Fourier Transform and its Applications

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Abstract: - This paper analyses Fourier transform used for spectral analysis of periodical signals and emphasizes some of its properties. It is demonstrated that the spectrum is strongly depended of signal duration that is very important for very short signals which have a very rich spectrum, even for totally harmonic signals. Surprisingly is taken the conclusion that spectral function of harmonic signals with infinite duration is identically with Dirac function and more of this no matter of duration, it respects Heisenberg fourth uncertainty equation. In comparison with Fourier series, the spectrum which is emphasized by Fourier transform doesn't have maximum amplitudes for signals frequencies but only if the signal lasting a lot of time, in the other situations these maximum values are strongly de-phased while the signal time decreasing. That is why one can consider that Fourier series is useful especially for interpolation of non-harmonic periodical functions using harmonic functions and less for spectral analysis.

Key-Words — signals, Fourier transform, continuous spectrum properties, Quantum Physics, Fourier series, discrete spectrum

1 Introduction

Non-harmonic periodical signals analytically described by a function which respects Dirichlet's conditions (the function which describes the signal is limited, has a finite number of discontinuities and finite extremes on the period's duration) are empirically known as numerical function and can be interpolated through an infinite series of harmonic functions, respectively Fourier series. If Fourier series is applied for periodical signals only, Fourier transform can be applied as well for periodical and non periodical signals which are considered as an extreme case of periodical signals. If the Fourier series identifies only discrete spectral components with pulsations equal with multiple of the pulsations of the periodical and non harmonic signal, the Fourier transform shows spectral components on the continuous domain of pulsations of a periodical or non periodical signal [1], [2], [3], [4]. Fourier transform makes that from a real function of time, which describes researched signal, to get a complex function having the pulsation as variable. This modulus of complex function is named either frequency characteristic if refers to attenuation properties of a medium whereby are transferred signals or spectral function if refers to spectral composition of a known signal [5]. The Fourier transform theory can be applied with the same results to any kind of signals [6] of very different technical field (electrical signals [7], radio electrical signals [8], [9], mechanical signals. Among Fourier transform results, the most surprising approaches to the Quantum Physics, mostly considered abstractly even for physicians.

2 Fourier transform

Although Fourier transform theory is well known, this must to be shortly sum up for emphasize some needed usage rules.

A periodical signal $f(t)$ of period T_0, and pulsation $\omega_0 = 2\pi/T_0$, can be spectrally analyzed with the form:

$$F(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt \qquad (1)$$

called, by definition, Fourier transform of function $f(t)$ and is symbolically written:

$$F(j\omega) = F\left[f(t)\right] \qquad (2)$$

The expressions of Fourier transforms were inferred for the case of non periodical signal. They are perfectly valid also for periodical signals, because can be ignored or considered as a no essential feature that the periodical signals values are regularly repeated at moments equal with one period.

From the form of expression (3) appears the necessity of knowing the real signal, depending of time, on an infinite domain, between plus and minus infinite. For a concrete signal, known between the finite moments t_1 and t_2, can be used the below argument:

$$f(t) = \begin{cases} 0, & t < t_1 \\ f(t), & t_1 \le t \le t_2 \end{cases} \Rightarrow$$

$$F(j\omega) = \int_{-\infty}^{t_1} 0dt + \int_{t_1}^{t_2} f(t)e^{-j\omega t}dt + \int_{t_2}^{+\infty} 0dt; 0, \quad t > t_2 \tag{3}$$

$so:$

$$F(j\omega) = \int_{t_1}^{t_2} f(t)e^{-j\omega t}dt$$

The transformations (1) and (3) make that a real function of real variable $f(t)$, to become a complex function $[F(j\omega)]$ with real and imaginary parts Re and Im. Using the writing ways of complex functions it gets:

$$\begin{aligned} Re[F(j\omega)] &= \int_{t_1}^{t_2} f(t)cos(-\omega t)dt; \\ Im[F(j\omega)] &= \int_{t_1}^{t_2} f(t)sin(-\omega t)dt \end{aligned} \tag{4}$$

The expression (4) allows finding out the modulus (amplitude) S called hereinafter spectral function and phase ϕ of complex function as real functions of pulsation:

$$\begin{aligned} S(\omega) &= \sqrt{\{Re[F(j\omega)]\}^2 + \{Im[F(j\omega)]\}^2} \\ \phi(\omega) &= arctan\frac{Im[F(j\omega)]}{Re[F(j\omega)]} \end{aligned} \tag{5}$$

