Effect of interactive normal and lateral Stiffness, damping and tip dimensions on the flexural and torsional vibration modes of rectangular AFM cantilevers

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Abstract: - In this paper the effect of interactive normal and lateral stiffness, damping and tip dimensions on the flexural and torsional vibration modes of an AFM1 rectangular cantilever is analyzed. A closed-form expression for the sensitivity of vibration modes is derived using the relationship between the resonant frequency and contact stiffness of cantilever and sample. Each mode has a different sensitivity to variations in surface stiffness. This sensitivity directly controls the image resolution. It is obtained an AFM cantilever is more sensitive when the contact stiffness is lower and the first mode is the most sensitive mode.

Key-Words: - Atomic Force Microscope (AFM), Vibration analysis, Flexural vibration, Torsional vibration, Cantilever

1 Introduction
The atomic force microscope (AFM) extends the high resolution capabilities of the scanning tunnelling microscope, and is now widely used for imaging the surface topography of conductors and insulators on an atomic scale [1]. An atomic force microscope uses a cantilever beam (probe) with a sharp tip at the free end of it to place the tip close proximity to the surface of a sample. The deflection of the probe, due to the interaction between the tip and the specimen is measured and used to map the topography or other properties of the surface and specimen. When the tip scans across a sample surface, it induces a non-linear tip-sample interaction force, that its analysis can help to increase the resolution of surface images. For convenient analysis, in general, the interaction force is assumed to be a linear model. In the few last years, many researches have studied the vibration response of an AFM cantilever, but for convenience only considered the interactive stiffness between the cantilever and the sample surface and neglated the interactive damping [2-4], but as shown by Chang et al [1], the effects of damping on the vibration response of an AFM cantilever is significant and can not be disregarded. Also some of studies that considered the interactive damping, neglated the dimensions of the tip [5]. In this paper, we considered both of normal and torsional damping and also stiffness and damping in longitudinal direction and also the dimensions of the tip, then sensitivity and resonant frequency of the flexural and torsional vibration of an AFM cantilever were analyzed.

2 Analysis
In this paper the atomic force microscope cantilever is considered as a rectangular elastic beam with uniform cross-section that has a length \( L \), thickness \( b \), and width \( a \), as shown in Fig.1. It is fixed at \( x = 0 \), while at \( x = L \), has a conical tip. The tip has a length \( h \), mass \( m \) and the distance between the lower edge of the cantilever and the centroid of the cross-section is \( h_c \). The tip interacts with the

1 Atomic Force Microscope
sample by linear springs $k_n$, $k_i$, $k_j$ and dashpots $c_n$, $c_i$, $c_j$. For normal interaction $k_n$, $c_n$ and for lateral interaction $k_i$, $c_i$, $k_j$ and $c_j$.

Fig.1: Schematic diagram of an AFM cantilever in contact with a sample

2.1 Flexural vibration analysis
The linear differential equation of motion for the flexural vibration of the cantilever beam with uniform cross-section, as shown by Weaver et al is [6]:

$$E I y^{iv}(x,t) + \rho A y''(x,t) = 0$$  \hspace{1cm} (1)

Where the primes and dots represent the corresponding derivatives with respect to space, $x$ and time, $t$, respectively. $E$ is the module of elasticity, $I$ is the area moment of inertia, $\rho$ is the volume density and $A$ is the rectangular cross-sectional area of the cantilever. The corresponding boundary conditions are:

$$y(0,t) = 0$$ \hspace{1cm} (2)

$$y'(0,t) = 0$$ \hspace{1cm} (3)

$$E I y''(L,t) = -m h c_1^2 y''(L,t)$$ \hspace{1cm} (4)

$$E I y''(L,t) = k_n y(L,t) + c_n y'(L,t) + m y(L,t)$$ \hspace{1cm} (5)

A general solution of Eqs. (1)-(5) can be expressed as:

$$y(x,t) = (a_1 \cos k x + a_2 \sin k x + a_3 \cosh k x + a_4 \sinh k x)e^{it}$$ \hspace{1cm} (6)

Where $a_i$, $i=1-4$, are constants determined from the boundary conditions, $\omega$ is the angular frequency, $k$ is the flexural wave number. From the above equations, the characteristic equation can be found:

$$C = \gamma^3 \left[ 1 - \frac{m h c_1^2 \gamma^3}{\rho A L^3} \right] \left( \sinh \gamma \cos \gamma + \cosh \gamma \sin \gamma \right) + \cos \gamma \cosh \gamma$$

$$- \left( \beta_n + i c_n L \gamma^2 \frac{1}{E I \rho A} - \frac{m \gamma^4}{\rho A L^3} \right)$$

$$\left[ \frac{m h c_1^2 \gamma^3}{\rho A L^3} \left( 1 - \cos \gamma \cos \gamma \right) - \sin \gamma \cosh \gamma + \cos \gamma \sinh \gamma \right] = 0$$  \hspace{1cm} (7)

