Coverage Problem Approach using Multi-Objective Scheme

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Abstract—The problem of coverage optimization has been proposed for a long time. In this group of difficulties, many settings have been confronted like placing facilities in order to increase the number of users that have access or placing antennas so that an area has a maximum availability or so that it is constantly monitored. In this paper we propose a generic solution for a group of these problems: placing elements so that the costs are minimized and the utility is maximized with the restriction of no redundancy (none of the radii can overlap). A genetic algorithm (using SPEA as a blueprint) was conceived and compared against the solver CPLEX solving a mixed integer quadratically constrained (MIQCP) problem. The results show that many scenarios can be resolved with the algorithm and gave superior results against the numeric solver.

Index Terms—Multi-objective optimization, MOEA.

I. INTRODUCTION

With the constant rise of new technologies, new challenges arise. In communications many problems have emerged like: how to increase transfer rates, improve the quality and reduce the transfer time of packets. All of these present great difficulties but many forget about the deployment: where to place a certain device has many implications, it can affect costs or improve the utilities. This is especially the case when we are dealing with apparatus like sensors, radio antennas and wireless LAN systems where coverage is also a concern. In order to provide the best service, many of these have to be put in a restricted area but they must also offer an appropriate amount of signal to comply with a certain demand. Much work has been undertaken in the subject of optimizing coverage. There is optimization for selecting sites of different types of facilities taking into account variables like cost and quality, placing access points offering good signal and for security systems. They have been useful in various scenarios and different approaches have been used like pruning, neighborhood search, genetic algorithms, voronoi tessellations and simulated annealing.

In this paper we propose a generic solution for a group of these problems: placing elements so that the costs are minimized and the utility is maximized. The main restriction planted is of no redundancy (the radius of two elements placed cannot overlap). This proves beneficial in several cases; placing emergency service facilities like ambulances, reforestation and afforestation where the trees have a rapid growth if they don’t overlap and sensor deployment in cases where no benefits are obtained with redundancy. Our work was carried out using a genetic algorithm due to the multi objective reasoning required and the need to present a group of solutions. The algorithm was compared to the mathematical model of the same problem using a mathematical optimization computer program. The analytic method of sum of weighs was used for the mathematical model.

This paper is structured as follows. Section 2 describes related works, Section 3 explains the problem, the algorithm used is detailed in Section 4, in Section 5 the results are portrayed and Section 6 contains the conclusions.

II. RELATED WORK

The traditional approach of coverage planning in various environments aims to achieve an optimal placement of elements such that the number of these is minimized while the coverage is maximized. A number of other works address the challenges of effectively deploying or choosing the location of different elements to increase the coverage and reduce costs. The placement of radio base stations was some of the first research undertaken concerned in deploying elements to augment coverage. In [6] examined the suitability of using a genetic algorithm for solving the problem of having a fixed number of base stations. Whereas in [1] they confront the issue by guaranteeing that a certain percentage of the traffic is can be served simultaneously reducing the costs of the placed base stations, they solved this by using a combination of computational techniques with bio inspired algorithms (genetic algorithm).

Similar papers have been published regarding the positioning of sensors in a limited space. The work elaborated in [2] places sensors in a grid complying with a minimum coverage of the field taking into account obstacles and boundaries that can block the signal or vision of the sensors. The solution implemented utilizes a proposed mate heuristic designed by the authors. On the other hand [9] proposes an optimization of
coverage in mobile networks where each one of these devices has a limited sensory range. The final result was obtained using voronoi tessellations to divide the space.

Mobile systems have become an essential part of every person’s life; because of this there is an ever greater effort to improve these. In [5] they embark the difficulties of WLAN. Here they optimize the placement of APs by improving the average signal quality in the entire service area and minimizing the areas with poor signal quality. They achieve a close to optimal result using heuristic algorithms; particularly: pruning and neighborhood search.

All the previously mentioned projects have described optimizations of placing devices that provide connectivity or that monitor the environment. But this sort of problem also exists in urban areas where different facilities need to be placed maximizing the coverage. An example is the emergency ambulances [4]: these facilities are placed maximizing the number of people covered using the distance from where it is placed and preserving low costs of where they are placed. Another example is the positioning of emergency warning sirens [8]; the service coverage is based on sound transmission from each siren. The model is resolved having a strict budget and reaching a large quantity of people.

III. PROBLEM STATEMENT

Given a surface $S$ we want to find a set of points $P$ in $S$, each one with an element of radius $R_p$, for each $p$ in $P$, such that each pair of element has no points in common (the area covered by two elements do not overlap) and all the elements cover a surface $A$ of $S$ ($A \subset S$) with a cost $C$. There is a cost function associated with each point in $S$ that defines the price of putting an element on each point. The problem consists on finding the set of elements that maximizes utility and minimizes the total cost, making it a multi objective optimization problem. In the case of antennas the utility is the coverage, whereas in the reforestation scenario the utility is the amount of pollution reduced by planting the different trees or the biodiversity that the new forest has.

A. Inputs

The inputs required for the problem are: the shape of the surface ($S$), the function $C(x,y,r)$ that defines the cost of putting an element of radius $r$ at point $(x,y)$ and the allowed ranges ($r_{\text{min}}$ and $r_{\text{max}}$) for all the elements.

$S \subset R^2$

$P = \{x | x \in S\}$

$C : S \rightarrow R$

$C(x,y) = \text{cost of putting an element at point } x,y$

$R: P \rightarrow R$

$R(x,y) = \text{radius of the antenna at } x,y$

B. Restrictions

The radius of each element should be in the range $r_{\text{min}}$ and $r_{\text{max}}$:

$(\forall p | p \in P : r_{\text{min}} \leq R(p) \leq r_{\text{max}})$

The area of each element cannot intersect:

$(\forall p_1, p_2 | p_1, p_2 \in P \land |p_1 - p_2| \leq R(p_1) + R(p_2))$

C. Decision Variables

The decision variables are the set $P$ of points that have elements and their radius.

D. Outputs

The outputs of the process are the set of elements with their utility and the cost of the set of elements. For the antennas, the utility would be the amount of covered surface.

E. Optimization Variables

The objective functions are the total cost and the cover area of $S$:

$z_1 = \sum_{p \in P} C(p, x, y, R(p))$

$z_2 = A(\{x | x \in S | (\forall p \in P : |p - x| \leq R(p))\})$

F. Solution

Two solutions where implemented to solve the problem, one consisting a multi objective evolutionary algorithm (MOEA) and other by writing the model in a numeric solver.

IV. MOEA SOLUTION

A meta-heuristic algorithm was implemented to solve the proposed problem. We use SPEA as the guideline to build a specific solution for the coverage problem. In this section we present the chromosome shaped along with the crossover and mutation mechanisms utilized.

In the subsequent figure the chromosome implemented is observed. The chromosome consists on a list of elements, each one with the information about its position and its radius. A set of chromosomes can be seen as a list of lists. Example: $\{\{0,2,3.0\}, \{00,20,5.0\}, \{20,3,20.0\}, \ldots, \{15,18,5.0\}\}$

![Fig. 1. Structure of a single chromosome.](image)
Based on the chromosome, the mutation function receives a chromosome and produces 2 new chromosomes. One chromosome is a copy of the input, except that an element between 0 and n is randomly chosen and it is moved to one of its neighboring positions. The second chromosome is built by randomly increasing or decreasing the element’s radius. Fig 2

\[ Z(x,y) = (F_1(x,y), F_2(x,y)) \]

\[ F_1(x,y) \text{ and } F_2(x,y) \text{ are the objective functions that compose the multi objective problem. } Z(x,y) \text{ is the set of feasible solutions.} \]

Objective function using the weighted sum problem:

\[ Z(w_1,w_2)(x,y) = w_1 * F_1(x,y) + w_2 F_2(x,y) \]

\( W_1 \) and \( W_2 \) are the assigned weights to the distinct objective functions.

The numeric solver CPLEX (we specifically solved a mixed integer quadratically constrained problem) alongside the weighted method were used to solve the coverage problem. The particular problem resolved consisted of a 5x5 matrix; numeric solver doesn’t converge with a larger problem.

In this case, we are using the weighted sum method to find as many Pareto optimal solutions to compare them to the genetic algorithm.

\[ A. \text{ Metrics} \]

The solutions produced by the numeric solver using numeric solver are part of the set \( Y_{\text{true}} \) whereas the ones given by the genetic algorithm belong to the \( Y_{\text{known}} \) set. To compare the results given by the two different methods a group of metrics will be used. Subsequently each one of these will be explained.