The definition domain of pulsation from the spectral function $S(\omega)$ is comprised between zero (negative pulsations don't make sense) and a maximum value ω_{max} which, for signals described by analytical functions, can be chosen of however high value depending on certain concrete criterion. For signals empirically taken as samples at constant period of time Δt, the value of ω_{max} is given by Shannon's sample theory, respectively:

$$\omega_{max} = \frac{\pi}{\Delta t} \tag{6}$$

We chose for the following analyze a pure harmonic signal which can exist alone or can appertain to a composite periodical signal:

$$f(t) = sin(\omega_o t) \tag{7}$$

which is quite known for $t \in [0, 2\pi/\omega_o]$. For the spectrum which is determined with Fourier transform using (3) and (5), the integration will be performed on variable durations between the limits $t_1 = 0$ and $t_2 = 2n\pi/\omega_o$ (*n* is a multiply of period $T_0 = 2\pi/\omega_0$) to have in view any influence of signal's duration, duration which doesn't appear at the Fourier series. After analytical calculating performing the expression for spectral function is getting:

$$S(\omega) = 2\omega_0 \left| \frac{sin\left(n\pi\frac{\omega}{\omega_o}\right)}{\omega^2 - \omega_o^2} \right| \tag{8}$$

3 Spectral function properties

The Fourier series gives for the signal (7) only a single spectral component, namely itself. The Fourier transform and the spectral function gives a large spectrum dependent of the signal duration *n* and presented in the figure 1 for *n=4*.

Analyzing the expression (8) is noticed some interesting issues:

- In point of abscise $\omega = \omega_0$, the amplitude has the value [11]:

$$S(\omega_0) = 2\omega_0 \lim_{\omega \to \omega_o} \left| \frac{sin(n\pi\omega/\omega_o)}{\omega^2 - \omega_o^2} \right| = n\frac{\pi}{\omega_o} \tag{9}$$

- The peak of frequency characteristic, respectively the maximum amplitude, appears for $\omega = \omega_0$ only if $n \to \infty$.
- The S amplitude in co-ordinate point $\omega = \omega_0$ is growing in the same time with signal's duration, so that for $n \to \infty$, $S \to \infty$.

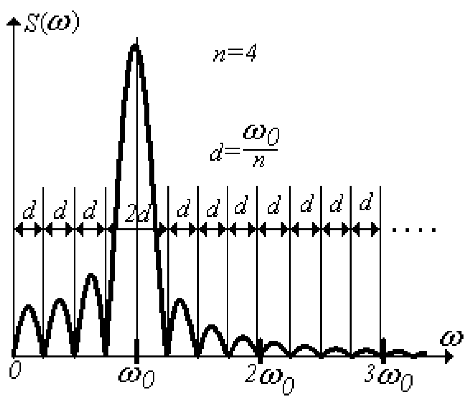

Fig. 1. Continuous spectrum of pure harmonic signal

- The signal (7) contains an infinite number of pulsations existing within a $2d$ width loop around $\omega = \omega_0$ and a series of d width loops one side and another of the $\omega = \omega_0$.
- The loop centered on the value $\omega = \omega_0$ has the biggest amplitude, the other loops amplitude being smaller the farer they are of $\omega = \omega_0$; the loops width decreases as signal duration increases.