Where $\gamma = k L$ is the normalized flexural wave number, $\beta_n = \frac{k_n}{E I / L^3}$ is the normal stiffness ratio between the normal contact stiffness and that of the cantilever. With substituting Eq.(6) into Eq.(1), we have:

$$E I k^4 - \rho A \omega^2 = 0$$ \hspace{1cm} (8)

Therefore the relationship between frequency and wave number is given by:

$$f = \frac{\gamma^2}{2 \pi L^2} \sqrt{\frac{E I}{\rho A}}$$ \hspace{1cm} (9)

The flexural sensitivity of the cantilever can be calculated from the frequency, which can be measured. The sensitivity of the mode of the cantilever changes significantly for small variations of stiffness as the cantilever crosses the sample. The relationship between frequency $f$ and normal contact stiffness $\beta_n$ can be expressed as:

$$\frac{df}{d\beta_n} = \frac{df}{d\gamma} \frac{d\gamma}{d\beta_n}$$ \hspace{1cm} (10)

Differentiation of Eq. (7) with respect to $\beta_n$ yields:
\[ \frac{d\gamma}{d\beta_n} = -\frac{\partial C/\partial \beta_n}{\partial C/\partial \gamma} \]  

(11)

And also differentiation of Eq. (9) with respect to \( \beta_n \) yields:

\[ \frac{df}{d\gamma} = \frac{\gamma}{\pi L^2} \sqrt{\frac{E I}{\rho A}} \]  

(12)

So with substituting Eq.(11) and Eq.(12) into Eq.(10), we have:

\[ \frac{df}{d\beta_n} = \frac{1}{2\pi L^2} \sqrt{\frac{E I}{\rho A}} \left[ 2\gamma \left( \frac{m h^2 \gamma^3}{\rho A L} (1 - \cos \gamma \cos \phi \gamma) \right) + \left(3\gamma^2 - \frac{m h^2 \gamma^3}{\rho A L} (\sin \gamma \cos \phi \gamma - \cos \gamma \sin \phi \gamma) \right) \right] \]

\[ - \left( \beta_n + ic_{n} L \right) \left( \frac{1}{E I \rho A} - \frac{m \gamma^4}{\rho A L} \right) \left[ \frac{m h^2 \gamma^3}{\rho A L} (\sin \gamma \cos \phi \gamma - \cos \gamma \sin \phi \gamma) \right]^{-1} \]

(13)

The sensitivity of flexural vibration can be written in normal form as:

\[ S_n = \frac{df / d\beta_n}{\pi L^2} \sqrt{\frac{E I}{\rho A}} \]  

(14)

2.2 Torsional vibration analysis

The linear differential equation of motion for the torsional vibration of the cantilever beam with uniform cross-section, as shown by Weaver et al is [6]:

\[ G \xi \varphi (x,t) - \rho I \ddot{\varphi} (x,t) = 0 \]  

(15)

Where the primes and dots represent the corresponding derivatives with respect to space, \( x \) and time, \( t \), respectively. \( \varphi \) is the angle of torsion, \( G \) is the module of shear, \( J \) is the polar moment of area, \( \rho \) is the volume density, \( \xi \) is the torsional parameter for rectangular bar and is given by:

\[ \xi = \frac{1}{3} \frac{b^3}{a} \left( 1 - 0.63 \frac{b}{a} \right) \]  

(16)

The corresponding boundary conditions are:

\[ \varphi(0,t) = 0 \]  

(17)

\[ G \xi \varphi'(L,t) = -k, h^2 \varphi(L,t) - c, h^2 \dot{\varphi}(L,t) - I, \dot{\varphi}(L,t) \]  

(18)

Where

\[ I_s = I + m (h_c + \frac{b}{2})^2 \]  

(19)

A general solution of Eq. (15) can be expressed as:

\[ \varphi(x,t) = (\alpha \sin \omega x e^{i\omega t}) \]  

(20)

Where \( \alpha \) is constant determined from the boundary conditions, \( \omega \) is the angular frequency, \( p \) is torsional vibration wave number. From above equations, the characteristic equation can be found:

\[ C = \gamma \cos \gamma + \left( \beta_i + ic_h^2 \gamma \left( \frac{1}{G \xi \rho I} - \frac{I_s \gamma^2}{L \rho L} \right) \right) \sin \gamma = 0 \]  

