MIMO Capacity Enhancement in Spatially Correlated Channels using Taguchi Method

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Abstract: - Higher system capacities can be achieved if multiple antennas are used on both sides of the wireless link, thus creating a multiple-input-multiple-output (MIMO) system. In this work, the maximization of MIMO system capacity in Rayleigh fading, spatially correlated channels involving practical antenna arrays is challenged through inter-element spacing optimization. The system capacity is evaluated using a proposed formula that takes into account both antenna mutual coupling and signal correlation. Capacity values turn out to outperform the ones obtained considering the conventional antenna array geometries.

Key-Words: -MIMO, capacity enhancement, spatial correlation, mutual coupling, genetic algorithms

1 Introduction
Wireless communication is currently characterized by the revolutionary interest in MIMO techniques. The employment of multiple antennas results in a significant improvement in capacity without any increase in the allocated power or bandwidth [1]. MIMO systems exploit the multipath structure of the propagation channel. However, correlations among channel coefficients are influenced by the antenna properties. As the antennas are collocated in a MIMO array, mutual coupling effects may occur. All these effects should be considered when designing an antenna array for MIMO systems. The antenna array system capacity is reduced due to both spatial correlation and mutual coupling. Signals received by two antennas are correlated due to insufficient separation or lack of rich scattering environment. The correlation coefficient is dependent on the distance between two antennas and the angular spectrum of the incoming signals. Mutual coupling is the other effect of antennas being closely collocated. [2,3]

The works related to capacity enhancement reported in literature focus on spatial correlation and mutual coupling in an independent manner [2,3]. Capacity formulas are given in [2] and [3] for mutual coupling and spatial correlation effects with a Rayleigh fading channel being assumed. In addition, the geometries considered are simple and namely uniform linear arrays are treated with spacing between the elements being optimized.

In this work, capacity maximization is challenged through optimizing the spacings between the elements of a linear array of half wave dipoles taking into account mutual coupling and spatial correlation simultaneously. A capacity formula is proposed to account for both mutual coupling and spatial correlation and is validated through comparing it with its independent counterparts. The optimization is carried out with the aid of the Taguchi method. This new tool has been favored for it guarantees reaching the global optimum solution compared to traditional local optimizers [6]. The results reveal an enhancement in system capacity relative to the approaches reported in literature which can be advantageous in modern wireless communication systems.

2 Problem formulation
A MIMO system, in its general form, is represented in Fig.1. The capacity of a MIMO channel system is given as the maximum data transfer rate for a given acceptable level of received signal and is given in bps/Hz. In its general form, the capacity of a MIMO channel having \( N_t \) transmitting antennas and \( N_r \) receiving antennas assuming the channel is unknown at the transmitter and known at the receiver with equal signal power is given as [8]:

\[
Cap = \log_2 \left[ \text{det}(I_{N_t} + \frac{SNR}{N_r} HH^H) \right]
\] (1)
Fig. 1 General MIMO system

Where $I_{N_s}$ is an $N_r \times N_r$ identity matrix, $\text{SNR}$ is the average received signal to noise ratio in dB, $H$ is the channel transfer matrix and the $H$ on the exponent is the complex conjugate transpose. For a spatially correlated channel, the channel transfer matrix is expressed as:

$$H = R_{\tau}^{\frac{1}{2}}GR_{\phi}^{\frac{1}{2}}$$  \hspace{1cm} (2)

Where $G$ is an $N_r \times N_r$ matrix with entries identically distributed (i.i.d) complex Gaussian zero-mean unit variance elements, $R_{\phi}$ ($N_s \times N_s$) and $R_{\tau}$ ($N_r \times N_r$) denote the correlation observed at the transmitter and receiver, respectively. The entries of these matrices are related to the spacing between the antenna elements (at the receiver or at the transmitter) by the approximate formula [10]:

$$r(d) \approx \exp(-23\lambda^2 d^2)$$  \hspace{1cm} (3)

Where:

d is the spacing between the antennas in wavelengths.

$\lambda$ is the angular spread which is the distribution of power in azimuth plane; value of 0 means a complete directional scenario and 1 represents a uniform spreading.

To take into account the mutual coupling effect, the following channel transfer matrix is used [5]:

$$H_{MC} = C_{R}HC_{\tau}$$  \hspace{1cm} (4)

Where $H$ is again an $N_r \times N_r$ matrix with entries identically distributed (i.i.d) complex Gaussian zero-mean unit variance elements. $C_{R}$ and $C_{\tau}$ are the coupling matrices at the receiver and transmitter, respectively. These matrices are computed as:

$$C = (Z_A + Z_R)(Z + Z_T I_N)^{-1}$$  \hspace{1cm} (5)

Where $Z_A$ is the self impedance of the element in isolation ($Z_A = 73 + 42.5j$ for a $\frac{\lambda}{2}$ dipole). $Z_R$ is the impedance of the receiver at each antenna element chosen to be the complex conjugate of $Z_A$ to obtain an impedance match for maximum power transfer. $I_N$ is the identity matrix and $Z$ is an $N \times N$ mutual impedance matrix whose entries for a side by side configuration of an array of dipoles are computed based on the induced EMF method presented in [1] and are given as:

$$Z_{m} = \begin{cases} 
30(0.5772 + j30S_{2}(2k)) & \text{if } m = n \end{cases}$$

$$= \begin{cases} 
30[2C(u_0) - C(u_0) - C(u_0)] & \text{if } m \neq n \\
- j30[2S_{2}(u_0) - S(u_0) - S(u_0)] & \text{if } m \neq n 
\end{cases}$$  \hspace{1cm} (6)

with $k$: the wave number;

if $d_h$ is the horizontal distance between the elements, then:

$$u_0 = kd_0$$  \hspace{1cm} (7)

$$u_1 = k(\sqrt{d_h^2 + l^2} + l)$$  \hspace{1cm} (8)

$$u_2 = k(\sqrt{d_h^2 + l^2} - l)$$  \hspace{1cm} (9)

And $C_i(u)$ and $S_i(u)$ are cosine and sine integrals defined as [8]:

$$C_i(u) = \int_{0}^{u} \cos(x) \frac{dx}{x}$$  \hspace{1cm} (10)

$$S_i(u) = \int_{0}^{u} \sin(x) \frac{dx}{x}$$  \hspace{1cm} (11)

This channel transfer matrix is normalized using:

$$\left\| H_{MC} \right\|_{F} = N_r N_s$$  \hspace{1cm} (12)

Where $\left\| \cdot \right\|_{F}$ is the Frobenius norm.

Notice that the channel transfer matrices in (2) and (4) take into account the spatial correlation and the mutual coupling in a separate manner. To account for both effects simultaneously, the proposed channel model is just the combination of the two models as:

$$H = C_{R}R_{\phi}^{\frac{1}{2}}HR_{\tau}^{\frac{1}{2}}C_{\tau}$$  \hspace{1cm} (13)

With the different matrices defined as before.

Using this formula, capacity value using (1) is aimed to be maximized through finding the inter-element spacing of the dipole arrays at both transmitter and receiver. The resulting capacity value is compared with different spacings for uniform situations using the same formula and
capacity values obtained using the individual formulas in (2) and (4).

3 The Taguchi method

Thanks to the rapid development of computer technology, many optimization techniques such as genetic algorithm (GA), particle swarm optimization (PSO), simulated annealing (SA), artificial neural network (ANN), and gradient-based techniques have been implemented by computer codes.

Taguchi methods belong to the class of optimization techniques named global optimizers while the more familiar, traditional techniques such as conjugate gradient and the quasi-Newtonian methods are classified as local optimizers. The distinction between local and global search of optimization techniques is that the local techniques produce results that are highly dependent on the starting point or initial guess, while the global methods are highly independent of the initial conditions. Though they possess the characteristic of being fast in convergence, local techniques have a direct dependence on the existence of at least the first derivative. In addition; they place constraints on the solution space such as differentiability and continuity, conditions that are hard or even impossible to deal with in practice. The global techniques, on the other hand, are largely independent of and place few constraints on the solution space [7].

Taguchi’s method was developed based on the concept of the orthogonal array (OA), which can effectively reduce the number of tests required in a design process [6]. It provides an efficient way to choose the design parameters in an optimization procedure.

Before presenting the Taguchi procedure, it is worth understanding what OAs are and how are they generated [6]. Let $S$ be a set of $s$ symbols or levels (the simplest symbols are integers 1, 2, 3…). A matrix $A$ of $N$ rows and $k$ columns with entries from $S$ is said to be an OA with $s$ levels and strength $t$ (0 $t$ $k$) if in every $N$×$t$ subarray of $A$, each $t$-tuple based on $S$ appears exactly the same times as a row. The notation OA($N$, $k$, $s$, $t$) is used to represent an OA.

3.1 Initialization procedure:

The optimization procedure starts with the problem initialization, which includes the selection of a proper OA and the design of a suitable fitness function. The selection of an OA($N$, $k$, $s$, $t$) mainly depends on the number of optimization parameters. In general, to characterize the nonlinear effect, three levels ($s = 3$) are found sufficient for each input parameter. Usually, an OA with a strength of 2 ($t = 2$) is efficient for most problems because it results in a small number of rows in the array.

3.2 Design of input parameters:

The input parameters need to be selected to conduct the experiments. When the OA is used, the corresponding numerical values for the three levels of each input parameter should be determined. In the first iteration, the value for level 2 is selected at the center of the optimization range.

Values of levels 1 and 3 are calculated by subtracting/adding the value of level 2 with a variable called level difference (LD). The level difference in the first iteration (LD1) is determined by the following equation:

$$LD_1 = \frac{Max - Min}{\text{Number of levels} + 1}$$

(14)

Where Max is the upper bound of the optimization range and Min is the lower bound of the optimization range.

3.3 Conduct Experiments and Build a Response Table

After determining the input parameters, the fitness function for each experiment can be calculated. These results are then used to build a response table for the first iteration by averaging the fitness values for each parameter $n$ and each level $m$ using the following equation:

$$F_{av} = \frac{s}{N} \sum_{i=1,OA(n,m)=m}^{f_n}$$

(15)

3.4 Identify Optimal Level Values and Conduct Confirmation Experiment

Finding the largest fitness value ratio in each column can identify the optimal level for that parameter.

When the optimal levels are identified, a confirmation experiment is performed using the combination of the optimal levels identified in the response table. This confirmation test is not repetitious because the OA-based experiment is a fractional factorial experiment, and the optimal combination may not be included in the experiment table. The fitness value obtained from the optimal combination is regarded as the fitness value of the current iteration.

3.5 Reduce the Optimization Range

If the results of the current iteration do not meet the termination criteria, the process is repeated in the next iteration. The optimal level values of the current iteration are used as central values (values of level 2) for the next iteration. To reduce the optimization range for a converged result, the $LDi$
is multiplied with a reduced rate \((rr)\) to obtain \(LD_{i+1}\) for the \((i+1)\)th iteration:

\[
LD_{i+1} = rr \times LD_i = RR(i) \times LD_i
\]  

(16)

Where \(RR(i)\) is called reduced function. When a constant \(rr\) is used, \(RR(i) = rr'\). The value of \(rr\) can be set between 0.5 and 1 depending on the problem. The larger \(rr\) is, the slower the convergence rate.

If \(LD_i\) is a large value, and the central level value is located near the upper bound or lower bound of the optimization range, the corresponding value of level 1 or 3 may reside outside the optimization range. Therefore, a process of checking the level values is necessary to guarantee that all level values are located within the optimization range. If an excessive situation happens, reassigning the level value for the parameter will be performed. A simple way is to use the boundary values directly.

3.6 Check the Termination Criteria

When the number of iterations is large, the level difference of each element becomes small from equation (15). Hence, the level values are close to each other and the fitness value of the next iteration is close to the fitness value of the current iteration. The following equation may be used as a termination criterion for the optimization procedure:

\[
\frac{LD_i}{LD_j} < \text{converged value}
\]  

(16)

Usually, the converged value can be set between 0.001 and 0.01 depending on the problem. The iterative optimization process will be terminated if the design goal is achieved or if equation (16) is satisfied.

4 Results and discussions

The MIMO system considered for the capacity maximization is a 5-input×5-output system. Both the transmitter and the receiver are equipped with a side-by-side lying arrays of half wavelength dipoles. The average SNR at the receiver is taken to be 20 dB.

As a first step, the value of the angular spread is sought. To do this, an investigation of the effect this parameter on the overall capacity using (2) and (3) is carried out. The results are shown in fig.2. The results demonstrate that maximum capacity is achieved when the value of the angular spread is 0.8485. So, for the remaining parts of the study, this value is considered hereafter.

Once the value of the angular spread has been fixed, there remains just to proceed with the capacity maximization using the Taguchi method procedure presented earlier since the only unknowns remaining for equations (1) through (13) are the spacings between the dipole which are allowed to vary from 0.25\(\lambda\) to 2\(\lambda\).

The algorithm was has been run with according to the specifications listed in the procedure \(s=3; t=2\).

At first, capacity maximization is challenged using the separate formulas (2) and (4). For the spatial correlation model in equation (2), the best capacity optimized by the GA was 15.1589 bps/Hz.

![Fig.2 Variation of the capacity vs. angular spread](image1)

![Fig.3 Variation of the capacity vs. element spacing for the spatial correlation only model](image2)
The corresponding capacity values for the uniform geometry at both transmitter and receiver are depicted in Fig. 3 with the best attainable capacity being 14.54 bps/Hz at a spacing of 1.2\(\lambda\). It is noticed that for the uniform geometry, the capacity behaviour versus element inter-spacing has no clear systematic change that the non-uniform geometries provide an enhancement in capacity at reduced array sizes at either the transmitter or receiver.

For the mutual coupling model in equation (4), the best capacity achieved by the Taguchi method is 7.7582 bps/Hz while the uniform configuration whose variations are shown in fig. 4 reveals a maximum capacity of 7.313 bps/Hz with a spacing of 0.4559\(\lambda\). Again, the Taguchi method reached capacity is better than the uniform geometry which in turns, demonstrates that the non-uniform geometry is a better choice though, one might increase the array spatial occupation at the transmitter or the receiver. For the uniform geometry, the variation of the capacity is again not systematic which dictates that there are no clear criteria for the choice of the spacing.

It can be noticed from the investigation involving the spatial correlation and the mutual coupling models separately that the spatial correlation model produces higher capacity values that can be roughly the double. This is explained by the fact that for large inter-element spacings, the value of the correlation coefficient is negligible which in turn, makes the correlation matrix higher order components to vanish producing a high rank matrix. Meanwhile, the effect of the mutual coupling still exists for larger spacings according to the formulas (5) through (11).

The next stage is to use the formula proposed in equation (13) to optimize the capacity with both spatial correlation and mutual coupling taken, simultaneously. The achieved capacity is found to be 10.043 bps/Hz. The corresponding capacity values for different inter-element spacings depicted in fig. 5 provide a maximum capacity of 9.629 bps/Hz at a spacing of 1.846\(\lambda\). One can easily notice that the achieved capacity for the non-uniform geometry is higher than the uniform one. In addition, the reduction in the spatial occupation of both the transmitter and receiver arrays is considerable which adds another advantage to the design. It should be noticed also that the capacity values obtained lie somewhere between the capacity values obtained using the individual formulas of equations (2) and (4). This is a logical result as many researchers report practical reached capacities that are less than the theoretical ones as the effect of the mutual coupling has been either excluded or underestimated. Table 1 summarizes the obtained inter-element spacings and the achieved capacities along with those obtained for uniform geometries and the total array lengths for a further measure of comparison.

![Fig.4 Variation of the capacity vs. element spacing for the mutual coupling only model](image)

![Fig.5 Variation of the capacity vs. element spacing for the mutual coupling and spatial correlation model](image)

5 Conclusion
The problem of capacity maximization through inter-element spacing determination in MIMO systems employing non-uniform arrays at both transmitter and receiver was investigated. A novel formula that takes into account both spatial...
correlation and mutual coupling has been proposed and used to evaluate capacity. The employment of Taguchi method guaranteed reaching the global optimum solution within the solution space. The results show that the designed systems outperform their uniform counterparts in terms of capacity. The designed arrays are characterized by a reduction in the overall array size which is beneficial as the space occupied is smaller.

References


Table 1 Resulting inter-element spacings for maximum capacity for both uniform and non uniform geometries.

<table>
<thead>
<tr>
<th>Channel model</th>
<th>Geometry</th>
<th>Ergodic Capacity Value (bps/Hz)</th>
<th>Element spacings (λ)</th>
<th>Total array length (λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial correlation (equation 2)</td>
<td>Uniform</td>
<td>14.54</td>
<td>1.279</td>
<td>5.1160</td>
</tr>
<tr>
<td></td>
<td>Non-Uniform</td>
<td>15.1589</td>
<td>Tx [0.4617 0.3875 1.2025 1.1348]</td>
<td>Rx [1.0463 1.6543 1.4674 1.5279]</td>
</tr>
<tr>
<td>Mutual coupling (equation 4)</td>
<td>Uniform</td>
<td>7.313</td>
<td>0.4559</td>
<td>Tx 3.1865</td>
</tr>
<tr>
<td></td>
<td>Non-Uniform</td>
<td>7.7582</td>
<td>Rx 5.6959</td>
<td></td>
</tr>
<tr>
<td>Both spatial correlation and mutual coupling (equation 13)</td>
<td>Uniform</td>
<td>9.629</td>
<td>1.846</td>
<td>Tx 5.5744</td>
</tr>
<tr>
<td></td>
<td>Non-Uniform</td>
<td>10.043</td>
<td>Rx 3.7784</td>
<td></td>
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</tbody>
</table>