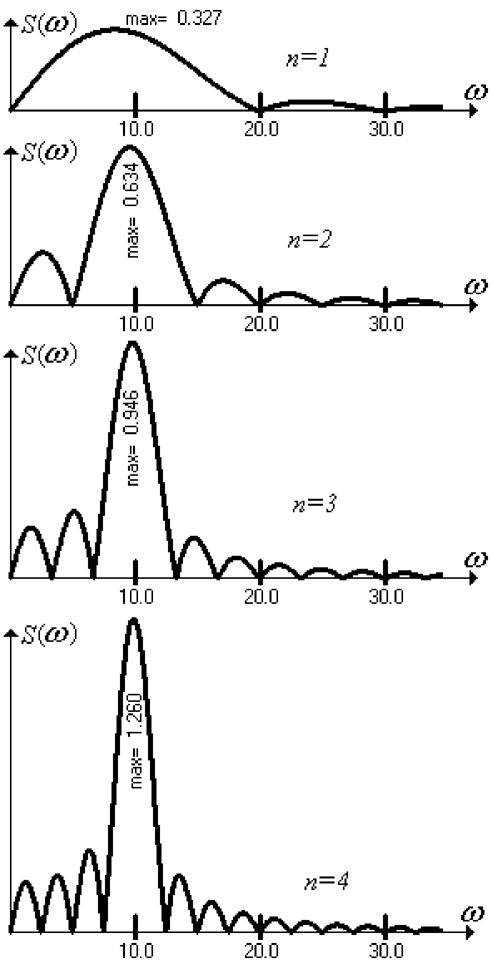

Fig. 2. Spectrum of signal with different duration

From those above a very interesting feature could be emphasized, figure 2 ($\omega_0 = 10\ s^{-1}$): as the pure harmonic signal is shorter, its spectrum is larger and the spectrum contains a very broad loop centered in $\omega = \omega_0$ called central loop and a series of loops with smaller and smaller amplitudes for pulsations farer and farer of $\omega = \omega_0$; as the signal duration increases the central loop width decreases and its amplitude increases and lateral loops became narrower and thicker, tending to get closer to the pulsation $\omega = \omega_0$.

If the signal is periodical but non-harmonic, than it contains a number of harmonic components which produces, each of them, a spectrum as is shown above, and each component's spectrum is cumulated into a global effect.

So it is explained why, in practice, an earthquake which takes for few ground oscillations causes walls, chimneys, pillars or many other construction elements collapsing with very different own pulsations. Even that a signal has a very long duration, its harmonics effect will have major variations while the signal proceeds over the object which is under its influence. In the beginning of respective signal's action, even after its first period will appear a lot of harmonics on a wide pulsations band, capable to stimulate a lot of oscillating systems with a wide variety of own frequencies and while the signal time of action is growing the harmonics tend to signal's own pulsation. In this way, the most dangerous time duration regarding the signal action is in its first periods, when the possibility to perturb is acting on the most wide spectrum possible.

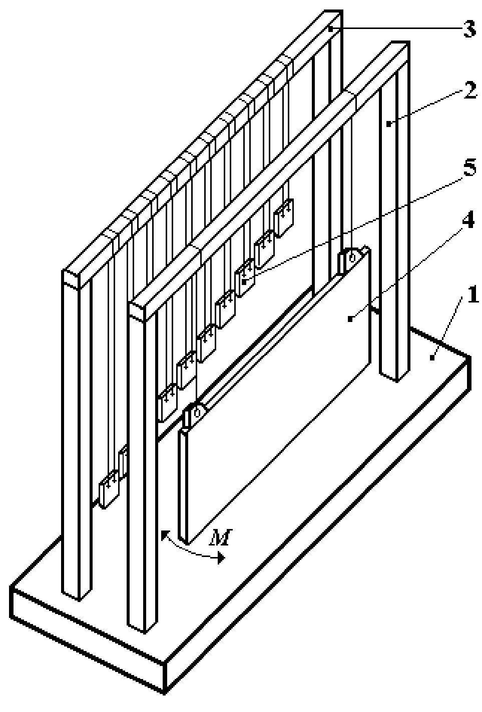

Fig. 3. Experimentally confirmation of the broad-band spectrum of the short signals

Figure 3 shows a very simple device which emphasize all above. On a holder 1 are fixed two identically frames 2 and 3. On the frame 2 is suspended a plate 4 using two equal wires of length l and on the frame 3 are suspended a lot of plates 5 (we used 10) using wires of different lengths. l_i, $l_1 > l$, $l_5 = l$, $l_{10} < l$. So, the plates are forming some pendulums with different own pulsations, $\omega_i = \sqrt{g/l_i}$, g=9.81 ms^{-2}. For a ratio $l_1/l_{10} = 2$ the

extreme pendulums will have pulsations in ratio $1:\sqrt{2}$, resulting that $l_1/l_5 = l_5/l_{10} = \sqrt{2}$. The other pendulums length l_i will vary as a square root function to oscillate on pulsations with equidistant values splited between ω_1 and ω_{10}.
If plate 4 is moved under the direction M, it will oscillate with its own pulsation and will stimulate through pressure waves the pendulums 5. Will be noticed very easy that at first oscillation of plate 4 will be stimulated and will oscillate all pendulums 5 and then, when oscillations number of plate 4 is rising in time only pendulums 5 with length equals with plate 4 will continue to oscillate.
This mechanical device was chosen to emphasize the influence of signal duration upon the spectrum because it produces slow oscillations, easy to notice and to study. Can be made more other devices, also electronic devices but these are more complicated and hard to be analyzed.

4 Fourier transform and quantum physics

In Quantum Physics exists Heisenberg's fourth equation of uncertainty [12], written like in following equation:

$$\delta W \cdot \delta\tau \geq h \qquad (10)$$

Where δW is a wave energy variation, $\delta\tau$ is wave duration, h is Plank constant. If (10) is divided by h and taking into account that wave s energy is $W = h\nu$, ν =frequency, is getting:

$$\delta\nu \cdot \delta\tau \geq 1 \qquad (11)$$

or

$$\delta\omega \cdot \delta\tau \geq 2\pi \qquad (12)$$

This means that a signal spectrum has a pulsations variation range $\delta\omega$ inversely proportional to its duration $\delta\tau$:

$$\delta\omega \geq \frac{2\pi}{\delta t} \qquad (13)$$

its spectrum being as broader and larger as it is shorter in time. This conclusion shows a first connection between Fourier transform and Quantum Physics.
To show another connection we have to weigh anchor the spectral function determination starting from the same pure harmonic signal (7) but integrated between negative and positive limits expressed as multiples of own period $T_0 = 2\pi/\omega_0$, to simplify the calculations:

$$F(j\omega) = \int_{-n_1T_0}^{n_2T_0} e^{-j\omega t} \sin(\omega_0 t)dt = \int_{-n_1T_0}^{n_2T_0} \cos(\omega t)\sin(\omega_0 t)dt - j\int_{-n_1T_0}^{n_2T_0} \sin(\omega t)\sin(\omega_0 t)dt =$$
$$= \left[-\frac{\cos(\omega_0+\omega)t}{2(\omega_0+\omega)}\Bigg|_{-n_1T_0}^{n_2T_0} - \frac{\cos(\omega_0-\omega)t}{2(\omega_0-\omega)}\Bigg|_{-n_1T_0}^{n_2T_0} \right] - \qquad (14)$$
$$- j\left[\frac{\sin(\omega_0-\omega)t}{2(\omega_0-\omega)}\Bigg|_{-n_1T_0}^{n_2T_0} - \frac{\sin(\omega_0+\omega)t}{2(\omega_0+\omega)}\Bigg|_{-n_1T_0}^{n_2T_0} \right] =$$
$$= Re\big(F(j\omega)\big) + j\,Im\big(F(j\omega)\big)$$

The calculations are simplified if $n_1 = n_2 = n$ for which $Re\big(F(j\omega)\big) = 0$, and spectral function becomes:

$$S(\omega) = 2\omega_0 \left| \frac{\sin(2\pi n\omega/\omega_0)}{\omega_0^2 - \omega^2} \right| \qquad (15)$$

In co-ordinate $\omega = \omega_0$, (15) becomes:

$$S(\omega_0) = \frac{2\pi An}{\omega_0} \qquad (16)$$

The spectral function is annulled in following co-ordinates:

$$\omega = \frac{k}{2n}\omega_0, \quad k = 0,1,2,3,\cdots, \quad k \neq 2n \qquad (17)$$

A typical graph of spectral function for $n=2$ could be seen in figure 4 and it looks like graph form figure 1, the difference is that n represents the pairs number of periods the signal lasts.
The calculation of the each loop surface of this function, for an infinite signal duration, starts with the calculation of main loop surface, between ω_1 and ω_2, where $\omega_1 = \omega_0 - \omega_0/2n$, $\omega_2 = \omega_0 + \omega_0/2n$).