(21)

Where \( \gamma = pL \) is the normalized torsional wave number, \( \beta_i = \frac{k, h^2}{G \xi / L} \) is the torsional stiffness ratio between the torsional contact stiffness and that of the cantilever. With the same calculation as part 2.1
the relationship between frequency $f$ and torsional contact stiffness $\beta_i$ can be expressed as:

$$
\frac{df}{d\beta_i} = \frac{1}{2\pi L} \sqrt{\frac{G \xi}{\rho J}} \sin \gamma \left( \gamma \sin \gamma - (1 + \beta_i) \cos \gamma - \frac{\gamma \sin \gamma + \gamma^2 \cos \gamma}{G \xi \rho J} \left( \frac{1}{c_i h^2} \right) \right) ^{-1} 
$$

The lateral contact stiffness was assumed as $k_i = 0.9k_n$. The damping effect of an AFM cantilever on the flexural sensitivity of three modes is shown in fig.2.

![Fig.2: The effect of damping on the normalized flexural modal sensitivity as a function of normal contact stiffness for an AFM cantilever.](image)

3 Results and discussion

In this paper, the sensitivity of the vibration modes of an AFM cantilever has been analyzed. The main purpose of this paper is to study the effect of tip dimensions and normal and lateral stiffness and damping on the sensitivity and resonant frequency of the vibration modes of an AFM cantilever. In order to know these effects, the AFM cantilever is considered as an elastic beam with uniform rectangular cross-section with below geometric and material parameters:

<table>
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<th>Parameter</th>
<th>Unit</th>
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</tr>
<tr>
<td>$G$</td>
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<td>$\mu$m</td>
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</tr>
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</tr>
<tr>
<td>$b$</td>
<td>$\mu$m</td>
<td>2</td>
</tr>
<tr>
<td>$h$</td>
<td>$\mu$m</td>
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<td>$2 \times 10^{-13}$</td>
</tr>
<tr>
<td>$c_n$</td>
<td>ns/m</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>$c_l$, $c_t$</td>
<td>ns/m</td>
<td>$10^{-5}$</td>
</tr>
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</table>

This figure shows when the value of $\beta_n$ is low, the sensitivity of damped status is lower than undamped status in each mode. The more the value of $\beta_n$, causes the smaller damping effect. The sensitivity of first mode is greater than second and third modes, at lower values of $\beta_n$. However, the sensitivity of first mode rapidly decreased when the normal stiffness ratio, $\beta_n$, increased. When the value of $\beta_n$ is about 30 and 67, the sensitivities of first mode are about 0.091 and 0.037 respectively. After the first point the sensitivity of second mode is greater than first mode, and after the second point the sensitivity of third mode is greater than first mode. Also when $\beta_n = 210.6$, the sensitivity of second mode is about 0.042.

Fig.3 shows the normalized sensitivity of mode 1 as function of the normal contact stiffness for an AFM cantilever at various damping coefficient values. When the value of $\beta_n$ is very small, the flexural sensitivity of mode 1 is lower with a higher damping coefficient. The higher damping
coefficient can influence the large range of $\beta_n$ value. The more value of $\beta_n$ causes the smaller damping effect. The flexural sensitivity of AFM cantilever can be influence by boundary conditions. The tip length $h$ and lateral contact stiffness produce a moment $M$ at the free end. The moment can change the sensitivity of the cantilever.

Fig.3: Comparison of the normalized flexural sensitivity of mode 1 as a function of normal contact stiffness for various normal damping coefficients.

Fig.4: Normalized flexural sensitivity of mode 1 as function of the normal cantact stiffness for an AFM cantilever at various tip lengths.

The normalized flexural sensitivities of mode 1 at various tip lengths, $h$, are shown in Fig.4. The effect of tip length is not very large. When $\beta_n$ has low values, but as $\beta_n$ reached about 100, increasing the tip length increases the sensitivity.

For torsional vibration, the same results were obtained. For example The damping effect of an AFM cantilever on the torsional sensitivity of three modes is shown in fig.5.

Fig.5: The effect of damping on the normalized torsional modal sensitivity as a function of torsional contact stiffness for an AFM cantilever.

4 Conclusion

According to the analysis, it is obtained that, for an AFM cantilever, the first mode is the most sensitive mode of flexural and torsional vibration, while when the contact stiffness is greater, the high-order vibration modes are more sensitive than the first mode. The effects of the tip length on the sensitivity are significant and are not negligible when the contact stiffness is greater.

References:


