Communication channel identification in frequency domain based on the Volterra model

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Abstract: An efficient technique of continuous communication channel experimental research for identification of amplitude-frequency characteristics (AFC) is given. The technique is Volterra model based and founded on the application of the approximating method of identification of nonlinear dynamic system. The identification is carried out by means of compiling the linear combinations of responses from researched system to the test polyharmonic signals with different amplitudes. Designed software-hardware tools, that realize the methodology of identification, is used for construction of informational model of communication channel in the form of first and second orders AFC on the basis of input-output experiment data, using harmonic and biharmonic test signals.

Key-Words: communication channels; telecommunication systems; nonlinear dynamic systems; identification; Volterra models; Volterra series; multidimensional transfer functions; multifrequency characteristics; polyharmonic signals

1 Introduction

One of the most important demands required from communication systems is an accuracy of information transmitted from source to recipient. In real conditions to fulfill this demands we have to eliminate an errors caused by external interference form communication channel (CC) at receiver entrance; internal noise of receiver; and signal distortion during transmission. In connection with this problems for the last ten years intensively developing field related to the methods of signals optimal reception which take into account characteristics of the hardware and CC [1]. The expediency of communication system application depends on how effectively its potential abilities are used.

The aim of this work is an identification of the continuous CC using Volterra model in the frequency domain, i.e. the determination of its multifrequency characteristics on the basis of input–output experiment data, using test polyharmonic signals.

2 Volterra models and identification of dynamical systems in the frequency domain

In general case “input–output” type ratio for nonlinear dynamic system can be presented by Volterra series [2, 3]:

\[ y[x(t)] = \sum_{n=1}^{\infty} y_n(x(t)) = \sum_{n=1}^{\infty} \int_0^\infty \cdots \int_0^\infty w_n(\tau_1, \tau_2, \ldots, \tau_n) \prod_{r=1}^n x(t - \tau_r) d\tau_r, \]  

(1)

where \( x(t) \) and \( y(t) \) are input and output signals of system respectively; \( w_n(\tau_1, \tau_2, \ldots, \tau_n) \) – weight function or \( n \)-order Volterra kernel; \( y_n[x(t)] \) – \( n \)-th partial component of object response.

In practice, Volterra series are replaced by polynomial and generally limited to several first members of the series. Identification of nonlinear dynamic system in the form of a Volterra series consists of determination of
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n-dimensional weighting functions \( w_n(\tau_1, \ldots, \tau_n) \) or their Fourier–images \( W_n(j\omega_1, \ldots, j\omega_n) \) — n-dimensional transfer functions, accordingly for system modeling in time or frequency domain [4, 5].

Identification of nonlinear system in frequency domain coming to determination of absolute value and phase of multidimensional transfer function at given frequencies — multidimensional AFC | \( |W_n(j\omega_1, j\omega_2, \ldots, j\omega_n)| \) and phase-frequency characteristics (PFC) \( \arg W_n(j\omega_1, j\omega_2, \ldots, j\omega_n) \) which are defined by formulas:

\[
|W_n(j\omega_1, j\omega_2, \ldots, j\omega_n)| = \sqrt{\left(\text{Re}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))\right)^2 + \left(\text{Im}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))\right)^2},
\]

\[
\arg W_n(j\omega_1, j\omega_2, \ldots, j\omega_n) = \arg \left\{ \frac{\text{Im}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))}{\text{Re}(W_n(j\omega_1, j\omega_2, \ldots, j\omega_n))} \right\},
\]

where \( \text{Re} \) and \( \text{Im} \) — accordingly real and imaginary parts of a complex function of \( n \) variables.

During the identification of a Volterra kernel of \( n \)-th order \((n+1)\) significant effect on accuracy is rendered adjacent members of a Volterra series. Therefore, it is necessary to apply the special methods, allowing minimizing this effect [6]. The idea of such method lays in construction such expression of system responses to \( N (N \geq n) \) test input signals with the given amplitudes that with certain accuracy (accurate within to the thrown terms members of series) would be equal to \( n \)-th member of a Volterra series:

\[
y_n[x(t)] = \sum_{j=1}^{N} c_j y[a_j x(t)] = \sum_{j=1}^{N} \left( \sum_{i=1}^{N} c_j a_j^i \right) \times \prod_{\tau=1}^{n} x(t - \tau) d\tau,
\]

where \( a_j \) — amplitudes of test signals, random nonzero and pairwise different numbers; \( c_j \) — real coefficients which are chosen so that in a right part (4) were converted in null all first \( N \) members, except \( n \)-th, and the multiplier at a \( n \)-multiple integral became equal to unit. This condition leads to a solution of the linear algebraic equations system concerning coefficients \( c_1, c_2, \ldots, c_N \):

\[
\sum_{j=1}^{N} c_j a_j^k = \delta_{n,k}, \quad \text{where} \ 1 \leq k \leq N,
\]

This system always has a solution, and the unique one, as the system determinant differs from Vandermonde determinant with only a multiplier \( a_1 a_2 \cdots a_N \). Thus, with any real numbers \( a_i \), that different from zero and pairwise different, it is possible to find such numbers \( c_j \) at which the linear combination (4) of system responses is equal to \( n \)-th member of a Volterra series accurate within to the thrown terms members of series.

It is possible to build numberless assemblage of modes for expressions (4), by taking various numbers \( a_1 a_2 \cdots a_N \) and defining (5) coefficients \( c_1, c_2, \ldots, c_N \) by them.

The choice of amplitudes \( a_j \) should provide the convergence of series (1) and an minimum error during extraction of a partial component \( y_n[x(t)] \) according to (4) defined by reminder of series (1) — members of degree \( N+1 \) and above. If \( x(t) \) — is a test effect with maximum admissible amplitude at which a series (1) converges, amplitudes \( a_j \) should be by their absolute values no more than unit: \( |a_j| \leq 1 \) for \( \forall j=1,2,\ldots,n \) [6].

The test polyharmonic effects for identification in the frequency domain representing by signals of such type:

\[
x(t) = \sum_{k=1}^{n} A_k \cos(\omega_k t + \varphi_k),
\]

where \( n \) — the order of transfer function being estimated; \( A_k, \omega_k \) and \( \varphi_k \) — accordingly amplitude, frequency and a phase of \( k \)-th harmonics. In research, it is supposed every amplitude of \( A_k \) to be equal, and phases \( \varphi_k \) equal to zero.

Thus, the test signal can be written in the complex form:

\[
x(t) = A \sum_{k=1}^{n} \cos(\omega_k t) = \frac{A}{2} \sum_{k=1}^{n} \left( e^{i\omega_k t} + e^{-i\omega_k t} \right).
\]

Then the \( n \)-th partial component in the response of system can be noted in an aspect:

\[
y_n(t) = A^k \sum_{k=0}^{n} \sum_{k=1}^{n} \sum_{k=1}^{n} e^{i\omega_k t} \times |W_n(j\omega_1, \ldots, j\omega_n)| \cos \left( \sum_{k=1}^{n} \omega_k \right) + \arg W_n(j\omega_1, \ldots, j\omega_n)
\]

here \( \left[ \right] \) means function of extraction of an integer part of number.

The component with frequency \( \omega_1 + \ldots + \omega_n \) is selected from the response to a test signal (6):
In [7] it is defined that during determination of multidimensional transfer functions of nonlinear systems it is necessary to consider the imposed constraints on choice of the test polyharmonic signal frequencies which provide an inequality of combination frequencies in output signal harmonics.

Described method was tested on a nonlinear test object – amplitude detector described by Riccati equation:

\[
\frac{dy(t)}{dt} + \alpha \cdot y(t) + \beta \cdot y^2(t) = u(t).
\]

Analytical expressions of AFC and PFC for the first and second order model where received:

\[
|W_1(j\omega)| = \frac{1}{\sqrt{\omega^2 + \alpha^2}}, \quad \arg W_1(j\omega) = -\arctg \frac{\omega}{\alpha},
\]

\[
|W_2(j\omega_1, j\omega_2)| = \frac{\beta}{\sqrt{(\omega_1^2 + \alpha_1^2)(\omega_2^2 + \alpha_2^2)} \cdot (\omega_1^2 + (\omega_2^2 + \alpha_2^2))},
\]

\[
\arg W_2(j\omega_1, j\omega_2) = -\arctg \frac{(2\alpha_1^2 - \omega_1^2, \omega_1^2)(\omega_2^2 + \alpha_2^2)}{\alpha(\omega_1^2 - \omega_2^2, \omega_2^2) - \alpha(\omega_1^2 + \alpha_2^2)^2}.
\]

Results (second order AFC and PFC) received after procedure of identification are presented on fig.1 (approximation order of the model \(N=2\) and \(N=4\)).

![AFC and PFC of the test object](image)

Fig.1. AFC and PFC of the test object: analytically calculated values (1), section estimation values with approximation order of the model \(N=2\) (2), \(N=4\) (3).

3 The technique and hardware-software tools of radiofrequency CC identification

Experimental research of an Ultra High Frequency range CC for the purpose of identification of its multifrequency performances, characterizing nonlinear and dynamic properties of the channel are fulfilled. The Volterra model in the form of the second order polynomial is used. Thus physical CC properties are characterized by transfer functions of \(W_1(j2\pi f)\) and \(W_2(j2\pi f_1, j2\pi f_2)\) – by the Fourier-images of weighting functions \(w_1(t)\) and \(w_2(t_1, t_2)\).

Implementation of identification method on the IBM PC computer basis has been carried out using the developed software in C++ language with the usage of such classes as CWaveRecorder, CWavePlayer, CWaveReader, CWaveWriter which allow to provide rather convenient interacting with MMAPI Windows. The software allows automating the process of the test signals forming with the given parameters (amplitudes and frequencies). Also this software allows transmitting and receiving signals through an output and input section of PC soundcard, to produce segmentation of a file with the responses to the fragments, corresponding to the CC responses being researched on test polyharmonic effects with different amplitudes.

In experimental research two identical S.P.RADIO A/S, T&T VHF–radio stations (a range of operational frequencies 146–174 MHz) and IBM PC with Creative SBLive! sound cards were used. Sequentially AFC of the first and second orders were defined. The method of identification with an order of approximation \(N=4\) was applied. Structure charts of identification procedure – determinations of the \(n\)-order AFC of CC are presented accordingly on fig.2. The general scheme of a hardware–software complex of the CC identification, based on the data of input–output type experiment is presented in fig.3.
The CC received responses to the test signals, compose a group of the signals, which amount is equal to the used order of approximation \( N \) \((N=4)\). In each following group the signals frequency increases by magnitude of chosen step. A cross-correlation was used to define the beginning of each received response. An information to form the test signals, amplitudes and corresponding to them coefficients given in [6] was used.

Maximum allowed amplitude in described experiment with use of sound card was 0.2V (defined experimentally). The used range of frequencies was defined by the sound card pass band, and frequencies of the test signals has been chosen from this range, taking into account restrictions specified above. Such parameters were chosen for the experiment: start frequency – 125 Hz; final frequency – 3750 Hz; a frequency change step – 125 Hz; to define AFC of the second order determination, an offset on frequency \( f_2 - f_1 \) was equal 6.25, 12.5, 25, 50, 100 and 200 Hz.

The weighed sum is formed from received signals – responses of each group (fig.2). As a result we get partial components of response of the CC \( y_1(t) \) and \( y_2(t) \). For each partial component of response a Fourier transform (the FFT is used), and from received spectra only an informative harmonics (which amplitudes represents values of required characteristics of the first and second orders AFC) are taken.

The first order amplitude-frequency characteristic \( |W_1(j2\pi f)| \) is received by extracting the harmonics with frequency \( f \) from the spectrum of the partial response of the CC \( y_1(t) \) to the test signal \( x(t)=A\cos2\pi ft \).

The second order AFC \( |W_2(j2\pi f_1,j2\pi f_2)| \) was received by extracting the harmonics with summary frequency \( f_1+f_2 \) from the spectrum of the partial response of the CC \( y_2(t) \) to the test signal \( x(t)=A\cos2\pi f_1 t+A\cos2\pi f_2 t \).

The results received after digital data processing of the data of experiments (wavelet “coiflet” de-noising) for the first and second order AFC are presented in fig.4 and 5.

Fig.4. AFC of the first order after wavelet “coiflet” 2\(^{nd}\) level de–noising.
4 Conclusions

The technique of experimental research of continuous CC of the telecommunication system is developed for the identification of its characteristics regarding to the nonlinear and dynamic properties on the basis of the Volterra models in the frequency domain. The technique is based on application of the approximating method of identification of the nonlinear dynamic system using the compilation of the linear combinations of the system responses to the input polyharmonic signals of different amplitudes.

The hardware-software tools are developed implementing the technique of identification and they are applied to create an information model of the CC in the form of the first and second orders AFC on the basis of the data of input-output type experiment with the usage of the test harmonic and biharmonic signals.

The received results of researches show essential nonlinearity of the CC that leads to distortions of the signals in a radio section, reduces important parameters of the telecommunication system: the reproduced signals accuracy, channel bandwidth, noise immunity.

In further research the received frequency characteristics of the CC will be used for the synthesis of the compensators of the nonlinearity distortions in the telecommunication system.

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