Abstract: - The current engineering problems often consider damped harmonic oscillator, resulting in second-order differential equations that so far have been solved by algebraic methods by hand; but with the advent of software to solve these equations in a quicker way you can optimize the time spent in the process. The purpose of this paper is to show how to apply MAPLE® for this classic problem, the linear solution of these equations and the generation of graphics for the system response, using some examples for this purpose.

Keywords: - Computational Physics, Differential Equations, Oscillations, Harmonic Motion, Maple.

1. Introduction
Vibrations and mechanical oscillations have been captured by man for long form different approaches and these have been studied and used in his work. In contemporary engineering these vibrations are present in different disciplines and problems, as simple as a moving mass, which may have an impact on their environment, as the shock absorbers of a car or a parallel plate capacitor; or a pendulum in motion which, in turn, may have another one attached to itself (double pendulum) or two (triple) or usually several, gaining complexity. The mathematical modeling of these problems generates differential equations both linear and non linear, which until now have been solved by manual methods, these being very tedious and costly in time; also, it has a component that is not possible to ignore: the error entered by the same person doing the work and human error. With the advent of CAS software (Computer Algebra Systems) a window is opened that shows the possibility of carrying out a more optimal and efficient solution. This article examines the utility of using this software, in this case Maple®, to solve the equations generated in the modeling of physical phenomena.

2. Problem
In figures 1 to 4 a list of some common basic problems in engineering is presented that where addressed in the present work to solve them by means of computational methods using CAS software, in this case with Maple®.
2.3. Electromechanical system with an RLC circuit whose capacitance is variable by the oscillation of a spring

Fig. 3

2.4. Electromechanical system with an RLC circuit with two variable capacitors by the oscillation of a spring, connected in parallel and other elements attached to these independently

Fig. 4

The above problems are the main but also took into account many other problems but will refrain from coming here due to space.

3. Method

One of the advantages of current computational methods is that they incorporate many of the mathematical methods developed and used to solve the equations that describe physical phenomena such as the vibrations addressed here. But it is important to note that not all of these methods have been implemented, because the CAS software is a constantly developing area that has achieved great progress, but still has some way to go.

The following are the methods used in this work:

3.1. Euler-Lagrange method

This is powerful enough to solve physics problems, besides being well publicized, but a small disadvantage is that it can become cumbersome. The CAS software simplifies the use of this method by reducing it to these steps:

First, a lagrangian is defined as the difference between the total kinetic energy and the potential energy of the system.

\[ L(\tau) = T(\tau) - V(\tau) \]  

Then this lagrangian is integrated over time to find the Action

\[ S = \int_{-\infty}^{\infty} L(\tau) \, d\tau \]  

Based on the principle of least action \( \delta S = 0 \) the functional derivative of the action is found using the command implemented in Maple \( \text{®} \), “Fundiff”, finding the Euler-Lagrange equations that describe the system.

3.2. Laplace transform Method

The following are the methods used in this work:
3.3. Numerical solution methods

4. Results
The equations and solutions to the problems addressed are outlined below. The parameters used were the same than in the figure in section 2 for each problem.

4.1. Spring mass oscillating system with two degrees of freedom
Kinetic energy
\[ T(\tau) = \frac{1}{2} M_1 \left( \frac{d}{d\tau} x_1(\tau) \right)^2 + \frac{1}{2} M_2 \left( \frac{d}{d\tau} x_2(\tau) \right)^2 \] (3)

Potential energy
\[ V(\tau) = \frac{1}{2} K_1 x_1(\tau)^2 + \frac{1}{2} K_2 \left( x_2(\tau) - x_1(\tau) \right)^2 \] (4)

Power dissipated by a buffer of the system
\[ P(\tau) = \frac{1}{2} C_1 \left( \frac{d}{d\tau} x_1(\tau) \right)^2 + \frac{1}{2} C_2 \left( \frac{d}{d\tau} x_2(\tau) - \left( \frac{d}{d\tau} x_1(\tau) \right) \right)^2 \] (5)

Lagrangian
\[ L(\tau) = \frac{1}{2} M_1 \left( \frac{d}{d\tau} x_1(\tau) \right)^2 + \frac{1}{2} M_2 \left( \frac{d}{d\tau} x_2(\tau) \right)^2 - \frac{1}{2} K_1 x_1(\tau)^2 - \frac{1}{2} K_2 \left( x_2(\tau) - x_1(\tau) \right)^2 \] (6)

Action
\[ S = \int_{\tau_1}^{\tau_2} \left[ \frac{1}{2} M_1 \left( \frac{d}{d\tau} x_1(\tau) \right)^2 + \frac{1}{2} M_2 \left( \frac{d}{d\tau} x_2(\tau) \right)^2 - \frac{1}{2} K_1 x_1(\tau)^2 - \frac{1}{2} K_2 \left( x_2(\tau) - x_1(\tau) \right)^2 \right] d\tau \] (7)

First Euler-Lagrange equation
\[ (K_1 + K_2)x_1(t) + (C_2 + C_1) \left( \frac{d}{dt} x_1(t) \right) - C_2 \left( \frac{d}{dt} x_2(t) \right) - K_2 x_2(t) + M_1 \left( \frac{d^2}{dt^2} x_1(t) \right) = 0 \] (8)

Second Euler-Lagrange equation
\[ K_2 x_2(t) + C_2 \left( \frac{d}{dt} x_2(t) \right) - x_1(t) K_2 + M_2 \left( \frac{d^2}{dt^2} x_2(t) \right) - C_2 \left( \frac{d}{dt} x_1(t) \right) = f(t) \] (9)

Solution
It can vary with the force applied to the system. With \( f(t) = A \delta(t) \) and initial conditions for \( x_1, x_2 \) and their velocities, the system response is obtained for each \( x \). The constants were defined to give a numerical solution, and then the results were plotted.

\[ M = \begin{bmatrix} 50 & 60 \\ 0.2 & 0.2 \end{bmatrix}, C = \begin{bmatrix} 10 & 10 \end{bmatrix}, A = 15 \]

4.2. Coupled pendulums oscillating system with three degrees of freedom
The triple pendulum system is described by the following equation:
\[ x_1(\tau) = L_1 \sin(\theta_1(\tau)) \] (11)
\[ y_1(\tau) = -L_1 \cos(\theta_1(\tau)) \] (12)
\[ x_2(\tau) = L_1 \sin(\theta_1(\tau)) + L_2 \sin(\theta_2(\tau)) \] (13)

\[ y_2(\tau) = -L_1 \cos(\theta_1(\tau)) - L_2 \cos(\theta_2(\tau)) \] (14)

\[ x_3(\tau) - L_1 \sin(\theta_1(\tau)) + L_2 \sin(\theta_2(\tau)) + L_3 \sin(\theta_3(\tau)) \] (15)

\[ y_3(\tau) = -L_1 \cos(\theta_1(\tau)) - L_2 \cos(\theta_2(\tau)) - L_3 \cos(\theta_3(\tau)) \] (16)

Kinetic energy
\[ T = \frac{1}{2} m_1 \left( \left( \frac{d}{d\tau} x_1(\tau) \right)^2 + \left( \frac{d}{d\tau} y_1(\tau) \right)^2 \right) + \frac{1}{2} m_2 \left( \left( \frac{d}{d\tau} x_2(\tau) \right)^2 + \left( \frac{d}{d\tau} y_2(\tau) \right)^2 \right) \] (17)

Potential energy
\[ V = m_1 g y_1(\tau) + m_2 g y_2(\tau) + m_3 g y_3(\tau) \] (18)

4.3. Electromechanical system with an RLC circuit whose capacitance is variable by the oscillation of a spring

Kinetic energy of the electromechanical system
\[ T(\tau) = \frac{1}{2} L \left( \frac{d}{d\tau} q(\tau) \right)^2 + \frac{1}{2} M \left( \frac{d}{d\tau} x(\tau) \right)^2 \] (21)

Potential energy of the electromechanical system
\[ V(\tau) = \frac{1}{2} K x(\tau)^2 + \frac{1}{2} \frac{q(\tau)^2}{C} \] (22)

Power dissipated by a buffer of the system
\[ P(\tau) = \frac{1}{2} R \left( \frac{d}{d\tau} q(\tau) \right)^2 \] (23)

Lagrangian
\[ L(\tau) = \frac{1}{2} L \left( \frac{d}{d\tau} q(\tau) \right)^2 + \frac{1}{2} M \left( \frac{d}{d\tau} x(\tau) \right)^2 - \frac{1}{2} K x(\tau)^2 - \frac{1}{2} \frac{(z-x(\tau)) q(\tau)^2}{A} \] (24)

Action
\[ S = \int_{-\infty}^{\infty} \left[ \frac{1}{2} L \left( \frac{d}{d\tau} q(\tau) \right)^2 + \frac{1}{2} M \left( \frac{d}{d\tau} x(\tau) \right)^2 - \frac{1}{2} K x(\tau)^2 - \frac{1}{2} \frac{(z-x(\tau)) q(\tau)^2}{A} \right] d\tau \] (25)

First Euler-Lagrange equation
\[ K x(t) - \frac{1}{2} \frac{q(t)^2}{A} + M \left( \frac{d^2}{d\tau^2} x(t) \right) = 0 \] (26)

Second Euler-Lagrange equation
\[ -\frac{1}{2} \frac{(z-x(t)) q(t)^2}{A} + L \left( \frac{d^2}{d\tau^2} q(t) \right) + R \left( \frac{d}{d\tau} q(t) \right) - f(t) \] (27)

Solution
It was not possible to find an analytical solution for this problem so it is solved by using numerical methods with \( f(t) = \epsilon_0 \sin(\omega t) \), the initial conditions and parameters of the system. Then the solution was plotted.

\[ q(0) = 0, D(q)(0) = 0, x(0) = 0, D(x)(0) = 0 \] (28)

\[ A = 20, z = 12, w = 15, K = 10, \]
\[ R = 0.5, L = 10, M = 5, \epsilon_0 = 100 \] (29)
4.4. Electromechanical system with an RLC circuit with two variable capacitors by the oscillation of springs, connected in parallel and other elements attached to these independent

Kinetic energy of the electromechanical system
\[ T(t) = \frac{1}{2} L_1 \left( \frac{d}{dt} q_1(t) + \frac{d}{dt} q_2(t) \right)^2 + \frac{1}{2} L_2 \left( \frac{d}{dt} q_1(t) \right)^2 + \frac{1}{2} L_3 \left( \frac{d}{dt} q_2(t) \right)^2 + \frac{1}{2} M_1 \left( \frac{d}{dt} x_1(t) \right)^2 + \frac{1}{2} M_2 \left( \frac{d}{dt} x_2(t) \right)^2 \]  
(30)

Potential energy of the electromechanical system
\[ U(t) = \frac{1}{2} \frac{q_1(t)^2}{C_1} + \frac{1}{2} \frac{q_2(t)^2}{C_2} + \frac{1}{2} K_2 x_1(t)^2 + \frac{1}{2} K_2 x_2(t)^2 \]
\[ C_1 = \frac{A_1}{z_1 - x_1(t)} \quad C_2 = \frac{A_2}{z_2 - x_2(t)} \]  
(31)

Power dissipated by a buffer of the system
\[ P(t) = \frac{1}{2} R_1 \left( \frac{d}{dt} (q_1(t) + q_2(t)) \right)^2 + \frac{1}{2} R_2 \left( \frac{d}{dt} q_1(t) \right)^2 + \frac{1}{2} R_3 \left( \frac{d}{dt} q_2(t) \right)^2 + \frac{1}{2} \frac{q_1(t)^2}{C_1} + \frac{1}{2} \frac{q_2(t)^2}{C_2} + \frac{1}{2} K_1 x_1(t)^2 + \frac{1}{2} K_2 x_2(t)^2 \]  
(32)

Then the Euler-Lagrange method was applied, defining the Lagrangian, the action and the Euler-Lagrange equations, but these are not shown here due to space constraints.

Solution
It was not possible to find an analytical solution for this problem so it is solved by using numerical methods with \( f(t) = \varepsilon_0 \sin(\omega t) \) and the initial conditions and parameters of the system. Then the solution was plotted.

\[ q_1(0) = 0, \quad D(q_1)(0) = 0, \quad D(q_2)(0) = 0, \]
\[ x_1(0) = 0, \quad D(x_1)(0) = 0, \quad D(x_2)(0) = 0 \]  
(33)

\[ A = \begin{bmatrix} 20 & 40 \end{bmatrix}, \quad B = \begin{bmatrix} 12 & 24 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \quad w = 15, \quad \varepsilon_0 = 100, \]
\[ L = \begin{bmatrix} 100 & 200 & 150 \end{bmatrix}, \quad K = \begin{bmatrix} 10 & 20 \end{bmatrix}, \quad F = \begin{bmatrix} 1, 1 \end{bmatrix}, \quad M = \begin{bmatrix} 5 & 10 \end{bmatrix} \]
(34)

In this case the charges one and two were the same, and the currents one and two were the same too.

5. Conclusions
- This study addressed issues of mechanical and electro-mechanical vibrations, seeking to show how to solve them through the use of CAS software, because in this way the process is optimized to make it more efficient.
- The differential equations encountered in modeling mechanical vibrations can be solved by several methods such as Laplace transform, which is very effective for linear equations.
- In everyday engineering is very common to find problems where modeling is performed using nonlinear differential equations, handling those is a bit more complicated but can be solved by the Euler-Lagrange powerful method, greatly facilitating the calculations.
- It is possible to find analytical solutions for linear differential equations, this being much more efficient by computational methods.
• For equations of higher degree it is not possible to find an analytical solution but they can be solved by using numerical methods giving values to the parameters involved in these.
• Through the use of computational methods it is possible to optimize the process of solving physical problems presented in everyday engineering, because normally this process is very costly in time, being necessary a person to carry it out manually.
• The engineers have found a great pillar of support in numerical computing software such as MATLAB®, being these more common than the software used in this work, Maple®, though the latter has the advantage of incorporating many mathematical methods for solving equations and is designed to work analytically obtaining general solutions for models in which is mathematically possible.
• It is expected that in the near future it will be possible to use the advances made by artificial intelligence and parallel computing in the processing of CAS software and in modern engineering applications.

6. References


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\(^{1}\) The image is not audit the author of the text, it was taken and modified for use purely academic with no commercial purpose.
\(^{2}\) Ibid.
\(^{3}\) The image is not audit the author of the text, it was taken and modified for use purely academic with no commercial purpose. Image was taken from Dare A. Wells. Theory and Problems of Lagrangian Dynamics, Mc Graw Hill, 1967.
\(^{4}\) Ibid.