Overall Non-dominated Vector Generation (ONVG) [10]

It is the sum of solutions in the Pareto front belonging to \( Y_{\text{known}} \).

\[ \text{ONVG} = |Y_{\text{known}}|_c \]

\( ||_c \) is the cardinality

Overall true Non-dominated Vector Generation (OTNVG) [3]

The number of solutions that make part of the Pareto front in \( Y_{\text{known}} \) and that are also true in \( Y_{\text{true}} \).

\[ \text{OTNVG} = \{|y| y \in Y_{\text{known}} \wedge y \in Y_{\text{true}}\}_c \]

Overall Non-dominated Vector Generation Ratio (ONVGR) [10]

It denotes the ratio between cardinality of \( Y_{\text{known}} \) and \( Y_{\text{true}} \). An ONVGR close to 1 is desired.

\[ \text{ONVGR} = \frac{\text{ONVG}}{|Y_{\text{true}}|}_c \]

Error Ratio (\( E \))[10]

This metric specifies ratio between the members of \( Y_{\text{known}} \) that aren’t part of \( Y_{\text{true}} \) and \( \text{ONVG} \). \( E = 0 \) is the ideal result.
Generational Distance (GD)\(^{[10]}\)

A large value of GD implies \(Y_{\text{Known}}\) is far from \(Y_{\text{True}}\). The best possible result is \(G = 0\).

\[
E = \frac{\sum_{i=1}^{N} \theta_i}{ONVG}
\]

\[\theta_i = \begin{cases} 1 & \text{if a solution in } Y_{\text{Known}} \text{ is also true in } Y_{\text{True}} \\ 0 & \text{otherwise} \end{cases}
\]

Where \(N\) is the number of solutions in \(Y_{\text{Known}}\) and \(d_i\) is the Euclidian distance between every solution in \(Y_{\text{Known}}\) and the closest member of \(Y_{\text{True}}\).

Spacing

This diversity metric indicates if the solutions in an objective space are evenly distributed. The closer to 0, the more evenly distributed the solutions are.

\[
S = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (d_i - \bar{d})^2}
\]

Where

\[
d_i = \min_j \left( |f_i^j(\chi) - f_j^i(\chi)| + |f_i^j(\chi) - f_j^i(\chi)| \right)
\]

\[i, j = 1, 2 \ldots N\]

And

\[
\bar{d} = \frac{1}{N} \sum_{i=1}^{N} d_i
\]

From the previous graph it can be deduced that the ONVG (53) is high compared to the number of solutions obtained by the analytical method, but the more interesting characteristics are: the OTNVG (4) is not equal to the number of solutions in \(Y_{\text{True}}\) because the genetic algorithm attains better solutions when these have higher costs, the ONVGR (7.57) is much larger than 1 because of the large ONVG and the error ratio (0.77) is high because there are far more solutions in \(Y_{\text{Known}}\) than in \(Y_{\text{True}}\). Equally the GD is high because of the nature between the cardinality in \(Y_{\text{Known}}\) and \(Y_{\text{True}}\). Lastly the spacing of the genetic algorithm is relative evenly distributed (2.3).

The times between each one of the methods utilized are considerable. Each one of the solutions found by the analytical method took approximately 2 hours: this is a total of 14 hours, whereas the time it took the genetic algorithm to find all 53 solutions was of 43 minutes.

VI. CONCLUSIONS AND FUTURE WORK

The problem of coverage has gained attention due to the constant emergence of new difficulties involved, from wireless networks to facilities and reforestation. The genetic algorithm has proven to be efficient at finding optimal results in the multi objective problem where the radius of the elements cannot overlap.

Comparing the genetic algorithm against the mathematical model program proves to be superior in every aspect. It does the task quickly and produces better results (a better Pareto); it provides far more solutions and the given Pareto is evenly distributed. The genetic algorithm can also handle larger problems; it is more scalable than the modeling program that couldn’t handle scenarios with a matrix larger than 5x5 while the genetic algorithm proved to do this task with matrixes as large as 25x25.

It would be interesting to compare the genetic algorithm against other well-known multi objective and mono objective algorithms like ant colony and simulated annealing. For future work more complex scenarios will be included like including obstacles and barriers.

REFERENCES


