The Markovian portfolio selection model with GARCH volatility dynamics

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Abstract: - This paper proposes and compare portfolio selection models under the assumption that the portfolios of returns follow a GARCH type process. We compute the price/return distribution at some future time approximating the GARCH process with a Markov chain. We present an ex-post comparison of portfolio selection strategies applied to some assets of the US Market. Since the optimization problems present more local optima, we implement an heuristic algorithm for the global optimum in order to overcome the intrinsic computational complexity of the models.

Key-Words: - GARCH models, Markov chains, portfolio selection, performance strategies, ex-post analysis.

1. Introduction

In this paper, we model the return portfolios with a Markov chain that account the GARCH evolution of the returns. In particular, we use the Duan and Simonato’s approximation of the returns evolution (Duan, Simonato 2001) in portfolios selection problems. Under this distributional hypothesis we compare the ex-post performance of some portfolio selection strategies.

There is a general consensus on the importance to model the time varying volatility (Engle [1982], Bollerslev [1986]) and the leverage effect (Black [1976]). Several empirical studies have showed that these statistical aspects serve to solve many biases between theoretical and empirical prices (see Bakshi, Cao and Chen [1997], Engle and Mustafa [1992], and Heston and Nandi [2000]). Because there is wide consensus that the variance of the financial asset returns is time variant, a great amount of efforts are directing to realize mathematical models which, by choosing the variance dynamics as the model corner-stone, should be effectively able to model financial prices. Surely the GARCH model is a reference instrument to study the volatility dynamics, and among its advantages there is its high flexibility to be suitable to capture the most important features of the financial variables. In this work we analyze the impact of choices based on the GARCH parametric characterization of financial asset series. It is to note that the passage from the GARCH parametric characterization of financial asset series to the computation of the price/return distribution at some future time is not immediate. In order to build portfolio

wealth distribution we use Duan and Simonato's GARCH approximation (Duan, Simonato 2001). Moreover, we extend the Duan and Simonato's ideas to other possible GARCH type models (see Glosten, et al.(1993), Nelson (1991)). As these authors explain many GARCH models and in particular the GARCH(1,1) models can be represented as a bivariate Markovian system (i.e., the state of the process is uniquely represented by price and variance states). This feature allows to approximate GARCH models by a discrete Markov chain. To build the transition matrixes we use the method discussed by Duan and Simonato (2001), Duan et al. (2003) for parametric Markovian processes. With parametric portfolio selection models the transition matrix depends on the parameters of the underlying multivariate Markov process and the parameters are functions of the portfolio weights. Therefore we should check for a global optimum for most of the portfolio selection problems. In the paper we use an optimization heuristic algorithm Angelelli and Ortobelli (2009) that reduces enormously the computational complexity with respect to other global optimization approaches like simulated annealing.

In the following empirical comparison, we present some portfolio selection strategies that use different GARCH models. All of them are based on the estimation of the distribution of the returns at future times under the assumption that the residuals of log returns portfolios follow a GARCH process. The paper is organized as follows. In Section 2 we show the models implemented. In Section 3 we discuss the Markovian approximation of portfolio value
and we formalize the portfolio selection model. In section 4 we perform an empirical comparison among different portfolio selection models. Finally, we briefly summarize the paper.

2. Portfolio Value With GARCH Volatility Dynamics.

Let us consider a discrete-time economy and $n$ risky assets with log returns $r_{i,t} = \left[ r_{i,1}, \ldots, r_{i,t+1} \right]'$. If we denote by $x = [x_1, \ldots, x_n]'$ the vector of the positions taken in the $n$ risky assets, then the portfolio log return during the period $[t, t+1]$ is given by

$$r_{p,t+1} = \sum_{i=1}^{n} x_i e^{r_{i,t+1}}.$$  

(1)

In particular, we assume that investors want to maximize the performance of their choices at a given future date $T$. Now we introduce the alternative GARCH volatility dynamics models implemented in this work.

Suppose that under the historical measure $P$ the daily portfolio log-return is described by the following relation:

$$r_{p,t} = \ln \left( \frac{W_{p,t}}{W_{p,t-1}} \right) = \mu + \sigma_{r_{p,t}} e_{r_{p,t}}$$  

(2)

where $W_p(x)$ is the portfolio value at time $t$, with asset position $x$, and $e_{r_{p,t}} \sim \phi \left[ 0, (1) \right]$ under $P$.

For convention we consider the initial portfolio wealth equal to 1 (i.e., $W_0 = 1$). In this work we use the standard GARCH(1,1) (see Bollerslev, T. (1986)) and some well-known extension of the standard GARCH(1,1). Each model can be represented as:

$$\sigma_{r_{i,t}} = f \left( \rho, \sigma, e_{r_{i,t}} \right)$$

where the relation expresses that the conditional variance at time $t+1$ is function of the lagged value of the variance ($\sigma$), the lagged shock ($e_{r_{i,t}}$) and a set of parameters ($\rho$).

The volatility dynamics models we consider are:

Model I: GARCH (1,1) (G11): (see Bollerslev, T. (1986))

$$\sigma_{i,t}^{2} = \omega + \alpha e_{i,t-1}^{2} + \beta \sigma_{i,t-1}^{2}$$  

(3)

where $\omega, \alpha > 0$ and $0 < \beta < 1$.

Model II: GJR-GARCH (GJR-G): (see Glosten, et al. (1993))

$$\sigma_{i,t}^{2} = \omega + \alpha e_{i,t-1}^{2} + \alpha \sigma_{i,t-1}^{2} e_{i,t} + \gamma \sigma_{i,t-1}^{2} I_{t}$$  

(4)

where $\omega, \alpha > 0$ and $0 < \beta < 1$ and

1 Generally, we assume the standard definition of log return between time $t$ and time $t+1$ of asset $i$ as $r_{i,t} = \ln \left( \frac{S_{i,t} + d_{i,t}}{S_{i,t}} \right)$, where $S_{i,t}$ is the price of the i-th asset at time $t$ and $d_{i,t}$ is the total amount of cash dividends paid by the asset between $t$ and $t+1$.

2 i.e., $E \left( \sigma_{i}^{2} \right) = h^r \forall t$ (and $E \left( \ln \left( \sigma_{i}^{2} \right) \right) = q^r$ for the E-G)

3 We use the GARCH property that: $E \left( f \left( \sigma_{i} \right) g \left( e_{i} \right) \right) = E \left( f \left( \sigma_{i} \right) \right) E \left( g \left( e_{i} \right) \right)$ where $f$ and $g$ are some measurable function, since $\sigma_{i}$ and thus each its function, is measurable with respect to the sigma-algebra at time $t-1$.

$\omega = \begin{cases} \omega & \text{if } \epsilon_{i} < 0 \\ 0 & \text{otherwise} \end{cases}$

Model III: E-GARCH (E-G): (see Nelson (1991))

$$\ln \left( \sigma_{i,t}^{2} \right) = \omega + \beta \ln \left( \sigma_{i,t-1}^{2} \right) + \alpha \left( \left| \epsilon_{i,t} \right| - \gamma \epsilon_{i,t} \right)$$  

(5)

where $\alpha > 0$ and $0 < \beta < 1$.

The parameters of the models are $\theta = (\mu, \rho)$ where $\mu$ is the constant drift term and $\rho = (\omega, \alpha, \beta, \gamma)$ is the parameter vector related to the variance dynamics.

In the model II and III the parameter $\gamma$ allows to model the asymmetric behavior of the variance, sometime called Black’s effect. It consists of a greater response of the variance when the news arrived in the market are negative ($\epsilon_{i} < 0$) than when the news are positive ($\epsilon_{i} > 0$).

All conditions on the parameters $\omega, \alpha$ are used to avoid theoretical inconsistence on the value for $\sigma_{i,t}^{2}$ (i.e., $\omega < 0$ should mean a potential negative value for the variance, while $\alpha < 0$ should mean that greater shock movements induce a decreasing variance), while the conditions on $\beta$ allow the variance process to be covariance-stationary. Let us consider the value of the stationary variance level in each model, supposing the weakly stationarity on $\sigma_{i,t}^{2}$ then we obtain:

$$h^r = \omega \left( 1 - \beta - \alpha \varepsilon_{i,t}^{2} \right)$$  

in the G11

$$h^r = \omega \left( 1 - \beta - \alpha \varepsilon_{i,t}^{2} \right) - \gamma \varepsilon_{i,t}^{2} I_{t}$$  

in the GJR

$$h^r = \exp \left( \frac{\omega - \alpha \varepsilon_{i,t}^{2} \left( \left| \epsilon_{i,t} \right| - \gamma \epsilon_{i,t} \right)}{1 - \beta} \right)$$  

in the E-G

Since we have to avoid the asymptotic divergence and the negativity of the variance process we need the following additional conditions:

• $\beta + \alpha \varepsilon_{i,t}^{2} < 1$ in the G11

• $\beta + \alpha \varepsilon_{i,t}^{2} + \gamma \varepsilon_{i,t}^{2} I_{t} < 1$ in the GJR

Note that in the normal innovation case (i.e., $e_{i,t} \sim N(0,1)$)

$$h^r = \frac{\omega}{1 - \beta - \alpha}$$  

in G11, $h^r = \frac{\omega}{1 - \beta - \alpha - \gamma / 2}$ in GJR,
\[ h^* = \exp\left(\frac{-\alpha \sqrt{2/\pi}}{1 - \beta}\right) \text{ in } E-G. \]

and the conditions are:

\[ \beta + \alpha < 1 \text{ in the } G11, \quad \alpha + \beta + \gamma/2 < 1 \text{ in the GJR-G.} \]

The GARCH(1,1) model is a benchmark model and it is used for model comparisons. The main characteristics of other GARCH models used can be summarized in:

1. Parsimonious model, only 4 parameters to model the variance dynamics
2. Two state variables: price and variance
3. Time varying variance: GARCH models drive the variance process.
4. Models are potentially able to explain the well-known stylised-facts as the “Leverage effect” (\( \gamma \) parameter) and the “Clustering effect” in the stochastic volatility (\( \beta \) parameter)

3. Portfolio Value Dynamics As Markovian Evolution Process

Duan and Simonato have shown that the GARCH(1,1) model can be represented as a bivariate Markovian system (i.e., the state of the process is uniquely represented by \( (W_t, \sigma^2_{t+1}) \), so the process is markovian of the first order).

This feature allows to approximate GARCH models by a discrete Markov chain. Duan and Simonato’s analysis can be extended to GJR-GARCH and E-GARCH models as we show here in the following. In particular, we present the Markov chain approximation of a GARCH (1,1) process (Duan, Simonato 2001) adapted to the work models.

Let us consider an underlying portfolio log-return modeled by the equation (2) or equivalently let us consider

\[ \ln(W_t) = \ln(W_t^0) + \mu + \sigma z_t, \]

where \( W_t \) denote the portfolio value at day \( t \). Let \( Q \) be some probability measure and \( \sigma_t \) be the variance modeled by G11, GJR or E-G. Let \( z_t \) be some random variable independently distributed with respect to the information up to time \( t-1 \), i.e.,

\[ z_t \sim \mathcal{N}(0,1). \]

Following Duan and Simonato’s suggestions, we form the partitions by using the logarithm of adjusted wealth and log variance for the two state variables considered. The adjusted wealth is used to reduce the dimension of the transition matrix by a wealth conversion. The logarithms of the values used are justified mainly for its better convergence behavior. The adjusted wealth is computed by \( W_t^* = e^{-\mu} W_t \) where \( \mu = \mu - h^*/2 \) and \( h^* \) is the stationary variance, the pre-adjusted wealth can be easily recover later. Also the unconditioned variance can be computed in all the GARCH model mentioned.

Note that in term of log-adjusted wealth the log return dynamics becomes:

\[ \ln\left(\frac{W^*_{t+1}}{W^*_{t}}\right) = \ln\left(\frac{W_{t+1}}{W_t}\right) - \frac{1}{2} \left( h^* - \sigma^2_{t+1} \right) + \sigma \varepsilon_t. \]

The unconditional expectation of the continuously compounded return on the adjusted wealth is zero, since \( E^0(\sigma^2_{t+1}) = h^* \) and \( E(\varepsilon_t) = 0 \).

Let \( p_t \) and \( q_t \) be the logarithm of the adjusted wealth (let us say log wealth) and the logarithm of the variance respectively (i.e., \( p_t = \ln(W^*_t) \) and \( q_t = \ln(\sigma^2_{t+1}) \)). Then the models can be rewritten with:

\[ p_t = p_{t-1} + \frac{1}{2} \left( h^* - e^{\sigma^2} \right) + \sqrt{e^{\sigma^2}} \varepsilon_t, \]

\[ q_{t+1} = \ln\left( \omega + \beta e^{\sigma^2} + \alpha e^{\sigma^2} e^2 \right) \text{ in the } G11 \text{ case,} \]

\[ q_{t+1} = \ln\left( \omega + \beta e^{\sigma^2} + \alpha e^{\sigma^2} e^2 + \gamma e^{\sigma^2} e^2 I_t \right) \text{ in the GJR case or} \]

\[ q_{t+1} = \omega + \beta q_t + \alpha \left( \varepsilon_t - \gamma e^2 \right) \text{ in the } E-G \text{ case.} \]

To find a states partition to approximate the GARCH process we use:

1) A log wealth partition centered on the logarithmic of the initial portfolio wealth: \( \{p_0 - I_p, p_0 + I_p\} \), where \( I_p \) is determined by studying the conditional behavior of the logarithm of the adjusted portfolio wealth over the investor time horizon \( T \):

\[ I_p = \delta_p \left(m \right) \sum_{t=1}^{T} E^0(\sigma^2_{t} | \phi_t) \] (6)

2) An analytical formula of the conditional variance of the log wealth can be derived for many GARCH processes.

3) Log variance partition: to form the partition we would study the conditional behavior of the logarithm of the variance \( q_t = \ln(\sigma^2_{t+1}) \). From the GARCH process features we know that there are two notable values of the variance:

a) the initial variance, which the process starts from,

b) the unconditional variance \( (h^*) \) to whom the process asymptotically is attracted.

Both these values have to be considered in the variance partition, but the second has increasing importance as we go far from the begin instant The partition center can be computed as:

\[ q^*_t = \ln \left( \tau - \min\left(\tau, T\right) \sigma^2_t + \min\left(\tau, T\right) h^* \right). \]

The value of \( \tau \) is a temporal index used to form the weights. As it increases as the relative weight of the unconditional variance respect to the initial variance increases. Then in the study of long-term horizon \( \tau \) has to be small. Anyway it is important to ensure that \( q^*_t \) belongs to the partition. The log variance partition is \( \{q^*_t - I_q, q^*_t + I_q\} \). In order to compute the width \( I_q \) of the partition it should be enough to study \( Var^0(q_t | \phi_t) \) but in G11 and GJR-G it could result analytically complex. We know by the Jensen inequality that
\[ V_{\text{ar}}^\delta (q_j | \phi_i) \leq \ln \left( V_{\text{ar}}^\delta (\sigma^2_i | \phi_i) \right) \] so Duan and Simonato propose to use a width:
\[ I_\epsilon = \ln \left( e^\delta + \delta (n) \right) - q_i. \] (7)

Only in the E-G case we have to note that the log variance partition can be constructed directly by the E-GARCH equation, because it expresses the variance in logarithmic terms: \[ q_i = I_\epsilon + q_i. \]

In particular in the models investigated we can write the variance as function of its lagged value, and two lagged wealth, i.e.: \[ q_{i,2} = \Phi(q_{i,1}, p_{i,1}, p_i). \]

First we recover \( \epsilon_{i+1} \) from the log price equation written one time forward:
\[ \epsilon_{i+1} = \frac{p_{i,1} - p_i + \frac{1}{2} \left( e^{\alpha_{i,1}} - h^i \right)^2}{\sqrt{e^{\alpha_{i,1}}}} \] and substituting in the log variance equation we obtain:
\[ \Phi^{E-G} (q_{i,1}, p_{i,1}, p_i) = \ln \left( \alpha + \beta e^{\alpha_{i,1}} \left( p_{i,1} - p_i + \frac{1}{2} \left( e^{\alpha_{i,1}} - h^i \right)^2 \right) \right) \]
\[ \Phi^{GJR} (q_{i,1}, p_{i,1}, p_i) = \ln \left( \alpha + \beta e^{\alpha_{i,1}} \left( p_{i,1} - p_i + \frac{1}{2} \left( e^{\alpha_{i,1}} - h^i \right)^2 \right) \right) \]
\[ \Phi^{GJR} (q_{i,1}, p_{i,1}, p_i) = \omega + \beta e^{\alpha_{i,1}} + \alpha \left( p_{i,1} - p_i + \frac{1}{2} \left( e^{\alpha_{i,1}} - h^i \right)^2 \right) \]

This implies a source of sparsity in the markovian transition matrix: for each combination of \( i,j,k \) it exists only an index \( l \) where the transition probability can be non zero. Thus we can rewrite the Markov transition probability as:
\[ p_{i}(j,k) = \begin{cases} \frac{p_{i,j} - p_i + \frac{1}{2} \left( e^{\alpha_{i,j}} - h^j \right)^2}{\sqrt{e^{\alpha_{i,j}}}} & \text{if } \Phi(q_{i,j}, p_{i,j}, p_i) \in D(l) \\ 0, & \text{otherwise} \end{cases} \]

The conditional probability can be computed as:
\[ p_{i,j}(k,l) = \begin{cases} \frac{c(k) - c(l)}{2} \left( e^{\alpha_{i,j}} - h^j \right) & \text{if } c(k) - c(l) < c(k+1) \leq c(l) - c(k-1) \\ 0, & \text{otherwise} \end{cases} \]

Clearly these transition probabilities can be easily computed for any classical distributional assumption on the innovations. In this paper we use innovations Gaussian distributed.

### 3.1 The dynamic portfolio selection problem

Once we have computed the transition matrix \( M \) we obtain the distribution at time \( T \) considering the power of the transition matrix \( M^T \). Therefore, given the state \((i,j)\) of the bivariate process (return, variance) corresponding to the k-th raw of the transition matrix, the distribution of the bivariate
process at time $T$ conditioned to start by $(i,j)$ state is given by the k-th raw of the matrix $M^T$. Thus, to get the probability the log wealth is in the state “$s$” after $T$ steps starting by $(i,j)$ state we have to sum the probabilities for the different variance states, i.e., $\pi_T([i,j],s) = \sum_{l=1}^{n} \pi_T([i,j],l,i)$ where $\pi_T([i,j],l)$ is the probability corresponding to the k-th raw of the matrix $M^T$ to go in the state $(s,l)$ after $T$ steps. Therefore, doing so we obtain the probability of the forecasted final wealth for any GARCH type model described.

In the portfolio selection problem we assume the initial wealth $W_0=1$ and all admissible wealth processes $W(x) = \{W_t(x)\}_{t \geq 0}$ depending on an initial portfolio $x \in S$ are defined on a filtered probability space $(\Omega,\mathcal{F},(\mathcal{F}_t)_{t \geq 0},\mathbb{P})$. The portfolio selection problem when no short sales are allowed, can be represented as the maximization of a functional $f: ([\Omega,\mathcal{F},\mathbb{P}] \rightarrow \mathbb{R}$ applied to the random final wealth $W(x)$ obtained with the portfolio weights belonging to the $n$-dimensional simplex

$$S = \{x \in \mathbb{R}^n \mid \sum_{i=1}^{n} x_i = 1; x_i \geq 0\}$$

i.e.,\[\inf_{x \in S} f(W(x))\]

Typical examples are the performance measure type functionals $f(X) = \frac{\rho_1(-X)}{\rho_2(X)}$ where $\rho_1(.)$ and $\rho_2(.)$ are two positive increasing functions of coherent risk measures (see Rachev et al. (2008) and the reference therein). These functionals are isotope with the monotony order, but $\rho_1(.)$ and $\rho_2(.)$ are consistent with risk averse preferences. Thus, the functional $f(X) = \frac{\rho_1(-X)}{\rho_2(X)}$ is not isotope neither with risk lover nor with risk averse preferences. We refer to Rachev et al. (2008) for further examples of the above measures. Here in the following we introduce the performance type measure used in the choice problem.

**Rachev ratio** This performance functional is defined as

$$RAC(x,y) = \frac{ETL_a(W_T(r_b - z(x)) - 1)}{ETL_b(W_T(z(x)) - r_b) - 1}$$

When the benchmark $r_b$ is the riskfree rate and $z(x)$ is the chosen portfolio gross return (i.e. $z(x) = \exp(r(x))$) and $W_T(z(x) - r_b) - 1$ is the final wealth at time $T$ we obtain investing in the excess return $z(x) - r_b$.

$ETL_a$ is the Expected Tail Loss or Average Value at Risk (AVaR) which is a coherent measure defined as

$$ETL_a(Y) = \frac{1}{\alpha} \int_0^\infty Y^{-\alpha} \cdot F_{Y^{-1}}(u) \text{d}u$$

where $F_{Y^{-1}}(u) = \inf \{ t \in \mathbb{P} / \mathbb{P}(Y \leq t) \geq u \}$ is the left inverse of the distribution function. Recall that the classic consistent estimator of expected tail loss is given by

$$ETL_a(Y) \approx \frac{1}{\Delta t} \sum_{t=1}^{T} Y_t^{-1} \cdot I_{\{Y_t \leq F_{Y^{-1}}(u)\}}$$

where $I_{\{Y_t \leq F_{Y^{-1}}(u)\}} = \begin{cases} 1 & \text{if} \ Y_t \leq F_{Y^{-1}}(u) \\ 0 & \text{otherwise} \end{cases}$. When the benchmark $r_b$ is the riskfree rate, $X$ is the portfolio return, and the numerator and the denominator are positive (negative), then the Rachev ratio is isotonic (consistent) with non-satiable preferences of investors who are neither risk averse nor risk lover (see Rachev et al. (2008)).

4. **An Ex-Post Empirical Comparison among GARCH type models**

In this section, we compare portfolio selection strategies based on the GARCH models introduced in the previous sections. We use 32 assets quoted on the US markets (NYSE and NASDAQ) from 01/02/97 till 06/14/2010 for a total of 3384 daily observations. We compare the performance of:

1) Rachev ratio under the hypothesis the log wealth follows a GARCH(1,1), or a GJR-GARCH or an E-GARCH.  
2) Sharpe ratio (see Sharpe 1994) under the assumption we consider historical iid returns.

We recalibrate daily the portfolio and for the dynamic strategies we use a temporal horizon $T=20$ working days.

Figure 1. Ex-post final wealth process when European strategies are applied with daily recalibration and temporal horizon $T=20$ days.

We forecast the future wealth using 28 states for the portfolio of returns and 6 states for its variance. As coefficients of AVaR in the Rachev ratio we use...
\[ \alpha = \beta = 0.05 \] The comparison consists in the ex post evaluation of the wealth produced by the strategies. For each strategy, we consider an initial wealth \( W_0 = 1 \) at the date 04/30/2009, and at the \( k \)-th recalibration (\( k = 0, 1, 2, \ldots \)), three main steps are performed to compute the ex-post final wealth:

**Step 1** Determine the market portfolio \( x_M^{(k)} \) that maximizes the performance ratio \( \rho(W(x)) \), i.e., the solution of the following optimization problem:

\[
\max_{x^{(k)}} \rho(W(x^{(k)}))
\]

s.t.

\[
(x^{(k)})_v = 1, \quad x^{(k)}_i \geq 0; \quad i = 1, \ldots, n.
\]

As shown by Angelelli and Ortobelli (2009) this type of problems could present more local optimum than we use the heuristic developed from them to approximate the global optimum.

**Step 2** Determine the ex-post final wealth given by:

\[
W_{t_{k+1}} = W_t \left( x_M^{(k)} \right) z^{(expost)}_{t_{k+1}}.
\]

where \( z^{(expost)} \) is the vector of observed gross returns between \( t_k \) and \( t_{k+1} \).

**Step 3** The optimal portfolio \( x_M^{(k)} \) is the new starting point for the \((k+1)\)-th optimization problem.

Steps 1, 2 and 3 are repeated until the observations are available and for each performance ratio.

The output of this analysis is represented in Figure 1. Figure 1 reports the ex-post wealth process using different GARCH models. In particular these results emphasize the good performance of the classic GARCH(1,1) model that in the last year present earnings of about the 100%. Instead the other GARCH models are almost never comparable to the classic one. However the comparison with static classic strategy is amazing and it suggests us that we should never use the classic strategies in portfolio choices. Thus, the empirical results show that volatility GARCH models could be very important in portfolio theory.

5. Conclusion

This paper examines the impact of GARCH type return evolution in portfolio selection problems. We describe how to approximate GARCH type processes with Markov chains and we deal the portfolio selection problem under these distributional assumptions. Thus we propose algorithms that permit to solve computationally complex problems in acceptable computational times. Finally, we propose an empirical comparison among the myopic portfolio selection models and those based on the GARCH approximation. The ex-post empirical comparison among GARCH approximation and those based on Markovian trees shows the greater predictable capacity of the latter.

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