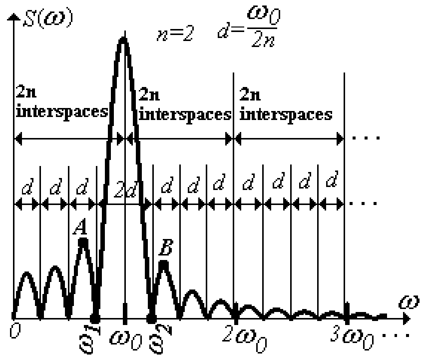

Fig. 4. Spectral function

Because $\int_{\omega_1}^{\omega_2} S(\omega)d\omega$ is transcendental can not be calculate on analytical way, so that it is solved using numerical method with data from table 1.

Table 1
Surface under the central loop as function of n

n	1	10	100	...	100000
Area	3.744....	3.7042....	3.7038....	...	3.70387410375....

It's noticed a very fast convergence to a transcendent number $3.70387410375...$, independent of ω_0.

The others loop surfaces, especially surfaces sum is very hard to be calculated with an analytical method and numerical calculations don't converge anymore, like in prior case. To solve this problem the majorising functions was used for (31) and their convergence study for *n* infinite values. Because (31) is a modulus function, two majorising functions S_{M1} and S_{M2} must be determined:

$$S_{M1} = 2\omega_0 \frac{1}{\omega_0^2 - \omega^2}, \quad \omega < \omega_0 \tag{18}$$

$$S_{M2} = 2\omega_0 \frac{1}{\omega^2 - \omega_0^2}, \quad \omega > \omega_0 \tag{19}$$

The value $\omega = \omega_0$ is not contained in (18) and (19). The single loop surface is:

$$\begin{aligned} Area\ \ S_{M1} &= 2\omega_0 \int_{\frac{k-1}{2n}\omega_0}^{\frac{k}{2n}\omega_0} \frac{d\omega}{\omega_0^2 - \omega^2} = \\ &= ln\frac{4n^2+2n+k-k^2}{4n^2-2n+k-k^2} \end{aligned} \tag{20}$$

$$Area\ S_{M2} = 2\omega_0 \int_{\frac{k-1}{2n}\omega_0}^{\frac{k}{2n}\omega_0} \frac{d\omega}{\omega^2 - \omega_0^2} = ln\frac{k^2+k+2n-4n^2}{k^2+k-2n-4n^2} \tag{21}$$

From (17) results that for (20) $k < 2n-1$ and for (21) $k > 2n$. Based on these, the arguments of logarithm function from (20) and (21) are positive and bigger than 1 for k and n finite values. At limit, these values became:

$$\begin{aligned} &\lim_{k\to\infty} Area\ S_{M1} = 0 \qquad for\ any\ \ n \\ &\lim_{n\to\infty} Area\ SA_{M1} = 0 \qquad for\ any\ \ k \\ &\lim_{k\to\infty} Area\ S_{M2} = 0 \qquad for\ any\ \ n \\ &\lim_{n\to\infty} Area\ S_{M2} = 0 \qquad for\ any\ \ k \end{aligned} \tag{22}$$

No one areas from (20) or (21) could be considered negative because the representative functions haven't negative values and taking into account the null values from (22) results that non-majorising functions values will be null, too. It can be drawn the conclusion that for a stationary harmonic signal of infinite duration ($n \to \infty$) and unitary amplitude, the total area of spectral function is independent of signal pulsation ω_0 and has a finite value, respectively 3.70387410375 ... . For $n \to \infty$, the spectral function central loop in $\omega = \omega_0$ is narrowing to zero and tend to infinite amplitude. In this situation, the spectral function properties became:

$$\begin{cases} \lim_{n\to\infty} S(\omega) = \begin{cases} +\infty & for\ \omega = \omega_0 \\ 0 & for\ \omega \neq \omega_0 \end{cases} \\ \lim_{n\to\infty} \int_0^{\infty} S(\omega)d\omega = 3.7038741037\cdots \end{cases} \tag{39}$$

In Quantum Physics, the Dirac function is symbolized with $\delta(t)$ and could be written like:

$$\begin{cases} \delta(t) = \begin{cases} +\infty & for\ t = t_0 \\ 0 & for\ t \neq t_0 \end{cases} \\ \int_{-\infty}^{\infty} \delta(t)dt = 1 \end{cases} \tag{40}$$

where t_0 is a random value of time. It is noticed that the spectral function limit for an infinite duration signal has the same properties with Dirac function, with the exception of the constant 3.70387410375 .... instead of the unit.

5 Conclusion

Fourier transform represents an extension and a generalization of Fourier series, as is shown into the above demonstrations. If Fourier series can be applied only for periodical functions analysis under the condition that the period to be already known, the Fourier transform can be applied to any function, periodical or no periodical, when the period is not known. These two

applications have in common the fact that both of them are providing information about spectral components of analyzed functions [13], [14], [15], [16]. About information's content, there are fundamental differences between these two applications. The most obvious and surprising difference appear in the most simple case of pure harmonic signal. If the signal is analyzed using Fourier series its spectrum contains only a single component or the signal itself. Analyzing the signal using Fourier transform one can get a spectrum with bands (figure 1) whose width depends of signal duration as number n of periods. So, the spectrum which Fourier series can get is a discreet one, while the Fourier transform provides a continue spectrum on many large or narrow bands depending of signal duration. Is very important to be noticed that for short signals, during only few periods, the continue spectrum has not the maximum value for its own pulsation ω_0 and that is why is very difficult to find the value of this spectrum frequency (figure 1). A very important case of short signals is represented by earthquake signals or some signals produced by industrial activities. For a correct evaluation, when one processes the measurements of spectrum determination or spectrum recording has to take into account the above aspects for the precise calculation of spectral components pulsations.
The facts are more complicated for the non-harmonic signals. A non-harmonic signal is formed very often by the sum of many harmonic signals with $\omega_{01}, \omega_{02}, \cdots \omega_{0n}$ pulsations of random values. In this case, an analysis using Fourier series will not be able to identify each components pulsation mean while Fourier transform is able to identify each $\omega_{01}, \omega_{02}, \cdots \omega_{0n}$ harmonics pulsation from spectral functions maximum under the condition that for each of them to be applied a correction given by signal duration.
An inconvenient of spectral function which results from Fourier transform is that for a longer signal the maximum value of each components spectrum is higher. One can say that the maximum values which indicate spectral components will not give the correct amplitude values of these components, because these values depending a lot of signal duration as (8) and (9) show.
For a spectral analysis of a non-harmonic periodical signal using Fourier series, the first difficulty to be passed with other methods is to find out the fundamental pulsation ω_0 of the signal. The spectral components have the pulsations $2\omega_0, 3\omega_0, \cdots n\omega_0$ indifferently of real structure of analyzed signal. That is why the Fourier series is especially used for periodical functions interpolation, as a mathematical artifice, than for spectral components analysis of a signal, as it was said long time ago [12].
Only Fourier transform can provide information regarding the spectrum, which compulsory is a continue spectrum, in case of non periodical signals.
Because to solve the expressions (1)-(5) for experimental signals is very laborious, it was settled a very fast way of Fourier transform, called Fast Fourier Transform (FFT). Recently, normal or fast Fourier transform are implemented on different software such as LabVIEW [17] or MATLAB [18]. But, from the authors experience results that is not indicated to use so called brand software because could appear no-permitted errors. For example, a signal processed in LabVIEW will not emphasize the influence of signal time length, so the result suggests that the signal could be artificial extended to indicate only maximum values of spectral components but not the enough wide bands which appear with short signals, too. The calculation modern technique reaches a work velocity which allows to program expressions (1)-(5) without spending much time, getting correct results at once [19], [20], [21], [22], [23], [24]. As we already shown, this aspect is very important in very short signal case, but dangerous too, as earthquakes or some industrial activities effects.
If the signal duration is long enough in comparison with its own period than the signal's effect is variable in time. During its first period the signal produces the widest possible spectrum and is able to excite oscillating systems with a lot of own pulsations on a very wide band, figure 2. As long as the signal continues to exist its spectrum is narrowed to the signal's own pulsation. For the signals with low pulsation (as mechanical ones) this effect is very intensive because its spectrum is superposed over many mechanical systems own pulsations, especially in constructions field.
Investigating the properties of spectral function in a deeper manner the author found that for a pure harmonic signal with an infinite duration, the difference between spectral function and Dirac's function appears as a constant value which in Dirac's function case has the conventional value 1. This similitude of the spectral function with an abstract and conventional function from Quantum Physics is fulfilled by the fact that spectral function respects the Heisenberg fourth equation of incertitude, valid into all Physics micro-universe. It is possible that continuing the investigations to be discovered more other surprises which could give different meanings to the actual signal investigation techniques.

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