the European Renaissance music was based on the medieval system of three hexachords:

- hexachordum natural: C-D-E-F-G-A
- hexachordum durum: G-A-B-C-D-E
- hexachordum molle: F-G-A-Bb-C-D

The tones hard B in hexachord durum and the soft Bb in hexachord molle are integral elements of the system. Transition from one hexachord to other hexachord – mutatio – caused a coequally using of the tone B or Bb depending of the type of hexachord. Conjunction of all three hexachords in Renaissance led to “eight” note system. Compositions of vocal polyphony of 16th century were written in six modes – Ionian, Dorian, Phrygian, Lydian, Mixolydian, Aeolian (Hypophrygian or Lorician was only theoretical and not used in practise) – in one area (e.g. in C/Do area Ioninan modus begin from C, Dorian from D, Phrygian from E, etc.). Vocal practice of tone joining (porrectus, torculus, scandicus, climacus) in the sense of musica ficta (falsa) uses three diesis (the sharp signs or augmented notes). In the C/Do area, for example, the combination of three hexachords (C/Do, D/Re, E/Mi, F/Fa, G/So, A/La, B/Ti or Bb/Ta) and three diesis (C#/Di, F#/Fi, G#/Si) led to using of the “eleven” note system!

Gesualdo through the chromatization enlarged this system into the “twelve” tone system (in C/Do area it is the note D# or Eb). Musicological attempts to analyse Gesualdo’s music from the tonal functional system (tonality) is not suitable because the functional dependence of all chords is not present. Tonal functional system was developed from cadences (ending sections) in modal music. In one modal composition several cadences in different modes and without modulation are possible.

If we analyse Gesualdo’s music of the tonal functional view we must explained it as a follow of permanent modulations. The analysis from a modal point of view is logical and clear.

In madrigal Moro lasso al mio duolo Gesualdo combines two areas C/Do area and A/La area with the dominance of the Aeolian modus. At the begin he uses a chromatic sequence in bass notes C#-C-B and a sequence of chords C# major, A minor, B major, G major, E major, A minor, which are in A/La area following chords: Mi major, Do, Re major, Ta, So and Do. The relation of last two
chords is the dominant – tonic relation and in C/Do area this cadence is in Aeolian mode.

Fig. 1: Gesualdo: Moro lasso al mio duolo

In the next development both areas (C/Do and A/La) and both modes (Aeolian and Ionian) mutual alternate. Madrigal ends in Aeolian Mode in C/Do area.

Gesualdo has extended the modal system through the chromatization at the end of 16th century and caused its gradual dissolution.

2.2.3 Modal and Tonal System
Preference of two modes (Ionian and Aeolian or hard contra soft or major contra minor) and development of functional relations in cadences (ending sections) evoked rise of the tonal system. Tonal system can be understood as a part of the modal system. The tonal key (in contrast to modal area with six modes) and relation between stabilization and destabilization of the main key become important.

Similarly as Gesualdo, Wagner has extended the tonal system through the chromatization at the end of 19th century. The consequence in both cases was the dissolution of the old system and origin of new systems.

The development of European music could be seen in a simplified form as a cycle of greater or smaller equilibrium between fixation and laxation of the tonal centre (tonic): baroque – combination of modal and tonal thinking; classicism – equilibrium between stabilisation and destabilisation; romanticism – lost of balance and greater destabilisation; impressionism – gradual dissolution of tonal functional system (instead of functional dependence of the chords the independent sequences of chords are possible). At the end of 19th and in the course of 20th century the origin of new forms of modal thinking on the higher level of the musical spiral development was the logical and lawful consequence.

2.3 Distribution of Tone Pitches
In previous studies [7], [8], [10] we have used the mathematical methods and computer programmes for testing the frequency distribution of the note pitches in the musical texts written by many composers from different historical periods and musical genres.

A given note will occur with frequency $f_1$ in a concrete musical text. Then the frequency $f_2$ of a functionally related sound will be a mathematical function of $f_1$. The next frequency $f_3$ will then be given by a function of $f_2$ etc.: $f_i = F(f_{i-1})$.

Many musical texts recorded MIDI was analysed by Reinhard Köhler’s Programme QUAMS and Altmann-Fitter 2.0. The results were surprising: The most adequate frequency distribution of pitch by all analysed musical texts in MIDI device corresponds to the negative hypergeometric distribution.

Than we have used the Ord’s criterion (it is a function of moments of a distribution) for characterization of musical texts and their development. We have shown that in four centuries of the development of rank-frequency distributions of pitch in European music there is a fixed location of this indicator in Ord’s scheme but within this area there is a slow but perpetual motion. In Fig. 1 we can see that the $<I, S>$ indicator of all compositions written by composers of diverse historical periods (Palestrina, Gesualdo, Monteverdi, Bach, Mozart, Beethoven, Liszt, Skrjabin, Schoenberg, Stravinsky, Shostakovich, Ligeti) lies really in the given area.

Fig. 2: Ord’s criterion for 272 musical texts
results achieved with parameters \( a = 152.3294, b = 1919.4999, c = 32.6030 \) yield \( R^2 = 0.89 \). The parameter \( b \) evidently represents the year in which the downward motion begins, \( a \) should be the maximal value at this \( b \). Thus one could apply this curve estimating two parameters empirically but a priori. If we put to each composer the year as a midpoint of his life we see that during the time as \( R_l \) also \( R_s \) they went through certain development. First they monotonously ascended and by Stravinsky they achieved the maximum and than they began to decrease. To show this process we have defined the indicator \( R_l^* R_s \). The result of fitting is shown in Table 1 and Fig. 3.

Table 1: Fitting the Lorentzian curve

<table>
<thead>
<tr>
<th>Composer</th>
<th>Year</th>
<th>( R_l^* R_s )</th>
<th>Lorentzian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Palestrina</td>
<td>1560</td>
<td>6.431</td>
<td>1.2426</td>
</tr>
<tr>
<td>Gesualdo</td>
<td>1587</td>
<td>3.221</td>
<td>1.4506</td>
</tr>
<tr>
<td>Monteverdi</td>
<td>1605</td>
<td>10.679</td>
<td>1.6196</td>
</tr>
<tr>
<td>Bach</td>
<td>1718</td>
<td>10.025</td>
<td>3.8862</td>
</tr>
<tr>
<td>Mozart</td>
<td>1774</td>
<td>11.185</td>
<td>7.2828</td>
</tr>
<tr>
<td>Beethoven</td>
<td>1799</td>
<td>45.904</td>
<td>10.3906</td>
</tr>
<tr>
<td>Liszt</td>
<td>1849</td>
<td>40.588</td>
<td>26.8381</td>
</tr>
<tr>
<td>Skrjabin</td>
<td>1894</td>
<td>101.407</td>
<td>94.5128</td>
</tr>
<tr>
<td>Schoenberg</td>
<td>1913</td>
<td>122.718</td>
<td>146.5063</td>
</tr>
<tr>
<td>Stravinskij</td>
<td>1927</td>
<td>174.187</td>
<td>144.6733</td>
</tr>
<tr>
<td>Shostakovitch</td>
<td>1940</td>
<td>98.574</td>
<td>109.6784</td>
</tr>
<tr>
<td>Ligeti</td>
<td>1965</td>
<td>30.514</td>
<td>51.6784</td>
</tr>
</tbody>
</table>

Fig. 3: Fitting the Lorentzian curve

2.4 Golden Section in Musical Texts

Golden ratio or golden section (proportion, cut, etc.) is well known in the art and also in music. It was interpreted in mathematics since ancient times. For the often occur not only in the art but also in the nature golden proportion is also called as the divine proportion.

In music we know many interpretation of golden section concerning the tone, interval, rhythmic, tectonic, form etc. organization (Lendvai investigated Bartók, Howat Debussy and Chopin, Haylock Beethoven, Counthy Satie and many other musicologist tried to find golden section in music; in Slovak music e.g. Vladimir Bokes rationaly applied the Fibonacci row for organization of many parameters in his compositions). The appearance of golden section is caused not only by rational conscious but also by irrational unconscious processes, “controlled” by order parameter in self-organizing systems.

The existence of golden ratio has been found also in natural language texts and also in musical texts in the distribution of pitches.

More than hundred compositions written by twelve composers from diverse historical periods (Palestrina, Gesualdo, Monteverdi, Bach, Mozart, Beethoven, Liszt, Skrjabin, Schoenberg, Stravinsky, Shostakovitch, Ligeti) were tested. The musical golden proportion impetus has a maximum located in the mid of the 17th century, a rise time \( T_{rise} \approx 75 \) years, and a decay time \( T_{fall} \approx 150 \) years, hence a width of about \( W = T_{rise} + T_{fall} = (75 + 150) \) years = 225 years, heralding and covering the brilliant epoch of Bach, Mozart, and Beethoven. On the other hand, the oldest composers belonging to the beginning of this motion are Palestrina, Gesualdo and Monteverdi. This development of golden proportion as revealed by the evolution of Log A (3 parameter impulse function fitting) can be seen in Fig. 4.
The quantification of different music parameters in future will probably yield the more exact picture about the development of European music and the possibility of more complex comparison with other kinds of music.

2.5 Stratification in Music
To other possibilities of searching regularities in musical texts belong the stratification which is a common phenomenon in nature and culture. In the study *Stratification in Musical Texts Based on Rank-Frequency Distribution of Tone Pitches* [12] the analysis of at least seven compositions by each composer of 13 selected composers from different style periods showed a rather variegated result given in Fig. 5.

Fig. 5: Monostratal to bistratal proportions for 13 composers

![Diagram showing stratification in music composers](image)

The interpretation may correspond with alteration of more diatonic or more chromatic character of compositional language in the music development. Palestrina, Monteverdi, Bach and Mozart – wrote more diatonic music, using the selection of the tone pitches. Gesualdo, Beethoven, Liszt, Skrjabin, Schoenberg, Stravinsky, Shostakovich wrote more chromatic music using often chromatization and all 12 tone pitches in their works.

3 Hypotheses and Implications
Today more and more musicologists and scientists are searching for regularities in music. Some of them investigate the correlation similarities between voices in music, e.g. Bach’s organ chorale settings for four voices [14], other are based on using probabilistic methods and models applications in music processing [2, 15].

To the most interesting investigations in music belongs search for the fractal nature of music which is a result of a recursive process where an algorithm is applied multiple times to process its precious output. There are hypotheses that all musical forms (both in macro and macro level) can be modeled with the fractal processes.

Testing of hypotheses based on quantitative computer method can lead to the implication and later to the confirmation that music as self-regulating and self-organizing system is based on invariant laws and rules which are not as visible as the variables given by individual approaches of composers.

4 Conclusion
In this contribution we wanted to underline the importance of interdisciplinary research and also the necessity of using quantitative methods. Searching for rules and discovery of laws belong to the main research purpose of each scientific discipline to which also musicology pretend.

We do not contradict the empirical and qualitative research in musicology which we only consider for not sufficient but which is necessary for formulation of hypotheses. The testing of these hypotheses is possible only by using computers and quantitative methods. In next future we would like...
to introduce more methods enabling investigation of musical texts and searching for invariant universal laws and rules.

References: