Fundamental Current Phasor Estimation Techniques
used in Fault Location

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Abstract: - In the paper are presented three algorithms for fundamental current phasor estimation: the Discrete Fourier Transform algorithm, the recursive least squares algorithm and an adaptive algorithm. The algorithms were tested using many simulated fault current signals, obtained from a 20 kV medium voltage test grid, transposed in the ATP program. It is analyzed the influence of different parameters on the output results of the presented methods, as the fault inception angle and the distance to the fault.

Key-Words: - Phasor estimation, Discrete Fourier Transform , recursive least squares, adaptive algorithm

1 Introduction

Nowadays the trend is to locate faults quickly, reliable and, if possible, without human intervention. This is made directly by using the samples from the faulted voltages and/or currents.

One of the fault location techniques processes the power frequency components of the fault signals [1], [2], one of the main advantage of this method being that they can be applied using the actual power network’s electronic devices. Good estimation in fault location can be achieved in the case of high precision in knowing the line’s parameters, sources’ power and power frequency voltages and currents phasors. This paper focuses on the last issue.

When a fault occurs, the voltage and current signals are severely distorted. Besides the power frequency component, the fault signals may contain harmonics and an exponentially decaying dc component. This makes the power frequency phasors difficult to be fast and accurately estimated and increases the errors of the fault location [1].

Among the phasor estimation techniques mentioned in the literature, the Discrete Fourier Transform (DFT) is the most popular. However, its performance can be adversely affected by the presence of the dc exponentially decaying component, especially in the fault current signals [1], [3].

2 Mathematical Models

For further considerations it is assumed that the fault signal, in the discrete form, is as follow:

\[ x[n] = X_0 e^{-\frac{n\Delta t}{\tau}} + \sum_{k=1}^{K} X_k \cos(k\omega_1 n\Delta t + \phi_k) \]  \hspace{1cm} (1)

where \( x[n] \) is a generic sample from the fault signal, \( X_0 \) and \( \tau \) are the magnitude and respectively the time constant of the dc exponentially decaying component, \( X_k \) and \( \phi_k \) are the magnitude and respectively the phase of the \( k \)th harmonic, \( k = 1, 2, \ldots, K \), \( K \) – number of considered harmonics, \( \omega_1 = \frac{2\pi}{T_1} \) is the fundamental angular frequency, \( T_1 \) is the period of the fundamental component, \( \Delta t = T_1/N \) is the sampling interval and \( N \) is the number of samples per fundamental period.

The purpose of the presented methods is to extract the phasor of the power frequency component using the samples \( x[1], x[2], \ldots, x[M] \) from the transient signals generated by the fault occurrence.

2.1 Discrete Fourier Transform algorithm

The phasor for the \( k \)th harmonic, from the fault signal (1), is computed using the following formula [5]:

\[ X_k[n] = \frac{2}{N} \sum_{i=1}^{N} x[n-i] \exp(-j\frac{ik2\pi}{N}) \]  \hspace{1cm} (2)

where \( x[n] \) is a generic sample from the fault signal with \( n = 1, 2, \ldots, M \) (\( M \) is the number of samples considered), \( k \) is the order of harmonic, \( N \) is the number of samples per fundamental period, equal in this case with the number of samples in the data.
window of the DFT algorithm, and \( i \) is the \( i \)-th sample in the data window.

If \( k = 1 \), (2) denote the phasor of the power frequency component as follows:

\[
X_1[n] = \frac{2}{N} \sum_{i=1}^{N} x[n-i] \left( \cos\left(\frac{2\pi n}{N}\right) - j \sin\left(\frac{2\pi n}{N}\right) \right)
\]  

(3)

where the exponential term was substituted with its sinusoidal equivalent [5]. The real and the imaginary parts of the phasor are given by the following formulas:

\[
\text{Re}\{X_1[n]\} = \frac{2}{N} \sum_{i=1}^{N} x[n-i] \cos\left(\frac{2\pi n}{N}\right) \tag{4}
\]

\[
\text{Im}\{X_1[n]\} = -\frac{2}{N} \sum_{i=1}^{N} x[n-i] \sin\left(\frac{2\pi n}{N}\right) \tag{5}
\]

The application of (4) and (5) consists of two steps. The first step is to obtain a weighted value of each sample by multiplying the value of the sample with a weight that is given by the terms of the sinusoids corresponding to the position of the sample in the data window. The second step is to add the weighted values of the \( N \) weighted samples (in the data window) to obtain the real and the imaginary parts of the phasor [5]. The magnitude and the phase of the fundamental phasor can be easily computed using (4) and (5).

