Methods for the Optimization of Some Functions of Interpolation with Application for Modelling of the Thermal Field in Machining Processes

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Abstract: - In this article, there are mainly presented two methods for the optimization of some functions of interpolation. A first method is that of minimizing functional operators, thus drawing conclusions on the importance of knots where the measurements are made, therefore allowing the number of knots to be reduced. A second method is that of genetic algorithms which is used to optimize an objective function, starting from a population of codifications of possible solutions for the problem, and where probabilistic transition rules are used instead of determinist ones.

Key-Words - Functional operator, genetic algorithm, spline functions, optimization, function of interpolation, objective function, genetic operators, thermal field, modelling.

1 Introduction
In practice, it has been established that the values measured during an experiment are not exact and, therefore, one can state that the function which interpolates the event depends on these values. Focussing on minimizing certain operators, from the necessary condition of extreme for a function with many variables, conclusions can be drawn on the importance of knots where the measurements are made, thus allowing their number to be reduced and, therefore, also reducing the necessary sensors for the experiment.[4],[5]

The genetic algorithms can be successfully used together with the methods of interpolation, regardless of dimensionality, in situations where the number of points of interpolation is relatively high and only the selection of a representative subset is desired. This subset of points is selected so as the error obtained using the entire multitude of points of interpolation when evaluating to be minimum.

Genetic algorithms are at the basis of a research direction specific to evolutionary calculus, a field which uses mathematical theories inspired from the biological paradigm of the evolution of life for fundamenting some search and optimization methods.[3]

2 Problem formulation
Optimizing some interpolating functions has the purpose of reducing the number of point’s knots where the measuring and implicitly the number of necessary sensors are made.

The first step by means of minimizing some form operators \( \int f^{(k)}(t)^2 dt \) (for the interpolating functions a single variable) \( \int \left( \frac{\partial^2 x}{\partial u^a \partial v^a} \right)^2 du dv \) where

\[ a_1 + a_2 = 2 \] (for the interpolating functions of two variables) or \( \int \int \left( \frac{\partial^a x}{\partial u^a \partial v^a} \right)^2 du dv dw \), unde

\[ a = a_1 + a_2 + a_3 \] (for the interpolating functions of three variables) we can obtain useful information on the importance of points.

The secondly by means of the genetic algorithm method those points considered to be the most representative are obtained.
3 Problem Solution

The use of spline functions as interpolating functions is based on certain linear convex combinations that ensure the counting, stability of the process. Considering the spline function as a function of values measured in knots, taking into account their extreme values, information about the importance of knots where the measuring is being made is obtained. By means of genetic algorithms, the more influential points are therefore subsequently determined.

3.1 Minimizing Some Functional Operators

Considering \( f : [a, b] \to \mathbb{R} \) as a function of interpolation with one variable, on the set of knots \( (t_i)_{i=1}^n \), \( f(t_i) = d_i, \forall i = 1, n \) and \( d = (d_1, d_2, ..., d_n) \) - the vector of control points. The goal is mainly to minimize the following homogeneous system is obtained:

\[
\int f^{(s)}(t) \, dt.
\]

Using a B-spline function as a function of interpolation, given by:

(1) \( f(t) = \sum_{i=1}^n d_i N_{i,k}(t) \), where \( N_{i,k}(t) \) - are basic B-spline functions recursively defined through:

(2) \( N_{i,1}(t) = \begin{cases} 1, & t \in [t_i, t_{i+1}] \\ 0, & \text{in rest} \end{cases} \)

(3) \( N_{i,k}(t) = \frac{t-t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t), \)

\( k \geq 2 \).

If it is considered:

(4) \( N_i(t) = (N_{i,k-1}(t), N_{i,k-1}(t), ..., N_{i-n+k-1}(t)), i = 0, n \)

then from the necessary conditions of extreme the following homogeneous system is obtained:

(5) \( A \cdot d = 0, d = (d_1, d_2, ..., d_n) \), where:

(6) \( A = \left( \int N_i^{(s)}(t) N_j^{(s)}(t) \, dt \right)_{i,j=1,n} \).

Conclusions can be drawn on the more influential (important) points of the measurements from studying system (5).

Using the same method, the following system is obtained for the second operator:

(9) \( M_1'' \cdot dM_1' = 0 \), where:

(10) \( M_1'' = (D_u^1)^T A''_1 D_v^1; M_1' = (D_v^1)^T A'_1 D_v^1 \)

(11) \( A''_1 = \left( \int \frac{\partial}{\partial u} \left( N_i^j(u) \right) \frac{\partial}{\partial u} \left( N_i^j(u) \right) du \right)_{i,j=1,m} \)

\( A'_1 = \left( \int \frac{\partial}{\partial v} \left( N_i^j(v) \right) \frac{\partial}{\partial v} \left( N_i^j(v) \right) dv \right)_{i,j=1,n} \)

(12) \( D_u^1 = \left( \frac{\partial d}{\partial u} \right)_{j=1,m} \); \( D_v^1 = \left( \frac{\partial d}{\partial v} \right)_{j=1,n} \).

As an analogy, the other two operators are minimized. Considering \( x \), a function of interpolation with three variables on the set of knots \( (u_i, v_j, w_k)_{i=1,m; j=1,n; k=1,p} \); \( x(u_i, v_j, w_k) = d_{ijk} \) and \( d = (d_{ijk})_{i=1,m; j=1,n; k=1,p} \), the matrix of control points. This function can be regarded as the tensorial product of a function with three
variables or the tensorial product of a function with one variable together with a function with two variables. In this case, the goal is to minimize certain operators such as:

$$\int \int \int \left( \frac{\partial^r x}{\partial u^{a_1} \partial v^{a_2} \partial w^{a_3}} \right)^2 du dv dw, \text{where } a = a_1 + a_2 + a_3 \text{ related to the elements of (matrix) d.}$$

For the operator $$\int \int \int (x(u, v, w))^2 du dv dw$$, from the necessary conditions of extreme, the following system is obtained:

(13) $$A_{i,j} d A_{j,k} A_{i,j} A_{k,w} = 0$$, where $$\bar{A}_i$$ represents the product of contraction of coefficient $$i$$.

Generally, the following system is obtained:

(14) $$M_{a_1}^r \bar{A}_i d \bar{A}_j M_{a_2}^r \bar{A}_k M_{a_3}^r = 0$$.

### 3.2 Genetic Algorithms Method

The genetic algorithms method is used for the optimization of an objective function starting from a population of codifications of possible solutions for the problem and using probabilistic transition rules instead of determinist ones, tom ove around in the space of solutions towards populations that contain the most appropriate solution.

A solution for the problem of optimization is represented as a vector called individual (cromozom), the elements of which are called genes.

To solve the problem of optimization, the genetic algorithms use a series of genetic operators which act upon the population, the cromozoms or the genes:

- **selection** - chooses individuals from the population in order to reproduce their characteristics;
- **crossing** - creates new members (offspring, successors) by combining alternative parts from two or more parents;
- **inversion** - inverses the order of the genes between two points from a cromozome;
- **mutation** - modifies randomly certain genes from the cromozom.

These operators allow the scanning of the space of solutions, the identification of the best solutions, and the spreading of their characteristics (genes) from one generation to another, until the desired solution is obtained, the one that optimizes (maximizes or minimizes) the objective function.

Two steps must be covered when using genetic algorithms. Firstly, the method for codifying the solutions for the problem and the function of evaluation (measuring) of the quality of every cromozom (every solution) are established. Then successive populations are generated until the most appropriate solution is obtained or until a definite number of evolutions are covered, evaluating the cromozoms from the population at every stage and applying the genetic operators for the selection and reproduction of the cromozoms elected using genetic operators. In a nut shell, this second step can be described as follows [Dumitrescu & Costin, 1996]:

**Algorithm.** The pseudocode of the genetic algorithms

**Step 1.** Initiates the population of cromozoms

**Step 2.** Evaluates every member of the population

**Step 3.** Repeats until the condition to stop is fulfilled

3.1. Selects the most suitable individuals for reproduction

3.2. Creates a new generation using genetic operators for crossing and/or mutation

3.3. Evaluates the new members of the population and replaces the weak members of the initial population with the members of the new generations.

**Observation.** The condition to stop the genetic algorithm (Step 3) can be dictated by the fact that the time to generate the new generations has expired, by a limited number of obtained generations or by a certain value of the objective function.

Genetic algorithms can be successfully used together with the methods of interpolation, regardless of the dimensionality, in the situations in which the number of points of interpolation is relatively high and only the selection of a representative subset is desired. This subset of points is selected so as the error obtained, using at the evaluation the entire multitude of points of interpolation, to be minimum.

Considering $$f : [a, b] \rightarrow \mathbb{R}$$ and the points of interpolation $$\Delta : a = x_1 < x_2 < \ldots < x_n = b$$, $$y_i = f(x_i) \forall i = 1 \ldots n$$, the method of the genetic algorithms can be applied for the selection of a subset of points from $$\Delta$$.

$$\Delta_{\min} : x_{i_1} < x_{i_2} < \ldots < x_{i_m} \text{ m < n, } i_1, \ldots, i_m \in \{1, \ldots, n\} \text{ so as the function } F \text{ which interpolates } f \text{ in the points of the subset above, to minimize the error:}$$

$$\sum_{i=1}^{n} (F(x_i) - f(x_i))^2 \text{ a min.}$$

### 3.3 Application

Modelling a thermal field from a hot-worked part

To carry out this experiment, the case of a hot-worked cylindrical part $$\Phi 60 \times 400 \text{ mm, made of OLC 45 steel,}$$

...
caught between the chuck and the tip of the lathe tailstock spindle was studied.

The thermal field was visualised with an infrared camera FLIR Systems ThermoVision A20M, having the following important characteristics:
- The measuring field: -20...+900°C;
- The thermal sensitiveness < 0.1°C;
- The image frequency: 50Hz;
- Detector: FPA microbolometer without cooling;
- Spectrum: 7.5-13 um;
- Measuring temperature: Spot, Area, Isotherm, Delta T;
- The resolution of the detector: 160x120 pixels;
- Analogical interface: PAL (standard), 0-5 V (temperature arie/spot);
- Video digital interface: Ethernet or FireWire;
- Control interfaces: RS232, FireWire/Ethernet;
- Interchangeable objective: 17 mm standard.

The acquisition and processing of the thermograms was achieved with the help of the specialised software ThermaCAM Researcher Professional.

The process of cutting was continuously filmed, afterwards a representative frame was selected (Fig. 1), for which the temperatures along the centre line of the processed part were determined, being considered as the center line; thus resulted the values from table 1.

Based on the values above, the modeling of the thermal field was achieved, using B-spline functions, in two ways: using the measured values in all the 61 points (the blue curve from Fig. 2) and using the measured values in only 30 points, selected through the genetic algorithms method as being the most representative (the red curve from image 2).
4 Conclusions
Even though in some practical problems it was observed that the method of genetic algorithms tends to converge towards a local optimum or that the time to obtain the solution is superior to other optimization algorithms, the rapidity and simplicity with which it locates a population of „good” solutions even for difficult search spaces, makes it many times more preferable in practical applications.

In the case of minimizing some functional operators, homogeneous systems are obtained from the necessary conditions of extreme. When analysing these homogeneous systems, a multitude of susceptible points are deducted as being more important (this condition is not also sufficient).

The practical experiment carried out shows that the methods of optimization presented can be successfully used for the interpolation of unidimensional fields or fields with many dimensions, this leading to the development of intelligent command systems.

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