Simulating the image quality of the human eye using the Zenike polynomials

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Abstract: The visual system of the human eye is a part of the central nervous system by which the human body sees and interprets the information provided by the visible light in order to build a representation of the world around. During the propagations of the light through the eye medium, the retinal image can be deteriorated by an eye which suffers from diseases and disorders. For retinal images the most important sources of image quality degradation are diffraction and optical aberrations. We simulate the effect of the diffraction on human vision. We present a description of the aberrations and we use the Zernike polynomials to simulate the wavefront aberrations of the eye. We compute the point spread function of an eye and we simulate its effects on the human vision.

Key-Words: diffraction, optical aberrations, Zernike polynomials, point spread function of the eye

1 Introduction

Light illuminates the scene and allows people to see spatially and to discern the location of the objects which are contained in the scene. Light can be understood as the aspect of luminous radiant energy which an observer perceives through its visual sensation. The visual system is the detector that transforms radiant power into luminous sensation [4,9]. In the process of spatial light capture by the human eye, the quality of the image of the scene depends on the diffraction of the pupil of the eye and of the aberrations of the cornea and lens [1,2,5]. Consequently, the eye can suffer from diseases and disorders.

The human eye is an imperfect optical instrument and there are not two individuals exactly alike or even their two eyes. Each individual has an optical eye print that is as unique as their own finger prints. We distinguish one person’s optical eye print from another depending on the imperfections of eye or the aberrations of eye. However, despite the fact that we can measure many orders of aberrations, the human eye can only distinguish aberrations up to the fourth or fifth order [1-3].

2 Functions used in describing the image quality of the eye

The common optical metrics are still used in vision science. Optical systems are typically described by the modulation transfer function (MTF) and the point spread function (PSF). The MTF accounts for the contrast degradation due only to the optics of the eye. The PSF describe the intensity distribution of a point source as imaged through the optical system. The optical aberrations produce further spread of the image of a point source [1-3].

2.1 The resolution of the optical system

The optical resolution is a measure of the ability of the optical systems to resolve the details present in
the scene. In optics spatial resolution is expressed as contrast or MTF. An optical system is typically judged by its resolving power, or ability to reproduce fine details in an image. One criterion, attributed to Lord Rayleigh, is that two image points are resolved if the central diffraction maximum of one is no closer than the first diffraction zero of the other. Rayleigh’s criterion applied to point images demands that the images are separated by a distance between centers of the dots [1,6].

![Fig.1 The Rayleigh’s criterion](image)

In fig. 1 we see three possible situations, corresponding to separations of 0.4, 0.5, and 0.6. The first case is not resolved because there is no evidence between the two peaks. The second case is barely resolved, and the third case is adequately resolved.

### 2.2 The Functions Describing the Optical and Retinal Image Quality of the Human Eye

From optics we know that one of the most important problems is that it is impossible to image a point object as a perfect point image. The human eye is a simple optical system. An optical system is made by a set of components (surfaces) through which the light passes. The quality of the system is analyzed in space by the point spread function (PSF) and in the spatial frequency by the modulation transfer function (MTF) [1-3]. The PSF gives the 2D intensity distribution of the image of a point source. PSF gives the physically correct light distribution in the image plane including the effects of aberrations and diffraction. The MTF characterizes the optical system functionality in spatial frequencies. Most optical systems are expected to perform a predetermined level of image integrity. MTF describe the image structure as a function of spatial frequency and is specified in lines per millimeter. It is obtained by the Fourier transform in the image spatial distribution or spread function.

When an optical system processes an image using incoherent light (sun light), the intensity in the image plane produced by a point in the object plane is called the impulse response function [1-3]:

$$g(x,y) = H[f(x,y)]$$  \hspace{1cm} (1)

where:

- $H$ is an operator representing a linear, position (or space) invariant system.

- The pattern of the input object intensity and the pattern of the output image intensity are related by the convolution equation:

$$g(x,y) = \int_{-\infty}^{+\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)d\alpha d\beta,$$  \hspace{1cm} (2)

where:

- \(\alpha\) and \(\beta\) are the spatial frequencies (line/mm).

$$h(x-\alpha, y-\beta) = H[\delta(x-\alpha,y-\beta)]$$  \hspace{1cm} (3)

is the impulse response of $H$; in optics, it is called the point spread function (PSF).

The PSF characterizes the image analyses in space but also we can characterize the image in frequency using the optical transfer function (OTF):

$$OTF(\alpha,\beta) = \frac{FT(PSF)}{FT(PSF)|_{\alpha=0,\beta=0}}.$$  \hspace{1cm} (4)

The MTF is a quantitative measure of the ability of an optical system to transfer various levels of detail from object to image. The MTF is defined as: the ratio of the contrast of the output image to that of the input image:

$$MTF = \frac{\text{contrast of output image}}{\text{contrast of input image}}.$$  \hspace{1cm} (5)

The OTF describes the response of the optical system to a known input and the relation between OTF and MTF is:

$$MTF(\alpha,\beta) = |OTF(\alpha,\beta)|.$$  \hspace{1cm} (6)

### 2.3 The aberrations of the eye

The wavefront of the light that is transmitted through an optical system is an imaginary surface that remains normal to the direction of propagation at all cross sectional points within the optical pathway. For a perfect eye focused at infinity, the wavefront of the light collected by the optics of the eye would be part of an aspheric surface, which would converge on the back of the eye to create a (diffraction-limited) spot on the retina. Because of the reciprocal behavior of light (i.e., it traverses the same path in either direction), it is possible to measure this wavefront from light that is scattered or reflected from the retina. The light scattered from the retinal surface it is collected by the lens and cornea, and is then projected out of the eye. A perfect,
An emmetropic eye would completely collimate this light. The resulting wavefront would be a perfect flat plane wavefront, perpendicular to the direction of propagation. Any deviations from a perfect plane wave are determined by the optical errors in the eye, which are called the wavefront error of the system [1-3, 5, 6].

The Zernike Polynomials are the functions that characterize the wave aberrations. These polynomials completely describe the aggregate effects of the cornea and lens when the light is passing through every location of the pupil. It defines how the phase of the light is affected after it has propagated through the optical system [1, 10].

2.4 The PSF with aberrations
The image of a point object formed by the optical system is given by the point spread function. It is defined as:

$$PSF(x, y) = \frac{1}{\lambda^2 d^2 A_p} |FT\left(p(x, y) \cdot e^{-\frac{2\pi i W(x, y)}{\lambda}}\right)|^2$$

(7)

where:
- $FT$ is the Fourier transform operator,
- $d$ is the distance from the exit pupil to the image,
- $A_p$ is the area of the exit pupil,
- $\lambda$ is the wavelength,
- $W(x, y)$ is the wave aberration function at the exit pupil.

$$P(x, y) = p(x, y) \cdot e^{-\frac{2\pi i W(x, y)}{\lambda}}$$

(8)

where:
- $P(x, y)$ is the generalized exit function.

The effect of aberrations on the OTF is to reduce the contrast of the image. There is no combination of optical aberrations that will increase contrast at any spatial frequency above the diffraction limit at that frequency [1-3].

Aberrations are the failure of the light rays emerging from a point object to form a perfect point image after the rays has passed through an optical system. Aberrations blur the image which was produced by the image-forming optical system. To describe the primary monochromatic aberrations, of rotationally symmetrical optical systems, we specify the shape of the wavefront emerging from the exit pupil. For each object point, there will be a quasi-spherical wave front converging toward the paraxial image point.

3 Sources of blur in retinal images
We investigate two sources of blur in retinal images. These sources are diffraction and aberrations of the eye. We use the formula of the point spread function (equation 3) to characterize their effects [2, 5].

3.1 The diffraction
Even for a perfect optical eye the quality of the image on the retina is not perfect because it is deteriorated by the diffraction that happens when the light enter in to the eye trough the pupil. Diffraction means that the light spreads whenever it passes through the pupil. Thus, diffraction is the ultimate limit on image quality in any optical system. The degree of spreading is greater for smaller pupils and shorter wavelengths of light. In Fourier optics the apertures functionality is characterized by the point spread function. The PSF has the shape of the Airy disk, but its size decreases as the pupil size increases (fig. 3 and fig. 4).

Using the convolution equation (2) which characterizes the effect of the point spread function, we analyze the diffraction of the pupil.

The pupil of the eye has a circular shape [1, 2, 7, 8]:

$$c(r) = \text{circ} \left( \frac{r}{r_0} \right)$$

(9)

where:
- $r$ is the circle radius,
- $r_0$ is the cut off radius.

The PSF or the Airy disc pattern is calculated as:
\[ c(x, y) = \lambda \frac{r_0}{r} J_1\left(\lambda \frac{r_0}{r}\right) \]  \hspace{1cm} (10)

where:
\( \lambda \) is the wavelength,
\( J_1 \) is the Bessel function of order one.

A perfect optical system is diffracted limited by the relation:
\[ d = 2.44 \lambda N \]  \hspace{1cm} (11)
\[ N = \frac{f}{d} \]  \hspace{1cm} (12)

where:
\( f \) is the focus length,
\( d \) is the aperture diameter.

The human eye is an imperfect optical instrument which can suffer of aberrations. The wavefront of the light that is transmitted through an optical system is an imaginary surface that remains normal to the direction of propagation at all cross sectional points within the optical pathway.

For a perfect eye focused at infinity, the wavefront of the light collected by the optics of the eye would be part of an aspheric surface, which would converge on the back (i.e., it traverses the same path in either direction) and it is possible to measure this wavefront from light that is scattered or reflected from the retina. The light scatters from the retinal surface; it is collected by the lens and cornea, and is then projected out of the eye to create a (diffraction-limited) spot of the retina [1-3,5,6,10].

A perfect, emmetropic eye would completely collimate this light. The resulting wavefront would be a perfect flat plane wavefront, perpendicular to the direction of propagation. Any deviations from a perfect plane wave are determined by the optical errors in the eye, and the results are called the wavefront errors of the system. In any real eye, the optical performance is worse than is predicted by the diffraction, due to the imperfections of the optics of the eye [1-3].

The best way to characterize the aberrations of the human eye is to use the Zernike polynomials. We use Zernike polynomials because have the same shape as the eye’s pupil.

The Zernike polynomials are defined as [1, 10]:
\[ Z^m_n(\rho, \theta) = N^m_n R^{|m|}_n(\rho) \cos(\theta, m), \]  \hspace{1cm} (13)
for \( m \geq 0 \), \( 0 \leq \rho \leq 1 \), \( 0 \leq \theta \leq 2\pi \),
\[ Z^m_n(\rho, \theta) = -N^m_n R^{|m|}_n(\rho) \sin(\theta, m), \]  \hspace{1cm} (14)
for \( m < 0 \), \( 0 \leq \rho \leq 1 \), \( 0 \leq \theta \leq 2\pi \)

where:
for a given \( n \); \( m \) can only take on values of \(-n, -n+2, -n+4, \ldots, n\),
\( R^{|m|}_n(\rho) \) is the radial polynomial,
\[ N^m_n = \sqrt{\frac{2(n+1)}{1+\delta_{m_0}}} \delta_m = 1 \text{ for } m=0, \delta_m = 0 \text{ for } m \neq 0, \]
\[ R^{|m|}_n(\rho) = \frac{(-1)^m}{s!} \frac{(n-s)!}{(0.5(n+|m|)-s)! (0.5(n-|m|)-s)!} \rho^{n-2s}. \]  \hspace{1cm} (15)

The wave aberration function can be express as a weighted sum of Zernike polynomials:
\[ W(\rho, \theta) = \sum_{k} \sum_{n=-n}^{n} W^m_n Z^m_n(\rho, \theta) = \]

\[
\sum_{k} \sum_{n=-n}^{n} \left[ -N^m_n R^{|m|}_n(\rho)\sin(m\theta) \right] + \]

\[
\sum_{k} \sum_{n=-n}^{n} \left[ N^m_n R^{|m|}_n(\rho)\cos(m\theta) \right] \]

(16)

where:

- \( k \) is the polynomial order of the expansion,
- \( W^m_n \) is the coefficient of the \( Z^m_n \) mode in expansion and is equal to rms wavefront error for that mode.

We can express the Zernike Polynomials in Cartesian coordinate:

\[ W(x, y) = \sum_{j=0}^{j_{\text{max}}} W_j Z_j(x, y) \]

(17)

where:

\[ W_j = W^m_n, \ Z_j(x, y) = Z^m_n(x, y), \ \sum_{n=0}^{n_{\text{max}}} j^2(n+2)+m \]

In table 1, we have the list of the first ten Zernike polynomials. In fig. 5 we have the representations of the Zernike polynomials of order two and in fig. 6 we have the PSF of the polynomials of order two [10].

<table>
<thead>
<tr>
<th>Nr.</th>
<th>M</th>
<th>N</th>
<th>Polynomials</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>Piston</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( 2\rho\cos(\theta) )</td>
<td>Tilt x</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>1</td>
<td>( 2\rho\sin(\theta) )</td>
<td>Tilt y</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
<td>( \sqrt{3}(2\rho^2-1) )</td>
<td>Power</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0</td>
<td>( \sqrt{6}\rho^2\cos(2\theta) )</td>
<td>Astigmatism x</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>0</td>
<td>( \sqrt{6}\rho^2\sin(2\theta) )</td>
<td>Astigmatism y</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>( 2\sqrt{2}(3\rho^2-2)\rho )</td>
<td>Coma x</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
<td>0</td>
<td>( 2\sqrt{2}(3\rho^2-2)\rho\sin(\theta) )</td>
<td>Coma y</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>0</td>
<td>( \sqrt{5}(6\rho^4-6\rho^2+1) )</td>
<td>Primary Spherical</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0</td>
<td>( \rho^3\cos(3\theta) )</td>
<td>Trefoil x</td>
</tr>
</tbody>
</table>

Table 1 List of first 10 Zernike polynomials

3.3 The aberrations of the retinal image

In order to simulate the effects of the aberrations, we need to make a convolution between the aberrations of the eye and the input image. The result on the output image is a blurred image [1-3,7,8]. The 2D convolution is useful because it provides a physical sense of the appearance of the objects other than points. In the spatial domain, the intensity distribution of the output image, \( I(x,y) \), is the convolution of the PSF\((x,y)\) with the intensity distribution of the input object, \( O(x,y) \):

\[ I(x,y) = \text{PSF}(x,y) * O(x,y) \]

(18)

In conformity with the equation (17) we can sum together different aberrations. In general a real eye suffers from many aberrations. It is more realistic to compute the point spread function of an eye which has the aberrations of order two and order three. Anyway, the aberrations from table 1 will not have the same intensity. We use the next values for the aberrations: 0.164 for astigmatism x, 0.12 for spherical aberrations, 0.135 for astigmatism y, 0.074 for coma x and 0.011 for terefoil x.

Aberrations affect the light rays that enter at the edge of the pupil more strongly than they affect the rays which enter in the center of the pupil. Diameter of the pupil of the eye is ranging between 3 mm and 8 mm [1-3]. For small pupils, the PSF approximates an Airy disk, but as the pupil size increases, aberrations take over and the PSF broadens and takes an irregular shape (fig. 8 a) and (fig. 9 a).
4 Conclusions
In this paper we simulate the effects of the diffraction and aberrations. We make the analysis of the aberrations of the human eye using the Zernike polynomials. Using the aberrations of order two we simulate the effects of the blur on the resolution of the retinal image. We sum together some Zernike polynomials, we compute the point spread function of an eye and we simulate its effects on the retinal image. We put all this simulation together in order to prove the complexity of the problems which are related to the perception of the spatial light distribution by the human eye. We demonstrate that the best situation happens when the eye has only the unavoidable diffraction and the worst situation when the eye has multiple aberrations.

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