The Density Altitude. Influence Factors and Evaluation

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Abstract: - The paper presents a method to determinate the density altitude with an electronic flight instrument system. A brief review of the flight altitudes is performed, and the calculus relations of the density altitude are developed. The first two atmospheric layers (0÷11 Km and 11÷20 Km) are considered. For different indicated barometric altitudes an evaluation of the density altitude, as a function of non-standards temperature variations and of dew point value, is realized.

Key-Words: - standard atmosphere; atmospheric layers; barometric formula; density-altitude; evaluation

1 Introduction
Regarded as one of the most important parameters that must known by the pilot during the flight, the altitude is defined as the distance between the centre of mass of the aircraft and the corresponding point on the surface of the Earth, considered by the vertical ground [1]. In general, at the board, the altitude is measured directly using the altimeter, but can be also calculated by means of complementary systems, such as, for example, the Air Data Computer (ADC), the Inertial Navigation System (INS) or Global Positioning System (GPS). Showing altitude information on board can be made directly by the measuring system or through an Electronic Flight Instrument System (EFIS). If it is determined using a GPS than it can be defined as the aircraft elevation from the reference geoid WGS 84 surface (World Geodetic System) [2]. Relative to the position of the ground point taken as a reference or to the corrections introduced in the measuring system, the flight altitude behaves different names [3]:
- true altitude ($H_{nm}$): is real altitude of aircraft above
mean sea level (MSL);

• relative altitude \( (H_r) \): is the altitude reported at an airfield level on which the aircraft performs take off or landing manoeuvres; its value depends on the altimeter adjustment.

• absolute altitude \( (H_a) \): in this case the reference point is considered at the intersection of the local vertical and the Earth surface, so its calculation takes into account the overflowed forms of relief. This altitude is considered to have the greatest importance for flight safety.

• barometric altitude (pressure altitude) \( (H_b) \): this is the altitude indicated by the altimeter when this is tuned on a basic pressure, so the reference point is positioned on a baric surface. If the reference pressure is 760 mmHg it corresponds to the mean sea level in the International Standard Atmosphere (ISA) [4].

• density altitude \( (H_d) \): is the barometric altitude corrected for non-standard temperature variations. This corresponds to the altitude at which the density of air is equal to ISA air density evaluated for the current flight conditions.

• indicated altitude \( (H) \): is the altitude displayed on the dashboard.

One of the altitudes of great importance, calculated from the barometric altitude, is the density altitude \( (H_d) \). Strongly influenced by changes in temperature and to a lesser extent by changes in air humidity, the real air density can provides to the pilot vital information for the flight safety. It is known that the density of air is the most important factor influencing the performances of aircraft both at the lift forces level, and at the thrust forces level generated by the propulsion systems. Density altitude is such a simple way to give the pilot information on air density, this parameter being calculated such as the pilot can make a coarse assessment of aircraft performance in the current flight conditions [5]. For example, the increase of the temperature or of the humidity of the outdoor environment in which is operated aircraft lead to lower air density [5], [6], which are as a consequence (as defined by the density altitude) the indication of a density altitude higher than the current indicated barometric altitude.

Therefore, the pilot should expect the following effects on the aircraft [5], [6], [7]:

• lift decreasing and the need to increase the climb speed to maintain the desired lift;

• decreasing of the propulsion systems power, and, thus, decreasing of the thrust forces;

• runway acceleration decreasing because of the low thrust;

• increasing of the take off distance and decreasing of the climb speed;

• decreasing of the aircraft maximum flight altitude, affecting their capacity to fly above the higher obstacles.

\section{2 Vapor Pressure Determination}

The density of the atmospheric air, considered dry, can be calculated with the relation

\[ \rho = \frac{p \mu}{RT} , \]  \hfill (1)

the static pressure \( p \) being measured directly. In relation (1) \( T \) is the air temperature, \( \mu \) - molecular mass of the air \((\mu = 0.0289644 \text{ kg/mol})\), and \( R \) is the gas universal constant \((R = 8.31432 \text{ N·m/(mol·K)})\). In real conditions, this density is affected by the humidity in the air, which is considered to be a mixture of dry air molecules and water vapor. So, the measured barometric pressure is in fact the mixture pressure not the dry air pressure. Thus, to determinate the density of the dry air the Dalton partial pressures law must be used. In this way, the mixture pressure is expressed as the sum of the dry air and water vapor \( p \), pressures

\[ p_m = p + p_v. \]  \hfill (2)

It results

\[ p = p_m - p_v, \]  \hfill (3)

and the relation (1) for the density calculation becomes

\[ \rho = (p_m - p_v) \mu / (RT) . \]  \hfill (4)

As a consequence, the dry air density calculation supposes to know the actual air pressure (the mixture pressure \( p_m \)), the water vapor pressure \( p_v \), and the local temperature \( T \) of the atmospheric air. The Mixture pressure and the temperature of local atmospheric air is determined relatively easily by placing the sensors outside the aircraft. Instead, the vapor pressure determination involves the performing of numerical calculations using information on relative humidity and dew point [8].

According to [8], two relations often used in the calculation of vapor pressure from the dew point:

a) The first relation shows a high accuracy and is based on a polynomial development

\[ p_v = \frac{C}{(c_1 + t)(c_1 + t)(c_1 + t)(c_1 + t)(c_1 + t)} \]  \hfill (5)

where \( t \) is the dew point expressed in °C, \( C = 610.78, c_1 = 0.99999683, c_1 = -0.90826951 \times 10^{-2}, c_2 = 0.78736169 \times 10^{-4}, c_3 = -0.61117958 \times 10^{-6}, c_4 = 0.4388418 \times 10^{-8}, c_5 = -0.29883895 \times 10^{-10}, c_6 = 0.2187442 \times 10^{-12}, c_7 = -0.1789232 \times 10^{-14}, c_8 = 0.1111251 \times 10^{-16}, c_9 = 0.03994571 \times 10^{-19}\).

b) The second relation, much easier, is generally used in applications that require a lower accuracy
\[ p_v = C \cdot 10^{(t/(c_1+t))} , \] (6)

with \( C = 610.78, c_1 = 7.5 \) and \( c_2 = 237.3 \). For both relations the pressure \( p_v \) measure unit is Pascal.

In Fig. 1 are represented graphically the dependences \( p_v = f(t) \) between vapor pressure and dew point, given by the relation (5), respectively the difference \( \varepsilon \) between the pressure \( p_v \) values calculated with relations (5) and (6). To be more suggestive the characteristics were designed for pressures represented in mmHg.

Dependence \( p_v = f(t) \) analysis combined with the equation (4) confirming earlier observations that the most disadvantageous circumstances for flying are those with high humidity (positive dew point). For these cases the vapor pressure can reach up to 42 mmHg, the growth being exponential. On the other hand, on observes that the differences between the two methods of calculation are very small (up to 0.012 mmHg). Analyzing both graphs shows that for positive dew points can be used without problems relation (6) in high accuracy applications. Instead, in such applications, for negative values of dew point, especially for those below \(-20^\circ C\), it is necessary to use formula (5) because the relative error caused by the simplified formula is high, reaching up to 4%.

![Graph](image)

**Fig. 1 Evaluation of formulas for calculating the vapor pressure**

Once pressure \( p_v \) determined, with formula (4) can be calculated the dry air density. Equalling the calculated density with the density of air in ISA is equivalent to combine the relation (1) with the formulas that give the humidity variations (between \(-50^\circ C\) and for different values of dew point \( T_o\)). Representing graphically the density altitude for the atmospheric layer 0, the following equations system is obtained

\[ T = T_o + \tau_o (H_b - H_{b0}), \] (9)

\[ p = p_o \left( 1 + \tau_o (H_b - H_{b0}) / T_o \right)^{-\mu p / (R T_o)}, \] (8)

Successively eliminating the temperature and the pressure between the three equations is obtained the formula for calculating the density altitude in the form (\( H_b \) becomes \( H_o \))

\[ H_o = H_{b0} - \left( T_o / \tau_o \right) \left( 1 - \left( p_o T_o / (R \mu) \right)^{R \mu / (p_o + R T_o)} \right), \] (10)

which, numerically, is

\[ H_o = 4.433076 \cdot 10^4 \cdot [1 - 0.953434 \cdot \rho^{0.234969}]. \] (11)

In previous relation the density values are entered in kg/m³, and the resulting density altitude values are in m. Representing graphically the density altitude for the change \( \Delta t \) of environmental temperature outside the aircraft (in the range \(-30^\circ C\) to \(-30^\circ C\) from the standard values in ISA) and for different values of dew point \( t \) (between \(-50^\circ C\) to \(-35^\circ C\)), the three-dimensional characteristics in Fig. 2 are obtained. These were presented for four values of air pressure considered to be in the mixture with water vapor \( p_{am} \). These values correspond to altitude readings on barometric altimeter equals to 0m, 3000m, 6000m and 9000m.

![Graphs](image)

**Fig. 2 Density altitude for layer 0 at the temperature and humidity variations**

### 3 Density Altitude Evaluation for the Atmospheric Layer 0

For atmospheric layer 0 (0÷11 km), the relation which gives the dependence of the temperature by the altitude is [1]

\[ T = T_o + \tau_o (H_b - H_{b0}), \] (7)

where \( T_o = 15^\circ C = 288.15K \) (temperature at the lower limit of the layer 0), \( \tau_o = -6.5^\circ C/km \) (temperature gradient of the layer 0), and \( H_{b0} = 0 km \) (altitude at the lower limit of the layer 0). In this situation, the barometric formula, that gives the dependence of the static pressure \( p \) by the altitude \( H_b \) is expressed under the form [1]

\[ p = p_o \left( 1 + \tau_o (H_b - H_{b0}) / T_o \right)^{-\mu p / (R T_o)}, \] (8)

\( p_o = 101325 N/m^2 = 760 mmHg \) is the static pressure for the altitude \( H_{b0} \). So, for the atmospheric layer 0, the following equations system is obtained

\[ \rho = p \mu / R T, \]

\[ T = T_o + \tau_o (H_b - H_{b0}), \]

\[ p = p_o \left( 1 + \tau_o (H_b - H_{b0}) / T_o \right)^{-\mu p / (R T_o)}. \] (9)
The maximum and the minimum values of the density altitude, calculated in the simulated conditions, are presented in Table 1.

Table 1 Maximum and minimum values of the density altitude for the conditions simulated in Fig. 2

<table>
<thead>
<tr>
<th>No.</th>
<th>Indicated barometric altitude [m]</th>
<th>Maximum value of the density altitude [m]</th>
<th>Minimum value of the density altitude [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1596.8135</td>
<td>-1159.4360</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>4799.5158</td>
<td>1834.7880</td>
</tr>
<tr>
<td>3</td>
<td>6000</td>
<td>8106.6341</td>
<td>4828.0847</td>
</tr>
<tr>
<td>4</td>
<td>9000</td>
<td>11595.6104</td>
<td>7820.2263</td>
</tr>
</tbody>
</table>

From Fig. 2 one observe that the maximum values are obtained for \( \Delta t=30^\circ C \) and \( t=35^\circ C \) and the minimum values for \( \Delta t=-30^\circ C \) and \( t=-50^\circ C \). Consistent with the values in Table I, from Fig. 2 can be remarked also a growing of the area curvature with the increasing of the indicated barometric altitude. According to the tabled maximum and minimum values, the difference between the density altitude and barometric altitude for this layer may even exceed 2500 m superposing certain conditions of temperature and humidity. Successively neglecting the effects of humidity and of non-standard change of environment temperature on density altitude, for the four previously considered cases, for the indicated barometric altitude, result the characteristics in Fig. 3 a., respectively in Fig. 3 b.

![Fig. 3 The evaluation of the influences of the non-standard temperature variations and humidity on the density altitude in the layer 0](image)

Fig. 3 The evaluation of the influences of the non-standard temperature variations and humidity on the density altitude in the layer 0

### 4 Density Altitude Evaluation for the Atmospheric Layer 1

For atmospheric layer 1 (11÷20 km), the relation which gives the dependence of the temperature by the altitude is [1]

\[
T = T_i + \tau_o (H_b - H_{b1}) = T_i, \quad (12)
\]

where \( T_i = -56.5^\circ C = 216.66 \text{K} \) (temperature at the lower limit of the layer 1), \( \tau_o = 0^\circ C/\text{km} \) (temperature gradient of the layer 1), and \( H_{b1} = 11 \text{km} \) (altitude at the lower limit of the layer 1). In this situation the barometric formula becomes [1]

\[
p = p_1 e^{-\frac{\tau_o (H_b - H_{b1})}{RT_i}}, \quad (13)
\]

\( p_1 = 22632.1 \text{N/m}^2 = 169.75 \text{mmHg} \) is the static pressure for the altitude \( H_{b1} \). Starting from the relations (1), (12) and (13), the following equations system is obtained

\[
\rho = \frac{p \mu}{RT},
\]

\[
T = T_i = -56.5^\circ C = 216.66 \text{K}, \quad (14)
\]

\[
p = p_1 e^{-\frac{\tau_o (H_b - H_{b1})}{RT_i}}. \quad (15)
\]

Successively eliminating the temperature and the pressure between the three equations is obtained the formula for calculating the density altitude in the form \( (H_b \text{ becomes } H_p) \)

\[
H_p = H_{b1} - (RT_i/\mu g) \ln[p RT_i/(\mu p_1)]. \quad (16)
\]

The density is entered in kg/m³, and the values of the density altitude results in m.

Considering the same values for the non-standard variation \( \Delta t \) of the environmental temperature outside the aircraft (-30°C÷30°C), the variation interval -50°C÷35°C for the dew point \( t \) at altitudes under 16000 m, and the variation interval -50°C÷25°C for the dew point \( t \) at altitudes over 16000 m, result the three-dimensional characteristics in Fig. 4. For the high altitudes in this layer the dew point values higher than 25°C temperature are irrelevant physically because in these situations the calculated vapor pressure becomes greater than the pressure of the mixture. The characteristics were presented for four values of air pressure considered to be in the mixture with water vapor \( (p_{\text{mix}}) \). These values correspond to altitude readings on barometric altimeter equals to 11000 m, 14000 m, 17000 m and 20000 m.

The maximum and the minimum values of the density altitude, calculated in the simulated conditions, are presented in Table II.

Table 2 Maximum and minimum values of the density altitude for the conditions simulated in Fig. 4

<table>
<thead>
<tr>
<th>No.</th>
<th>Indicated barometric altitude [m]</th>
<th>Maximum value of the density altitude [m]</th>
<th>Minimum value of the density altitude [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11000</td>
<td>13633.6955</td>
<td>10056.6798</td>
</tr>
<tr>
<td>2</td>
<td>14000</td>
<td>18048.2587</td>
<td>13057.7078</td>
</tr>
<tr>
<td>3</td>
<td>17000</td>
<td>20657.6839</td>
<td>16059.3595</td>
</tr>
<tr>
<td>4</td>
<td>20000</td>
<td>26301.9825</td>
<td>21666.2166</td>
</tr>
</tbody>
</table>

From Fig. 4 one observe that the maximum values are obtained at \( \Delta t=30^\circ C \) and \( t=35^\circ C \) for \( H_p=11000 \) m and \( H_b=14000 \) m, respectively at \( \Delta t=30^\circ C \) and \( t=25^\circ C \) for \( H_p=17000 \) m and \( H_b=20000 \) m. The minimum values are obtained at \( \Delta t=-30^\circ C \) and \( t=-50^\circ C \) for all four situations.
According to the maximum and minimum values above table, the difference between the density altitude and barometric altitude for this layer may even exceed 6000 m superposing certain conditions of temperature and humidity.

For the other layers, formulas for calculating the density altitude in the layer 1 are derived similarly, but they are less important for aviation.

5 Conclusions
A method to calculate the density altitude starting from the static pressure, non-standard air temperature variations and dew point information was presented. In this way, mathematical relations were developed for the both 0 and 1 atmospheric layers (0=11km, respectively 1=20 km).

For the layer 0, values between -30°C±30°C for non-standard temperature variations, and between -50°C±35°C for the dew point, was considered. Also, the evaluation was performed for four barometric altitude values: 0m, 3000m, 6000m and 9000m. The maximum deviation of the density altitude from the indicated barometric altitude value is 2595.6104 m, and was obtained for $\Delta t=30°C$ and $t=35°C$ at $H_b=9000m$.

For the layer 1, values between -30°C±30°C for non-standard temperature variations was considered. For the dew point, values between -50°C±35°C for barometric altitudes fewer than 16000 m, and between -50°C±25°C for barometric altitudes over 16000 m, was considered. The evaluation was realised for the next values of the indicated barometric altitude: 11000 m, 14000 m, 17000 m, and 20000 m. In this case, the maximum deviation of the density altitude from the indicated barometric altitude value is 6301.9825 m, and was obtained for $\Delta t=30°C$ and $t=25°C$ at $H_b=20000m$.

For both layers was depicted the individual effects of the non-standard temperature variation, respectively of the air humidity, on the density altitude (Fig. 3, respectively Fig. 5).

References: