Stabilization and Tracking in Lorenz Chaotic System using Optimal Generalized Backstepping Method

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Abstract— This study deals with the control chaos using generalized backstepping method. This new method to control nonlinear systems was called generalized backstepping method because of its similarity to backstepping but its abilities to control more systems than backstepping. Generalized backstepping approach consists of parameters which accept positive values. The parameters are usually chosen optional. The system responses are different for each value. It is necessary to select proper parameters to obtain a good response because the improper selection of the parameters lead to inappropriate responses or even may lead to instability of system. Genetic algorithm can select appropriate and optimal values for the parameters. This selected fitness function is for minimizing the least square error. Fitness function forces the system error to decay to zero rapidly that it causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach to its minimum value. This hybrid makes an optimal backstepping controller.

Keywords— Lorenz chaos, Lyapunov, Generalized Backstepping Method, Genetic Algorithm.

I. INTRODUCTION

In recent years, there has been many interest in the control of chaos in nonlinear dynamical systems, as in [1] proposed an adaptive method to control nonlinear systems was called generalized backstepping method (GBM) because of its likeness to backstepping but its abilities to control more systems than backstepping. This adaptive method was applied to Lorenz system. Saverio Mascolo proposes a great benefit of its method in [2] that it has the flexibility to build the control law by avoiding cancellations of practical nonlinearities. As a result, the aims of stabilization of chaotic motion and tracking of a reference signal are achieved with a reduced control effort. C. Wang and S. S. Ge in [3] consider the problem of controlling chaos in the Lorenz system. Firstly they show that the Lorenz system can be changed into a nonlinear system in the so-called general strictly feedback form and then backstepping design is used to control the Lorenz system with key parameters unknown. In [4] also authors propose chaos control with an additive, inequality forced, and scalar control input is inspected. The control aim is to stabilize the unstable system equilibrium or balance. Over the last decade different impressive methods have been proposed and utilized [5–9] to achieve the control and stabilization of chaotic systems.

This paper investigates combination of the methods backstepping and genetic algorithm. New control method is more optimize than current backstepping method. This method is used for synchronization the Lorenz chaos. Backstepping method designs a controller for Lorenz chaos, this controller has some parameters which accept the positive values. Usually these values are selected arbitrary. With respect to various values of parameters, controlled system indicates various reactions. Selection the wrong parameters cause unfavorable reaction so that can cause system instability. Genetic algorithm optimizes this controller so that appropriate and optimal values for these parameters are selected. For this purpose, genetic algorithm minimizes Fitness function to hereby finds lowest exist value for the function. Also fitness function finds lowest values for minimizing total squares error. Fitness function causes error of system to decay to zero, quickly. So the system has short and optimal setting time which has less overshoot than previous works.

Designed controller is used easier and has smaller control signal than previous works, so that will not cause saturation of operators. In addition, this controller is more economical and more optimize than previous controllers. Also by selection the various fitness functions can be achieved to other favorable results.

II. GENERALIZED BACKSTEPPING METHOD

Generalized backstepping method will be applied to a certain class of autonomous nonlinear systems which are expressed as follow:

$$\begin{cases} 
\dot{X} = F(X) + G(X)\eta \\
\dot{\eta} = f_\eta(X,\eta) + g_\eta(X,\eta)u
\end{cases}$$  

(1)

In which $\eta \in \mathbb{R}$ and $X = [x_1, x_2, ..., x_n] \in \mathbb{R}^n$. In order to obtain an approach to control these systems, we may need to prove a new theorem as follow.
Theorem: Suppose Equationation.1 is available, then suppose the scalar function $\Phi_i(x)$ for the $i_{th}$ state could be determined in a manner which by inserting the $i_{th}$ term for $\eta$, the function $V(x)$ would be a positive definite (3) with negative definite derivative.

$$V(X) = \frac{1}{2} \sum_{i=1}^{n} x_i^2$$  \hspace{1cm} (2)

Therefore the control signal and also the general control Lyapunov function of this system can be obtained by (3) and (4).

$$u = \frac{1}{g_0(X, \eta)} \left[ \sum_{i=1}^{n} \Phi_i(X) \right]$$

$$- \sum_{i=1}^{n} k_i (\eta - \Phi_i(X)) \leq 0 \hspace{1cm} (3)$$

$$V_i(X, \eta) = \frac{1}{2} \sum_{i=1}^{n} x_i^2 + \frac{1}{2} \sum_{i=1}^{n} [\eta - \Phi_i(X)]^2$$  \hspace{1cm} (4)

Proof: (1) can be represented as the extended form of (5).

$$\dot{x}_i = f_i(X) + g_i(X)\eta; i = 1, 2, ..., n \hspace{1cm} (5)$$

By $u_0 = f_0(X, \eta) + g_0(X, \eta)u$ and adding $g_i(X)\Phi_i(X)$ to the $i_{th}$ term of (5) and (7) would be obtained.

$$\dot{x}_i = [f_i(X) + g_i(X)\Phi_i(X)] + g_i(X)\eta$$

$$\eta = u_0 \hspace{1cm} (7)$$

Now we use the following change of variable.

$$z_i = \eta - \Phi_i(X) \Rightarrow \dot{z}_i = u_0 - \Phi_i(X) \hspace{1cm} (8)$$

$$\Phi_i(X) = \sum_{j=1}^{n} \Phi_j(X)$$

Therefore (7) would be obtained as follows:

$$\dot{z}_i = [f_i(X) + g_i(X)\Phi_i(X)] + g_i(X)\eta$$

$$\dot{z}_i = u_0 - \Phi_i \hspace{1cm} (9)$$

Regarding that $z_i$ has $n$ states, $u_0$ can be considered with $n$ terms. So (11) would be established as follows.

$$u_0 = \sum_{j=1}^{n} u_j$$  \hspace{1cm} (11)

Therefore, the last term of (10) would be converted to (12).

$$\dot{z}_i = u_i - \Phi_i(X) = \lambda_i \hspace{1cm} (12)$$

Control Lyapunov function would be considered as (13).

$$V_i(X, \eta) = \frac{1}{2} \sum_{i=1}^{n} x_i^2 + \frac{1}{2} \sum_{i=1}^{n} z_i^2$$  \hspace{1cm} (13)

This is a positive definite function. Now it is sufficient to examine negative definitively of its derivative.

$$\dot{V}_i(X, \eta) = \sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i} f_i(X) + g_i(X)\Phi_i(X)$$

$$+ \sum_{i=1}^{n} \frac{\partial V(X)}{\partial x_i} g_i(X) + \sum_{i=1}^{n} z_i \lambda_i$$

In order to negative definitively $\dot{V}_i(X, \eta)$, it is sufficient that the value of $\lambda_i$ would be selected as:

$$\lambda_i = - \frac{\partial V(X)}{\partial x_i} g_i(X) - k_i z_i \hspace{1cm} (15)$$

Therefore, the value of $\dot{V}_i(X, \eta)$ would be obtained as:

$$\dot{V}_i(X, \eta) = \sum_{i=1}^{n} z_i [f_i(X) + g_i(X)\Phi_i(X)]$$

$$- \sum_{i=1}^{n} k_i z_i \leq -W(X) - \sum_{i=1}^{n} k_i z_i$$

Which indicates that the negative definitively status of the function $V_i(X, \eta)$. Consequently, the control signal function, using (7), (9) and (11) would be converted to (17).

$$u_0 = \sum_{i=1}^{n} \frac{\partial \Phi_i}{\partial x_i} f_i(X) + g_i(X)\eta$$

$$- \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i [\eta - \Phi_i(X)] \hspace{1cm} (17)$$

Therefore (3) and (4) can be obtained. Now, considering the unlimited region of positive definitively of $V_i(X, \eta)$ and negative definitively of $\dot{V}_i(X, \eta)$ and the radically unbounded space of its states, global stability gives the proof.

III. GENETIC ALGORITHM

The most of optimization algorithms are based on the gradient of the cost function, so for the ill choice of the initial point or the interval search. These algorithms can be misled on
the locally optimum and can’t achieve the globally optimum. For this problem, a class of optimization algorithm, like genetic algorithms, is developed to avoid this constraint.

In its most general usage, genetic algorithms refer to a family of computational models inspired by evolution. These algorithms start with many initial points in order to cover all search intervals and encode a potential solution to a specific problem on a simple chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implantation of genetic algorithms begins with a population of chromosomes randomly bred. We evaluate each chromosome by using the objective function called Fitness function. In order to apply the genetic reproductive operations called crossover and mutation, we select, randomly, two individuals called parents and we apply the crossover operation, if its probability reaches, between parents by exchanging some of their bits to produce two children. A mutation is the second operator applied on the single children by inverting its bit if the probability reaches. After this stage we obtain two population: a parent population and a children population, the individual who has a goodness solution is preserved [11].

The genetic algorithms are used to search the optimal parameter \( k_i \) \( (k_j, j = 1, 2 \) is positive constant) in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response. The fitness function used is

\[
f = \frac{1}{n} \sum (x_i - x_a) \]

\( i \) is system state and \( x_a \) is favorite mood for \( x_i \). Based on the system purpose for placing the states at zero value \( x_a \) is equal with zero.

IV. CONTROL LORENZ SYSTEM

The Lorenz system is described as:

\[
\begin{align*}
\dot{x} &= ax - xy - bz \\
\dot{y} &= cy - xz - y \\
\dot{z} &= xy - bz
\end{align*}
\]  

(19)

Where \( a = 10, b = 8/3, c = 28 \) are system parameters and \( x, y, z \) are state and initial values \((x, y, z) = (10, -10, 10)\). The generalized backstepping method is used to design a controller. In order to control Lorenz system we add a control input \( u \) to the third equation of (19). Then the controlled Lorenz system is:

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= cx - xz - y \\
\dot{z} &= xy - bz + u
\end{align*}
\]  

(20)

In order to convert Lorenz system to the general state of (1), the change variable \( r = y - x \) should be carried out. Therefore (20) would be converted to (21).

\[
\begin{align*}
\dot{r} &= (c - 1)y - (a + e)r - (y - r)z \\
\dot{y} &= (c - 1)y - cr - (y - r)z \\
\dot{z} &= (y - r)y - bz + u_i
\end{align*}
\]  

(21)

It is sufficient to establish (22) and (23) to use theorem.

\[
\begin{align*}
\phi_1(r, y) &= c - 1 \\
\phi_2(r, y) &= c
\end{align*}
\]  

(22) (23)

So the control signal and Lyapunov function will be as:

\[
\begin{align*}
u_i &= y^2 - r^2 - k_1(z - \phi_1) - k_2(z - \phi_2) - (y - r)y + bz \\
V(r, y, z) &= \frac{1}{2}r^2 + \frac{1}{2}y^2 + \frac{1}{2}(z - \phi_1)^2 + \frac{1}{2}(z - \phi_2)^2
\end{align*}
\]  

(24) (25)

After using the genetic algorithm obtained these optimal parameters: \( k_1 = 0.116, k_2 = 4.498 \). According to controller (24) the system (21) has been stabilized at the point \((0,0,0)\). The results are shown in Fig. 1, 2 and 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>size population</td>
<td>100</td>
</tr>
<tr>
<td>maximum of generation</td>
<td>300</td>
</tr>
<tr>
<td>prob. crossover</td>
<td>0.75</td>
</tr>
<tr>
<td>prob. mutation</td>
<td>0.001</td>
</tr>
<tr>
<td>Search interval de</td>
<td>[0,1,10]</td>
</tr>
</tbody>
</table>

Table I: Show the Genetic Algorithm

![Fig. 1 time response of x in (20) with (24)]

![Fig. 2 time response of y in (20) with (24)]
In order to control Lorenz system to the origin point \((0,0,0)\), can add a control input \(u_2\) to (19) then

\[
\dot{x} = a(y - x) \\
\dot{y} = cx - xz - y + u_2 \\
\dot{z} = xy - bz
\]

(26)

In order to theorem, it is sufficient to establish (27) and (28).

\[
\phi_1(x, z) = 0 \\
\phi_2(x, z) = 0
\]

(27) (28)

So the control signal and Lyapunov function will be as:

\[
u_2 = -(a + c)x - (k_1 + k_2 - 1)y
\]

(29)

\[
V(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}z^2 + \frac{1}{2}(y - \phi_1)^2 + \frac{1}{2}(y - \phi_2)^2
\]

(30)

By using the genetic algorithm obtained these optimal parameters: \(k_1 = 8.234, k_2 = 4.52\). The results are shown in Fig. 4, 5, 6 and 7.

V. TRACKING DESIRED TRAJECTORY

In this section a control law \(u_2\) so that \(x\) can track any desired trajectory \(r(t)\) would be obtained. If \(x\) is deviation between the output \(x\) and \(r(t)\) then \(\bar{x} = x - r(t)\). Therefore (26) would be converted to (31).

\[
\dot{\bar{x}} = a(y - \bar{x} - r) - \dot{r} \\
\dot{y} = c(\bar{x} + r) - (\bar{x} + r)\bar{z} - y + u_2 \\
\dot{\bar{z}} = (\bar{x} + r)y - bz
\]

(31)

It is sufficient to establish (32) and (33) to use theorem.

\[
\phi_1(\bar{x}, z) = r + \frac{\dot{r}}{a} \\
\phi_2(\bar{x}, z) = 0
\]

(32) (33)

So the control signal and Lyapunov function will be as:

\[
u_2 = -a\bar{x} - k_1(y - \phi_1) - k_2(y - \phi_2) - c(\bar{x} + r) + y
\]

(34)

\[
V(\bar{x}, y, z) = \frac{1}{2}\bar{x}^2 + \frac{1}{2}z^2 + \frac{1}{2}(y - \phi_1)^2 + \frac{1}{2}(y - \phi_2)^2
\]

(35)

After using the genetic algorithm obtained these optimal parameters: \(k_1 = 9.983, k_2 = 0.132\).

To track \(r(t) = Sin t\) Fig. 8 and 9 can be obtained.
VI. DISCUSSION

By comparing the figures the following results can be obtained:

• In the generalized backstepping method in relation to the backstepping method, the system states are stabilized by a more limited control signal. Consequently, it is less possible that the control signal to be saturated.

• In the generalized backstepping method in relation to the backstepping method [12], synchronization will be accomplished in a much shorter time and overshoot.

Considering the results obtained from simulations, the much more efficiency of generalized backstepping method in relation to the backstepping method will be demonstrated.

VII. CONCLUSION

In this study, a new method to control nonlinear systems is presented. The method proposed which is called the generalized backstepping, by feedbacking the dynamics of system and without eliminating the nonlinear dynamics, a controller is designed. The designed controller consists of parameters which accept positive values. The controlled system presents different behavior for different values. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system.

Genetic algorithm optimizes the controller to gain optimal and proper values for the parameters. For this reason GA minimize the fitness function to find minimum current value for it. On the other hand fitness function finds minimum value to minimize least square errors.

By this approach the setting time and overshoot reach to their minimum values that are demonstrated to have more optimal values when compared with previous methods.

REFERENCES