### 2.2 Recursive Least Squares algorithm

Considering the same \( x[n] \) as a generic sample from the fault signal, \( u[n] \) as a generic filter input sample and \( w \) being the filter’s window, like in Fig. 1, it can be written the output of the filter \( y[n] \) as:

\[
y[n] = \sum_{k=0}^{N-1} w_k u[n-k] \tag{6}
\]

where the length of the filter window \( w \) is \( L \) and the length of the vector of the input samples \( u \) is \( M \), the same as the length of the vector of the fault signal samples \( x \), where \( M > L \). The purpose of the RLS algorithm is that to recursively find the parameters \( \{w(0), w(1), w(2), \ldots, w(L-1)\} \) such as to minimize the sum of the error squares [4]:

\[
\varepsilon[n] = \sum_{i=1}^{n} \lambda^{n-i} (x[i] - y[i])^2 \tag{7}
\]

where \( 0 < \lambda < 1 \) is the forgetting factor and \( i = 1, 2, \ldots, n \). Replacing (6) in (7) it results the following formula:

\[
\varepsilon[n] = \sum_{i=1}^{n} \lambda^{n-i} \left( x[i] - \sum_{k=0}^{L-1} w_k [n] u[i-k] \right)^2 \tag{8}
\]

which has the solution in the following form:

\[
w[n] = \Phi^{-1}[n] \psi[n] \tag{9}
\]

where the following notations were made [4]:

\[
\Phi[n] = \sum_{i=1}^{n} \lambda^{n-i} u[i] u^T[i] \tag{10}
\]

\[
\psi[n] = \sum_{i=1}^{n} \lambda^{n-i} u[i] x[i] \tag{11}
\]

Regarding (9), it is desired to find a recursive in time way to compute \( w[n] \), using the information already available at the time \( n-1 \). Writing the variables \( \Phi[n] \) and \( \psi[n] \) as functions of \( \Phi[n-1] \) and \( \psi[n-1] \) it results:

\[
\Phi[n] = \lambda \Phi[n-1] + u[n] u^T[n] \tag{12}
\]

\[
\psi[n] = \lambda \psi[n-1] + u[n] x[n] \tag{13}
\]

Looking at (9), it is necessary to compute the quantity \( \Phi^{-1}[n] \) that is done using the matrix inversion formula showed in [4]. After applying the aforementioned formula, for \( \Phi^{-1}[n] \) it results:

\[
\Phi^{-1}[n] = \lambda^{-1} \Phi^{-1}[n-1] - \\
\frac{\lambda^{-2} \Phi^{-1}[n-1] u[n] u^T[n] \Phi^{-1}[n-1]}{1 + \lambda^{-1} u^T[n] \Phi^{-1}[n-1] u[n]} \tag{14}
\]

Further, the following notation are used:

\[
P[n] = \Phi^{-1}[n] \tag{15}
\]

\[
k[n] = \frac{\lambda^{-1} \Phi^{-1}[n-1] u[n]}{1 + \lambda^{-1} u^T[n] \Phi^{-1}[n-1] u[n]} \tag{16}
\]
Replacing (15) and (16) in (14) results the formula:

\[ P[n] = \lambda^{-1} P[n-1] - \lambda^{-1} k[n] u[n] T[n] P[n-1] \]  

(17)

Replacing (17) in (9) and using simple mathematical calculus it results:

\[ w[n] = w[n-1] + k[n] (x[n] - u[n] T[n] w[n-1]) \]  

(18)

For the initialization of the algorithm, the parameters \( w[0] \) and \( P[0] \) can be chosen as being: \( w[0] = 0 \) and \( P[0] = I \), where \( I \) is the identity matrix [4].

2.3 The adaptive algorithm

The algorithm starts from the classic full-period Fourier algorithm and adaptively suppresses the dc decaying component from the fault signal using the least error squares method [1], [6]. Regarding (1), it can be written using the least error square method as:

\[ x[n] = \begin{bmatrix} w_x[n] \\ u_x[n] \\ u_0[n] \end{bmatrix} \begin{bmatrix} w_x[n] \\ u_x[n] \\ u_0[n] \end{bmatrix} T[n] + e[n] \]  

(19)

or in the matrix form:

\[ X = UW + E \]  

(20)

where \( u_x[n] = \cos(n2\pi/N) \), \( u_s[n] = \sin(n2\pi/N) \) and \( w_0[n] = \exp(-nT_1/N) \) with \( n = 1, 2, ..., M \) being a generic sample, \( T_1 \) is the fundamental period, \( \tau \) is the time constant of the dc exponentially decaying component and \( N \) is the number of samples per fundamental period. In (20) \( X \) is the vector of measurements, \( U \) is the signal model matrix, \( W \) is the vector of the estimated parameters and \( E \) is the vector of errors. The terms \( u_x[n] \) and \( u_s[n] \) are the weighting coefficients of the fundamental components of the phasor \( w_x[n] \) and \( w_s[n] \), and \( w_0[n] \) is the weighting coefficient of the dc exponentially decaying component \( w_0[n] \).

Considering that the value of \( \tau \) is known, the vector of the estimated parameters \( W \) is defined as:

\[ W = PU^T X \]  

(21)

with \( P = (U^T U)^{-1} \). Due to the presence of the dc exponentially decaying weighting component in the signal model matrix, the matrix \( P \) is a full matrix with off-diagonal terms depending on this exponential function, which makes difficult to estimate the parameters of vector \( W \) [6]. For simplification of the algorithm consider the weighting least error squares, with the signal model matrix defined as follows:

\[ U_G^T = U^T G \]  

(22)

where \( G \) is a weighting matrix which is chosen to minimize the estimation error, but in this approach \( G \) is chosen to assure adequate simplification of the algorithm. Replacing (22) in (21) results the formula:

\[ W = P_G U_G^T X \]  

(23)

where \( P_G = (U_G^T U)^{-1} \). For a generic sample \( n \), the terms in each row of the signal model weighted matrix \( U_G \) are \( u_x[n] = \cos(n2\pi/N) - d_s[n], u_s[n] = \sin(n2\pi/N) - d_s[n] \) and \( u_0[n] = \exp(-nT_1/N) \). The functions \( d_s \) and \( d_r \) are determined by imposing the condition that the matrix \( P_G \) should be diagonal, in its final form [7]. Thereby the relations for computing \( d_s \) and \( d_r \) are as follows:

\[ d_s[n] = D \cos \left( \frac{2\pi}{N} n + \delta \right), \]  

(24)

\[ d_r[n] = D \sin \left( \frac{2\pi}{N} n + \delta \right) \]

where:

\[ D = \frac{1 - \exp(-T_1/N\tau)}{\sqrt{\left( \cos \frac{2\pi}{N} - \exp \left( -\frac{T_1}{N\tau} \right) \right)^2 - \sin^2 \frac{2\pi}{N}}} \]  

(25)

\[ \delta = \tan^{-1} \left( \frac{\sin \frac{2\pi}{N}}{\cos \frac{2\pi}{N} - \exp \left( -\frac{T_1}{N\tau} \right)} \right) \]  

(26)

From (23) it can be derived the algorithm for calculation of the orthogonal components of the fundamental phasor:

\[ w_x[n] = w_x[n] - \delta_s[n], w_s[n] = w_s[n] - \delta_s[n] \]  

(27)

where the quantities \( w_x[n] \) and \( w_s[n] \) are the components of the fundamental phasor computed
with standard full-period Fourier algorithm, similar to (4), (5):

\[ w_c[n] = \frac{2}{N} \sum_{j=0}^{N-1} x[n-j] \cos \frac{2\pi}{N} j, \]

\[ w_s[n] = -\frac{2}{N} \sum_{j=0}^{N-1} x[n-j] \sin \frac{2\pi}{N} j \]

(28)

\[ \delta_c[n], \delta_s[n] \] are the correction functions:

\[ \delta_c[n] = d_c[n]w_o[n], \delta_s[n] = d_s[n]w_o[n] \]

(29)

where the dc decaying component is computed as:

\[ w_o[n] = \frac{2}{N} \sum_{j=0}^{N-1} x[n-j] \]

(30)

Regarding (24), (25) and (26) it is observed that \( d_c \) and \( d_s \) depend on the angle \( a = 2\pi/N \) defined by a given sampling frequency and on the exponential function \( r = \exp(-T_f/Nr) \) which depends on the time constant \( r \). The parameter \( r \) at each sample can be estimated from the measurement as:

\[ r = r[n] = \frac{w_o[n]}{w_o[n-1]} \]

(31)

Equation (31) was obtained taking into account (30) where \( w_o[n] \) stands for a sum of \( N \) elements of geometric progression with multiplier \( r \). For stabilizing the estimator (31), the results can be discriminated according to the following principle:

\[ r_{\text{min}} < r[n] < r_{\text{max}} \]

(32)

where \( r_{\text{max}} = 1 \) and \( r_{\text{min}} = \exp(-T_f/Nr_{\text{min}}) \) with \( r_{\text{min}} \) being the assumed minimum value of the dc exponentially decaying time constant.

Finally, the adaptive algorithm for phasor estimation of the fundamental component from fault signal results as having the following steps:
a) calculate the orthogonal components and a dc exponentially decaying component using (28), (30);
b) calculate the parameter \( r \) using (31), (32);
c) determine \( d_c \) and \( d_s \) using (24), (25) and (26);
d) determine the corrections \( \delta_c \) and \( \delta_s \) using (29);
e) obtaining the corrected values of the orthogonal components of the fundamental phasor using (27).

### 3 Results and comments

The results obtained refer to the simulations of many single phase-to-ground faults in a medium voltage 20 kV grid, depicted in Fig. 2, with the nominal frequency fixed at 50 Hz. The sampling frequency for the fault signal (1) results from Fourier spectral analysis applied to the fault voltages and currents. For registered fault signals and for ATP simulations it can be shown that the amplitude of the components having frequencies higher than 500 Hz can be neglected [7]. Thereby, in order to satisfy the Nyquist criteria, the appropriate sampling frequency is chosen of 1000 Hz, this corresponding to the real digital relays that process the signal of \( N = 20 \) samples per power frequency period.

The main components of the one-line diagram of the grid are: a 110 kV finite power source, a 16 MVA step-down transformer and three feeders that are composed by cable segments and aerial segments. The monitored feeder is that noted with II. The length of each segment in meters is shown in each box drawn in Fig. 2, under the word “Cable” or “Aerial.”

The neutral of the grid is treated in two ways: by a Petersen coil with a 10 % over-compensating current (EC), or by a 3 \( \Omega \) resistor (R). When distance to the fault is taken as parameter, some of the fault locations are chosen in points 032 and 034, these being the crossing points from cable segment to aerial segment.

Taking as parameter the distance to the fault and the fault inception angle, the authors have simulated different faults, considering two cases of grids’ neutral grounding: the neutral treated by the extinguishing coil EC and the neutral treated by the resistor R.

In Fig. 3 and 4 there are shown comparatively the time evolutions outputs of the three previously presented algorithms in the case of magnitude and phase estimation of the power frequency current phasor, when the fault is at the beginning of the feeder II, in the point 027, where DFT refers to the output of the discrete Fourier transform algorithm, RLS refers to the output of the recursive least squares algorithm and ADP refers to the output of the adaptive algorithm. It can be seen that the DFT and RLS algorithms present some oscillations in the outputs of the magnitude and phase estimation, while the outputs of the ADP algorithm are stable after almost one cycle after the fault’s occurrence.

Considering the fault inception angle \( \psi_F \) as being a parameter, for the fault having the impedance of 10 \( \Omega \) that occurs in the point 027, some numerical results for the power frequency current phasor
estimation are those shown in Table 1 and Table 2. In Table 3 there are shown the results obtained when the distance to fault along feeder II, \(d_F\), measured from BUS (see Fig. 2) is taking as parameter and it is assumed that the fault inception angle is equal with \(\frac{\pi}{2}\) rad and the fault’s impedance is of 10 \(\Omega\).

\[
error = 100 \cdot \frac{|X_{\text{stabDFT}} - X_{\text{estimation}}|}{X_{\text{stabDFT}}} \tag{33}
\]

As results from the tables of results, the magnitude and the phase of the fundamental current phasor are analyzed after one fundamental cycle, one and a half of fundamental cycle and two fundamental cycles, after the moment of the fault’s occurrence. The estimation errors were computed using (33), where \(X_{\text{stabDFT}}\) is the output of the DFT algorithm in the stabilized fault regime (when it is supposed that the fault signal contains only the power frequency component) and \(X_{\text{estimation}}\) is the output of each algorithm presented in section 2.

Looking at the results shown in the tables it is clearly seen that the adaptive algorithm provides very accurate results after one and a half cycle, with less than 0.9 % errors in the case of magnitude estimation and less than 0.2 % errors in the case on phase estimation. When the neutral grounding system is a current limitation resistor, it is observed that the estimation errors of the adaptive and the DFT algorithms are comparable after one and a half cycle from the moment of the fault’s occurrence, this fact being a consequence of a small value for the time constant of the exponentially dc decaying component.

**Table 1** - The fundamental current phasor estimation errors taking as parameter \(\varphi_F\), when the neutral is treated by the extinguishing coil

<table>
<thead>
<tr>
<th>(\varphi_F)</th>
<th>No. of cycles</th>
<th>Magnitude error [%]</th>
<th>Angle error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi/4) rad</td>
<td>1</td>
<td>6.37, 9.63, 6.37</td>
<td>0.01, 0.01</td>
</tr>
<tr>
<td>(\pi/2) rad</td>
<td>1</td>
<td>4.58, 8.21, 4.47</td>
<td>0.17, 0.17</td>
</tr>
<tr>
<td>(\pi/4) rad</td>
<td>1</td>
<td>11.0, 6.80, 11.0</td>
<td>0.01, 0.01</td>
</tr>
<tr>
<td>(\pi/2) rad</td>
<td>1</td>
<td>11.0, 6.80, 11.0</td>
<td>0.01, 0.01</td>
</tr>
</tbody>
</table>

**Table 2** - The fundamental current phasor estimation errors taking as parameter \(\varphi_F\), when the neutral is treated by the resistor

<table>
<thead>
<tr>
<th>(\varphi_F)</th>
<th>No. of cycles</th>
<th>Magnitude error [%]</th>
<th>Angle error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi/4) rad</td>
<td>1</td>
<td>3.21, 30.69, 0.16</td>
<td>6.41, 6.83, 0.07</td>
</tr>
<tr>
<td>(\pi/2) rad</td>
<td>1</td>
<td>13.22, 38.26, 1.16</td>
<td>10.82, 0.09</td>
</tr>
</tbody>
</table>
Table 3 - The fundamental current phasor estimation errors taking as parameter $d_F$, when the neutral is treated by the resistor

<table>
<thead>
<tr>
<th>$d_F$</th>
<th>No.of cycles</th>
<th>Magnitude error [%]</th>
<th>Angle error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DFT</td>
<td>RLS</td>
</tr>
<tr>
<td>0.032</td>
<td>1</td>
<td>10.60</td>
<td>37.30</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.74</td>
<td>22.12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.02</td>
<td>13.11</td>
</tr>
<tr>
<td>0.034</td>
<td>1</td>
<td>11.74</td>
<td>37.12</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.19</td>
<td>21.98</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.01</td>
<td>13.07</td>
</tr>
<tr>
<td>0.039</td>
<td>1</td>
<td>11.92</td>
<td>37.11</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.22</td>
<td>21.98</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.04</td>
<td>13.10</td>
</tr>
<tr>
<td>0.048</td>
<td>1</td>
<td>12.07</td>
<td>37.05</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.28</td>
<td>21.96</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.05</td>
<td>13.09</td>
</tr>
</tbody>
</table>

When the neutral is compensated with the considered Petersen coil, it is observed that the estimation errors provided by the DFT algorithm differ from those provided by the adaptive algorithm, this fact being a consequence of a big value for the time constant of the exponentially dc decaying component. Regarding the recursive least squares algorithm, after many simulation with different values for the forgetting factor $\lambda$, the optimal value was set at $\lambda=0.95$. Looking at tables above it can be seen that the outputs of the RLS algorithm present large errors comparatively with the other two-presented algorithms. In the case of magnitude estimation, for all the three presented algorithms, it is observed the tendency of the error to decrease with the time elapsing from the fault’s occurrence. This remark is not available in the case of the phasors’ arguments, where the estimation error of the RLS algorithm stabilizes it self at a value that considerably differs from the values of estimation error provided by the other two algorithms.

Concluding, when the purpose is to estimate the fundamental current phasor, the most suitable to use is the adaptive algorithm because it provides accurate results after short time from fault’s inception.

4 Conclusions

In the paper are presented three algorithms used for the estimation of the power frequency phasor of the fault transient current, as follows: the Discrete Fourier Transform algorithm, the recursive least squares algorithm and an adaptive algorithm. The algorithms were tested using simulated results obtained by transposing the medium voltage 20 kV grid presented in Fig. 2, into the ATP program. The occurrence of many different phase-to-ground faults was simulated, taking as parameter the fault’s inception angle and the distance from the substations’ busbars to the fault.

The obtained results lead to the general conclusion that the adaptive algorithm is the most suitable for the estimation of the power frequency phasors of the transient currents generated by the occurrence of the shunt faults.

References: