Bayes estimators of Modified Weibull distribution

VASILE PREDA
University of Bucharest
Faculty of Mathematics and Computer Science
Academiei Street, No. 14, 010014, Bucharest
Romania
preda@fmi.unibuc.ro

ALINA CONSTANTINESCU
University "Valahia"
Department of Mathematics
Targoviste
ROMANIA
alinaconsta@yahoo.com

EUGENIA PANAITESCU
"Carol Davila" University of Medicine and Pharmacy
Department of Medical Informatics and Biostatistics
Eroilor Sanitari Street, No.8, 050471, Bucharest
ROMANIA
e.panaitescu@yahoo.com

Abstract: In this paper, we have obtained the Bayes estimators of Modified-Weibull distribution scale and shape parameters using Lindley's approximation (L-approximation) under various loss functions. The proposed estimators have been compared with the corresponding MLE for their risks based on corresponding simulated samples.

Key–Words: Bayesian estimation, Lindley's approximation, Maximum likelihood estimates, Modified Weibull distribution, Monte Carlo simulation.

1 Introduction

The Weibull distribution is one of the most popular widely used models of failure time in life testing and reliability theory. The Weibull distribution has been shown to be useful for modeling and analysis of life time data in medical, biological and engineering sciences.

The three-parameter Modified-Weibull has a distribution function of the form:

$$f(x) = \alpha x^{\beta-1}(\beta+\lambda x)e^{\lambda x-\alpha x^{\beta}e^{\lambda x}} \tag{1}$$

with $x \geq 0$, $\alpha, \beta, \lambda > 0$ and cumulative distribution function

$$F(x) = 1 - e^{-\alpha x^{\beta}e^{\lambda x}}, x \geq 0, \alpha, \beta, \lambda > 0 \tag{2}$$

Here α is the scale parameter, and β and λ are the shape parameters.

For a random sample $x = (x_1, x_2, ..., x_n)$ of size n form (1) the likelihood function is

$$L(\alpha,\beta,\lambda|x) = \prod_{i=1}^{n} f(x_i) = \alpha^n e^{\lambda\sum\limits_{i=1}^{n} x_i - \alpha\sum\limits_{i=1}^{n} x_i^{\beta}e^{\lambda x_i}} \cdot$$

$$\cdot \prod_{i=1}^{n}\left[x_i^{\beta-1}(\beta+\lambda x_i)\right] \tag{3}$$

and taking the logarithm we get

$$l(\alpha,\beta,\lambda|x) = n\ln\alpha + \lambda\sum_{i=1}^{n} x_i - \alpha\sum_{i=1}^{n} x_i^{\beta}e^{\lambda x_i} +$$

$$+(\beta-1)\sum_{i=1}^{n}\ln(x_i) + \sum_{i=1}^{n}\ln(\beta+\lambda x_i) \tag{4}$$

2 Maximum likelihood estimation of the parameters

The maximum likelihood estimate of parameters of the Modified-Weibull distribution is obtained by differentiating the log of the likelihood and equating to zero. The three normal equations thus obtained are given below:

$$\frac{n}{\alpha} - \sum_{i=1}^{n} x_i^{\beta}e^{\lambda x_i} = 0$$

$$-\alpha\sum_{i=1}^{n} x_i^{\beta}\ln(x_i)\cdot e^{\lambda x_i} + \sum_{i=1}^{n}\ln(x_i) + \sum_{i=1}^{n}\frac{1}{\beta+\lambda x_i} = 0$$

$$-\alpha\sum_{i=1}^{n} x_i^{\beta+1}e^{\lambda x_i} + \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\frac{x_i}{\beta+\lambda x_i} = 0$$

But the last two equations are not solvable. Therefore the MLE does not exist in a nice closed form for β and λ.

However, the maximum likelihood estimator of the Modified-Weibull distribution can be obtained by iterative procedures. We propose here to use a bisection or Newton-Raphson method for solving the above-mentioned normal equations which give the MLE of β and λ, $\hat{\beta}_{ML}$ and $\hat{\lambda}_{ML}$.Then, the MLE of α is

$$\hat{\alpha}_{ML} = \frac{n}{\sum_{i=1}^{n} x_i^{\hat{\beta}_{ML}} e^{\hat{\lambda}_{ML} x_i}} \tag{5}$$

3 Bayesian estimation of the parameters

In Bayesian estimation, we consider three types of loss functions. The first is the squared error loss function (quadratic loss) which is classified as a symmetric function and associates equal importance to the losses for overestimation and underestimation of equal magnitude. The second is the LINEX (linear-exponential) loss function which is asymmetric, was introduced by Varian in [14]. These loss functions were widely used by several authors; among of them [11], [1], [7], [12], [13] and [6]. This function rises approximately exponentially on one side of zero and approximately linearly on the other side. The third is the generalization of the Entropy loss used by several authors where the shape parameter c is taken equal to 1. This more general version allows different shapes of the loss function.

The squared error loss (SEL) function is as follows

$$L_{BS}(\phi^*, \phi) \propto (\phi^* - \phi)^2 \tag{6}$$

Under the assumption that the minimal loss occurs at $\phi^* = \phi$, the LINEX loss function (LINEX) for can be expressed as

$$L_{BL}(\Delta) \propto e^{c\Delta} - c\Delta - 1, c \neq 1 \tag{7}$$

where $\Delta = (\phi^* - \phi)$, ϕ^* is an estimate of ϕ. The sign and magnitude of the shape parameter c represents the direction and degree of symmetry, respectively. (If $c > 0$, the overestimation is more serious than underestimation, and vice-versa.) For c close to zero, the LINEX loss is approximately SEL and therefore almost symmetric.

The posterior expectation of the LINEX loss function (7) is

$$E_\phi[L(\phi^* - \phi)] \propto e^{c\phi^*} E_\phi[e^{-c\phi}] - c(\phi^* - E_\phi[\phi]) - 1 \tag{8}$$

where $E_\phi(\cdot)$ denotes the posterior expectation with respect to the posterior density of ϕ. By a result of Zellner in [15], the (unique) Bayes estimator of ϕ, denoted by ϕ^*_{BL} under the LINEX loss function is the value ϕ^* which minimizes (8). It is

$$\phi^*_{BL} = -\frac{1}{c} \ln \left\{ E_\phi[e^{-c\phi}] \right\} \tag{9}$$

provided that the expectation $E_\phi[e^{-c\phi}]$ exists and is finite. The problem of choosing the value of the parameter c is discussed in [2].

The modified Linex loss i.e the General Entropy loss (GEL) is defined as:

$$L_{BE}(\phi^*, \phi) \propto \left(\frac{\phi^*}{\phi}\right)^c - c \log\left(\frac{\phi^*}{\phi}\right) - 1 \tag{10}$$

where ϕ^* is an estimate of parameter ϕ. It may be noted that when $c > 0$, a positive error causes more serious consequences than a negative error. On the other hand, when $c < 0$, a negative error causes more serious consequences than a positive error.

The Bayes estimate ϕ^*_E of under general entropy loss (GEL) is given as

$$\phi^*_{BE} = [E_\phi\{\phi^{-c}\}]^{-\frac{1}{c}} \tag{11}$$

provided that $E_\phi\{\phi^{-c}\}$ exists and is finite. It can be shown that, when $c = 1$, the Bayes estimate (11) coincides with the Bayes estimate under the weighted squared-error loss function. Similarly, when $c = -1$ the Bayes estimate (11) coincides with the Bayes estimate under squared error loss function.

For a Bayesian estimation, we need prior distribution for the parameters α, β and λ. Hence, gamma prior may be taken as the prior distribution for the scale parameter of the Modified-Weibull distribution. It is needless to mention that under the above-mentioned situation, the prior is a conjugate prior. On the other hand, if all the parameters are unknown, a joint conjugate prior for the parameters does not exist. In such a situation, there are a number of ways to choose the priors. We consider the use of piecewise independent priors for all the parameters, namely a non-informative prior for the shape parameters and a natural conjugate prior for the scale parameter (under the assumption that shape parameter is known). Thus the proposed priors for parameters α, β and λ may be taken as

$$g_1(\alpha) = \frac{b^a \alpha^{a-1} e^{-b\alpha}}{\Gamma(a)}, \alpha > 0, a, b > 0 \tag{12}$$

$$g_2(\beta) = \frac{1}{\beta}, \beta > 0 \quad (13)$$

and

$$g_3(\lambda) = \frac{1}{\lambda}, \lambda > 0 \quad (14)$$

respectively, to give the joint prior distribution for λ,β and α as

$$g(\alpha, \beta, \lambda) = \frac{b^a \alpha^{a-1} e^{-b\alpha}}{\beta\lambda\Gamma(a)}, \alpha > 0, \beta, \lambda > 0, a, b > 0 \quad (15)$$

Substituting $L(\alpha, \beta, \lambda|x)$ and $g(\alpha, \beta, \lambda)$ from (3) and (15) respectively we get the joint posterior $P(\alpha, \beta, \lambda|x)$ as

$$P(\alpha, \beta, \lambda|x) = K\frac{\alpha^{n+a-1}}{\beta\lambda} e^{\lambda\sum_{i=1}^{n} x_i - \alpha\left(b + \sum_{i=1}^{n} x_i^{\beta} e^{\lambda x_i}\right)} \cdot$$

$$\cdot \prod_{i=1}^{n} \left[x_i^{\beta-1}(\beta + \lambda x_i)\right] \quad (16)$$

where

$$K^{-1} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \frac{\alpha^{n+a-1}}{\beta\lambda} e^{\lambda\sum_{i=1}^{n} x_i - \alpha\left(b + \sum_{i=1}^{n} x_i^{\beta} e^{\lambda x_i}\right)} \cdot$$

$$\prod_{i=1}^{n} \left[x_i^{\beta-1}(\beta + \lambda x_i)\right] d\lambda d\beta d\alpha$$

It may be noted here that the posterior distribution of (α, β, λ) takes a ratio form that involves an integration in the denominator and cannot be reduced to a closed form. Hence, the evaluation of the posterior expectation for obtaining the Bayes estimator of α, β and λ will be tedious. Among the various methods suggested to approximate the ratio of integrals of the above form, perhaps the simplest one is Lindley's approximation method [5], which approaches the ratio of the integrals as a whole and produces a single numerical result. Thus, we propose the use of Lindley's approximation [5] for obtaining the Bayes estimator of α, β and λ. Many authors have used this approximation for obtaining the Bayes estimators for some lifetime distributions; see among others, [3] and [4].

In this paper we calculate $E(\theta_i|x)$ and $E(\theta_i^2|x)$ in order to find the posterior variance estimates given by $Var(\theta_i|x) = E(\theta_i^2|x) - (E(\theta_i|x))^2$, $i = 1, 2, 3$, where $\theta_1 = \alpha$, $\theta_2 = \beta$, $\theta_3 = \lambda$.

If n is sufficiently large, according to [5], any ratio of the integral of the form

$$I(x) = E[u(\theta_1, \theta_2, \theta_3)|x] =$$

$$= \frac{\int_{(\theta_1,\theta_2,\theta_3)} u(\theta_1, \theta_2, \theta_3) e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)} d(\theta_1, \theta_2, \theta_3)}{\int_{(\theta_1,\theta_2,\theta_3)} e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)} d(\theta_1, \theta_2, \theta_3)}$$

where

$u(\theta) = u(\theta_1, \theta_2, \theta_3)$=function of θ_1, θ_2 or θ_3 only

$L(\theta_1, \theta_2, \theta_3)$=log of likelihood

$G(\theta_1, \theta_2, \theta_3)$=log of joint prior of θ_1, θ_2 and θ_3 can be evaluated as

$I(x) = u(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)+$
$+(u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5)+$
$\frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13})+$
$+B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23})+$
$+C(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})]$

where

$\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ are the MLE of θ_1, θ_2, respective θ_3
$a_i = \rho_1\sigma_{i1} + \rho_2\sigma_{i2} + \rho_3\sigma_{i3}, i = 1, 2, 3$
$a_4 = u_{12}\sigma_{12} + u_{13}\sigma_{13} + u_{23}\sigma_{23}$
$a_5 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22} + u_{33}\sigma_{33})$
$A = \sigma_{11}L_{111} + 2\sigma_{12}L_{121} + 2\sigma_{13}L_{131}+$
$+2\sigma_{23}L_{231} + \sigma_{22}L_{221} + \sigma_{33}L_{331}$
$B = \sigma_{11}L_{112} + 2\sigma_{12}L_{122} + 2\sigma_{13}L_{132}+$
$+2\sigma_{23}L_{232} + \sigma_{22}L_{222} + \sigma_{33}L_{332}$
$C = \sigma_{11}L_{113} + 2\sigma_{12}L_{123} + 2\sigma_{13}L_{133}+$
$+2\sigma_{23}L_{233} + \sigma_{22}L_{223} + \sigma_{33}L_{333}$

and subscripts 1,2,3 on the rigth-hand sides refer to θ_1, θ_2, θ_3 respectively and
$\rho_i = \frac{\partial\rho}{\partial\theta_i}, i = 1, 2, 3, u_i = \frac{\partial u(\theta_1,\theta_2,\theta_3)}{\partial\theta_i}, i = 1, 2, 3$
$u_{ij} = \frac{\partial^2 u(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j}, i, j = 1, 2, 3,$
$L_{ij} = \frac{\partial^2 L(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j}, i, j = 1, 2, 3$
$L_{ijk} = \frac{\partial^3 L(\theta_1,\theta_2,\theta_3)}{\partial\theta_i\partial\theta_j\partial\theta_k}, i, j, k = 1, 2, 3$
and σ_{ij} is the (i, j)-th element of the inverse of the matrix $\{L_{ij}\}$, all evaluated at the MLE of parameters.

For prior distribution (14) we have
$\rho = \ln g(\alpha, \beta, \lambda) = a\ln b + (a-1)\ln\alpha - b\alpha - \ln\beta - \ln\lambda - \ln\Gamma(a)$
and then we get
$\rho_1 = \frac{a-1}{\alpha} - b$, $\rho_2 = -\frac{1}{\beta}$, $\rho_3 = -\frac{1}{\lambda}$.

We can deduce the values of the Bayes estimates of various parameters in what follows.

a) The Bayes estimators under the squared error loss function of the parameters

(I) If $u(\hat{\alpha}, \hat{\beta}, \hat{\lambda}) = \hat{\alpha}$ then

$\hat{\alpha}_{BS} = \hat{\alpha} + \frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{11} - \frac{1}{\beta}\sigma_{12} - \frac{1}{\lambda}\sigma_{13} +$
$+ \frac{A\sigma_{11}+B\sigma_{21}+C\sigma_{31}}{2}$

(II) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\beta}$ then
$\hat{\beta}_{BS} = \hat{\beta} + \frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{21} - \frac{1}{\beta}\sigma_{22} - \frac{1}{\lambda}\sigma_{23} +$
$+ \frac{A\sigma_{12}+B\sigma_{22}+C\sigma_{32}}{2}$

(III) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\lambda}$ then
$\hat{\lambda}_{BS} = \hat{\lambda} + \frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{31} - \frac{1}{\beta}\sigma_{32} - \frac{1}{\lambda}\sigma_{33} +$
$+ \frac{A\sigma_{13}+B\sigma_{23}+C\sigma_{33}}{2}$

b) The Bayes estimators under the Linex loss function of the parameters

(I) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = e^{-c_1\hat{\alpha}}$ then
$\hat{\alpha}_{BL} =$
$\hat{\alpha} + \log\Big[1 - c_1\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{11} - \frac{1}{\beta}\sigma_{12} - \frac{1}{\lambda}\sigma_{13} -$
$+ -\frac{c_1}{2}\sigma_{11} + \frac{A\sigma_{11}+B\sigma_{21}+C\sigma_{31}}{2}\Big)\Big]$

(II) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = e^{-c_2\hat{\beta}}$ then
$\hat{\beta}_{BL} =$
$\hat{\beta} + \log\Big[1 - c_2\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{21} - \frac{1}{\beta}\sigma_{22} - \frac{1}{\lambda}\sigma_{23} -$
$+ -\frac{c_2}{2}\sigma_{22} + \frac{A\sigma_{12}+B\sigma_{22}+C\sigma_{32}}{2}\Big)\Big]$

(III) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = e^{-c_3\hat{\lambda}}$ then
$\hat{\lambda}_{BL} =$
$\hat{\lambda} + \log\Big[1 - c_3\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{31} - \frac{1}{\beta}\sigma_{32} - \frac{1}{\lambda}\sigma_{33} -$
$+ -\frac{c_3}{2}\sigma_{33} + \frac{A\sigma_{13}+B\sigma_{23}+C\sigma_{33}}{2}\Big)\Big]$

c) The Bayes estimators under the general entropy loss function of the parameters

(I) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\alpha}^{-c_1}$ then
$\hat{\alpha}_{BE} =$
$\Big[\hat{\alpha}^{-c_1}\Big[1 - \frac{c_1}{\hat{\alpha}}\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{11} - \frac{1}{\beta}\sigma_{12} - \frac{1}{\lambda}\sigma_{13} -$
$-\frac{c_1+1}{2\hat{\alpha}}\sigma_{11} + \frac{A\sigma_{11}+B\sigma_{21}+C\sigma_{31}}{2}\Big)\Big]\Big]^{-\frac{1}{c_1}}$

(II) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\beta}^{-c_2}$ then
$\hat{\beta}_{BE} =$
$\Big[\hat{\beta}^{-c_2}\Big[1 - \frac{c_2}{\hat{\beta}}\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{21} - \frac{1}{\beta}\sigma_{22} - \frac{1}{\lambda}\sigma_{23} -$
$-\frac{c_2+1}{2\hat{\beta}}\sigma_{22} + \frac{A\sigma_{12}+B\sigma_{22}+C\sigma_{32}}{2}\Big)\Big]\Big]^{-\frac{1}{c_2}}$

(III) If $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\lambda}^{-c_3}$ then
$\hat{\lambda}_{BE} =$
$\Big[\hat{\lambda}^{-c_3}\Big[1 - \frac{c_3}{\hat{\lambda}}\Big(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{31} - \frac{1}{\beta}\sigma_{32} - \frac{1}{\lambda}\sigma_{33} -$
$-\frac{c_3+1}{2\hat{\lambda}}\sigma_{33} + \frac{A\sigma_{13}+B\sigma_{23}+C\sigma_{33}}{2}\Big)\Big]\Big]^{-\frac{1}{c_3}}$

Also, let $u(\hat{\alpha},\hat{\beta},\hat{\lambda}) = \hat{\alpha}^2$. Then
$E(\hat{\alpha}^2|x) = \hat{\alpha}^2 + 2\hat{\alpha}(\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{11} - \frac{1}{\beta}\sigma_{12} - \frac{1}{\lambda}\sigma_{13}) +$
$+\sigma_{11} + \hat{\alpha}\frac{A\sigma_{11}+B\sigma_{21}+C\sigma_{31}}{2}$

Hence the posterior variances are given by
$Var(\hat{\alpha}|x) = E(\hat{\alpha}^2|x) - (E(\hat{\alpha}|x))^2 =$
$=\sigma_{11} -$
$\Big[\frac{a-1-b\hat{\alpha}}{\hat{\alpha}}\sigma_{11} - \frac{1}{\beta}\sigma_{12} - \frac{1}{\lambda}\sigma_{13} - \frac{A\sigma_{13}+B\sigma_{23}+C\sigma_{33}}{2}\Big]^2 <$
$< \sigma_{11} = Var(\hat{\alpha})$

Similary $Var(\hat{\beta}|x) < \sigma_{22} = Var(\hat{\beta})$ and
$Var(\hat{\lambda}|x) < \sigma_{33} = Var(\hat{\lambda})$

4 Numerical Findings

The estimators $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\lambda}$ are maximum likelihood estimators of the parameters of the Modified-Weibull distribution; whereas $\hat{\alpha}_{BS}$, $\hat{\alpha}_{BL}$, $\hat{\alpha}_{BG}$, $\hat{\beta}_{BS}$, $\hat{\beta}_{BL}$, $\hat{\beta}_{BG}$, and $\hat{\lambda}_{BS}$, $\hat{\lambda}_{BL}$, $\hat{\lambda}_{BG}$ are Bayes estimators obtained by using the L-approximation for squared error, Linex and general entropy loss function respectively. As mentioned earlier, the maximum likelihood estimators and hence risks of the estimators cannot be put in a convenient closed form.

Therefore, risks of the estimators are empirically evaluated based on a Monte-Carlo simulation study of samples. A number of values of unknown parameters are considered. Sample size is varied to observe the effect of small and large samples on the estimators. Changes in the estimators and their risks have been determined when changing the shape parameter of loss functions while keeping the sample size fixed.

Different combinations of prior parameters α, β and λ are considered in studying the change in the estimators and their risks. The results are summarized in following tables.

It is easy to notice that the risk of the estimators will be the function of sample size, population parameters, parameters of the prior distribution (hyper parameters), and corresponding loss function parameters.

In order to consider the wide variety of values, we have obtained the simulated risks for sample sizes N=20, 40, 60 and 100.

The various values of parameters of the distribution considered are scale parameter α=0.2 (.3) 1.4, shape parameters β=0.6 (.2) 1.2, λ=0.2 (.3) 1.4, and loss parameter $c_i = \pm 1.5, \pm 1.1, \pm 0.5$ and 0.1 with $i = 1, 2, 3$.

Prior parameters a and b are arbitrarily taken as 1 respectively 2. After an extensive study of the results thus obtained, conclusions are drawn regarding the behavior of the estimators, which are summarized below. It may be mentioned here that because of space restrictions, all results are not shown in the tables. Only a few are presented here to demonstrate the effects found and the conclusion drawn.

However, in most of the cases, the proposed Bayes estimator is better than the Maximum Likelihood Estimator (MLE).

Table 1: The effect of sample size on risks of the estimator of α when true vales are $\alpha = 1.2$, $\beta = 0.8$, $\lambda = 0.7$.

		20	40	60	100
	α_{ML}	0.279	0.132	0.103	0.083
	α_{BS}	0.481	0.330	0.243	0.035
c=-0.5	α_{BL}	0.210	0.125	0.111	0.050
c=0.5	α_{BL}	0.824	0.363	0.198	0.154
c=1	α_{BL}	0.322	0.149	0.109	0.094
c=-0.5	α_{BG}	0.794	0.483	0.316	0.035
c=0.5	α_{BG}	23.18	3.157	0.743	0.050
c=1	α_{BG}	2.473	0.998	0.489	0.041

Table 2: The effect of sample size on risks of the estimator of β when true vales are $\alpha = 1.2$, $\beta = 0.8$, $\lambda = 0.7$.

		20	40	60	100
	β_{ML}	0.062	0.019	0.018	0.011
	β_{BS}	0.034	0.036	0.032	0.006
c=-0.5	β_{BL}	0.028	0.016	0.019	0.007
c=0.5	β_{BL}	0.149	0.045	0.027	0.019
c=1	β_{BL}	0.074	0.022	0.019	0.012
c=-0.5	β_{BG}	0.054	0.046	0.036	0.006
c=0.5	β_{BG}	0.463	0.116	0.051	0.007
c=1	β_{BG}	0.146	0.072	0.044	0.007

Table 3: The effect of sample size on risks of the estimator of λ when true vales are $\alpha = 1.2$, $\beta = 0.8$, $\lambda = 0.7$.

		20	40	60	100
	λ_{ML}	0.554	0.201	0.131	0.099
	λ_{BS}	0.267	0.149	0.122	0.042
c=-0.5	λ_{BL}	0.233	0.111	0.103	0.058
c=0.5	λ_{BL}	0.891	0.332	0.185	0.142
c=1	λ_{BL}	0.628	0.227	0.141	0.108
c=-0.5	λ_{BG}	0.217	0.111	0.099	0.045
c=0.5	λ_{BG}	0.235	0.092	0.081	0.054
c=1	λ_{BG}	0.221	0.096	0.086	0.050

Table 4: The effect of variation of α on risks of the estimator of α when true vales are $\beta = 0.8$, $\lambda = 0.7$.

True value		0.2	0.5	0.8	1.1
	α_{ML}	0.005	0.023	0.069	0.111
	α_{BS}	0.010	0.025	0.051	0.070
c=-0.5	α_{BL}	0.007	0.019	0.046	0.063
c=0.5	α_{BL}	0.005	0.038	0.127	0.246
c=1	α_{BL}	0.005	0.025	0.077	0.128
c=-0.5	α_{BG}	0.012	0.030	0.062	0.091
c=0.5	α_{BG}	0.021	0.070	0.184	0.302
c=1	α_{BG}	0.016	0.045	0.098	0.163

Table 5: The effect of variation of β on risks of the estimator of β when true vales are $\alpha = 0.5$, $\lambda = 0.5$.

True value		0.6	0.8	1	1.2
	β_{ML}	0.028	0.036	0.065	0.083
	β_{BS}	0.037	0.042	0.055	0.064
c=-0.5	β_{BL}	0.030	0.033	0.051	0.057
c=0.5	β_{BL}	0.028	0.047	0.097	0.164
c=1	β_{BL}	0.028	0.037	0.070	0.095
c=-0.5	β_{BG}	0.041	0.048	0.063	0.065
c=0.5	β_{BG}	0.055	0.072	0.105	0.121
c=1	β_{BG}	0.048	0.060	0.081	0.085

Table 7: The effect of variation of λ on risks of the estimator of λ when true vales are $\alpha = 0.7$, $\beta = 0.8$.

True value		0.2	0.5	0.8	1.1
	λ_{ML}	0.012	0.046	0.091	0.187
	λ_{BS}	0.007	0.043	0.124	0.103
c=-0.5	λ_{BL}	0.007	0.040	0.087	0.075
c=0.5	λ_{BL}	0.018	0.066	0.139	0.359
c=1	λ_{BL}	0.013	0.049	0.099	0.222
c=-0.5	λ_{BG}	0.218	0.027	0.085	0.075
c=0.5	λ_{BG}	3.550	13.17	2.327	0.064
c=1	λ_{BG}	0.025	0.023	0.061	0.064

Table 8: Effect of loss parameters on risks of the estimator of α, β, λ when true vales are $\alpha = 1.2$, $\beta = 0.8$, $\lambda = 0.7$.

	BL			BG		
c	α	β	λ	α	β	λ
-1.5	0.060	0.008	0.303	0.067	0.012	0.064
-1.1	0.059	0.010	0.082	0.070	0.010	0.056
-0.5	0.076	0.020	0.067	0.084	0.009	0.055
0.1	0.145	0.039	0.144	0.122	0.008	0.059
0.5	0.258	0.060	0.201	0.182	0.008	0.062
1.1	0.705	0.107	0.269	0.478	0.010	0.070
1.5	1.633	0.153	0.297	1.708	0.017	0.075

5 Conclusion

The performance of the proposed Bayes estimators has been compared to the maximum likelihood estimator. On the basis of these results, we may conclude that for positive c, i.e., overestimation is more serious than underestimation, the Bayes estimators SEL and GEL of λ performs better than the corresponding maximum likelihood estimator and the Bayes estimators LINEX. The maximum likelihood estimators and Bayes estimators SEL of α and β are better for small and moderate sample sizes; whereas risks of the Bayes

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estimator GEL of all parameters population perform better than any of the estimators for a very large sample sizes.

For negative c, the Bayes estimators LINEX and he maximum likelihood estimators of α and β performs better than the Bayes estimators SEL and GEL. For parameter λ, the Bayes estimators LINEX and GEL are better for small and moderate sample sizes; whereas risks of the Bayes estimators GEL and LINEX of all parameters of population perform better than any of all the estimators for a very large sample sizes.

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