Pressures Distribution for Eccentrically Loaded Rectangular Footings on Elastic Soils

IVAN ADRIAN
Department of Steel Structures and Structural Mechanics
Civil Engineering Faculty
“POLITEHNICA” University of Timisoara
Str. Ioan Curea nr. 1, Timisoara
ROMANIA
adrian.ivan@ct.upt.ro

Abstract: In this paper a rigid foundation of a rectangular shape is analyzed under the action of a vertical load placed anywhere on the foundation area. A general method of analysis for calculating the pressure intensity corresponding to a given settlement for eccentrically loaded square and rectangular footings resting on elastic soils has been found through a comprehensive numerical procedure. The common assumption of linear contact pressure in footing-soil interface is adopted for the solutions. Special attention has been given to the cases where there are inactive parts of foundation, without contact with soil. The solution for these cases is not yet given in any Romanian codes. The procedure has been made clear by giving an illustrative example.

Key-Words: foundation, rectangular footing, eccentrically loaded footings, linear surface pressure, contact pressure, active area

1 Introduction

The design of a rectangular footing usually starts with an initial assumption of the most suitable dimensions of it. With these dimensions under serviceability conditions normally the bearing capacity of the ground should not be exceeded. Once the suitable base size has been established the footing may be designed for bending, shear and punching shear at ultimate load conditions. Under the action of a vertical load placed anywhere on the foundation area tension between the footing and the elastic soil in most cases may not appear. That’s why sometimes inactive parts of foundation without contact with soil will develop. In such cases the active part of footing will be in full compression, but the maximum pressure should be checked not to exceed the bearing capacity of the ground soil. More than that the active area of the footing is not allowed to be less than 80% of the initial base area. The numerical procedure developed in this paper solves the problem of finding the maximum ground pressure and the size of the active area in the case of eccentrically loaded rectangular footings with linear pressure distribution. It should be noted that the solution for these cases considering a linear pressure distribution is not yet given in any Romanian codes. Footings are commonly assumed to act as rigid structures. As a consequence the vertical settlement of the elastic soil beneath the base must have a planar distribution because a rigid foundation remains plane when it settles. A second assumption is that the ratio of pressure to settlement is constant. So the pressure distribution below a rigid footing is considered to be linear. Neither of these assumptions is strictly valid, but each is generally considered sufficiently accurate for ordinary problems of design.

2 Pressures distribution computation

A vertical load placed anywhere on the foundation area is consider to act on the rectangular shape foundation. As the cases with the entire footing area in compression where treated in detail by Jarquio [1], Vitone and Valsangkar [2], Algin [3] and others in this paper special attention has been given to those cases with inactive parts of the initial base of foundation. The following notations are used:

B – width of footing area
L – length of footing area
e_x – eccentricity distance along x direction
e_y – eccentricity distance along y direction
M_x – bending moment with respect to x axis
M_y – bending moment with respect to y axis
p_i – pressure magnitude at point i of the footing area (compression)bending moment with respect to x axis
p_{max} – maximum compressive pressure
V – volume of the pressure distribution diagram
x0g – centre of gravity distance along x direction
y0g – centre of gravity distance along y direction
$aria_t$ – initial total area of the footing
aria – remaining active area the footing
I_{xg} – moment of inertia with respect to x central axis

(second moment of area)

I_{yg} – moment of inertia with respect to y central axis

i_{xg} – radius of gyration with respect to x central axis

i_{yg} – radius of gyration with respect to y central axis

I_x – moment of inertia with respect to y principal axis

I_y – moment of inertia with respect to y principal axis

i_x – radius of gyration with respect to x principal axis

i_y – radius of gyration with respect to y principal axis

x_s – x distance of the corner of central core with respect to x principal axis

y_s – y distance of the corner of central core with respect to y principal axis

Many foundations must resist not only vertical loads but also moments and lateral loads about both axes. When all loads are transferred to the footing we may consider that they are produced by a resultant load *F* located eccentrically at distances e_x and e_y from the centroid of the base of the rectangular footing. The method presented here can be used directly to calculate the maximum ground compressive pressure and the size of the active area in the case of eccentrically loaded rectangular footings with linear pressure distribution for all introduced load combinations. There is no need to check if the eccentric load *F* is inside or outside of the central core (*L/6, B/6*).

The method is based on the bending theory. In the first instance the geometric characteristics of the initial rectangular footing with respect to a central system axes are found. Then it is assumed that the resultant load *F* is placed in position e_x and e_y from the centroid of the base. This is equivalent with taking in consideration the effects of all applied loads (including bending moments) in a very simple and effective way. We have

$$e_x := \frac{M_y}{N_z} \qquad e_y := \frac{M_x}{N_z} \tag{1}$$

where N_z is the total applied vertical load.

If e_x and e_y represent a point inside the central core the entire initial footing area will be in compression with the characteristic values of the pressures in the corners given by

$$p_{1234} := \frac{N_z}{aria_t}\cdot\left(1+\frac{x_i\cdot e_x}{i_y^{\,2}}+\frac{y_i\cdot e_y}{i_x^{\,2}}\right) \tag{2}$$

where x_i is the x coordinate of the corresponding corner and y_i is the y coordinate of the corresponding corner.

When the resultant load *F* is placed outside the core according to (2) tensions should appear at least at one corner. But tensions cannot develop between the footing and the elastic soil. That's why a part of the initial rectangular base becomes inactive (without contact with soil). The problem to be solved is to find the edge of the active area diagonal opposite to load *F*. This edge represents the zero contact pressure line. The active area is totally in compression. The aim of the proposed iterative numerical computing method is to find the active area such that the total volume of the pressure distribution diagram to equalize the resultant load *F* and the centroid of this volume to be located along the direction of *F*.

The first assumption for the zero contact pressure line is the neutral axis given by bending theory. The neutral axis is computed using its cutting distances on x and y axes as

$$x_n := -\frac{i_{xg}^{\,2}\cdot i_{yg}^{\,2}-i_{xyg}^{\,4}}{e_{xt}\cdot i_{xg}^{\,2}-e_{yt}\cdot i_{xyg}^{\,2}} \tag{3}$$

and

$$y_n := -\frac{i_{xg}^{\,2}\cdot i_{yg}^{\,2}-i_{xyg}^{\,4}}{e_{yt}\cdot i_{yg}^{\,2}-e_{xt}\cdot i_{xyg}^{\,2}} \tag{4}$$

Then the active area is considered that part of the initial rectangular base having inside the resultant load *F* and the neutral axis as one edge.

The pressure distribution is computed using the geometric characteristics of this assumed active area using equation (2).

If any tension is detected then the actual neutral axis is translated along a direction normal to it in order to reduce the active area, to increase the maximum pressure and in this way to modify the position of the centroid of the total volume of the pressure distribution diagram moving it closer to *F* direction. This new neutral axis becomes the assumed zero contact pressure line. The changed position of the neutral axis is obtained by modifying the coordinates of its ends as follows

$$x(y) := \frac{x_b - x_a}{y_b - y_a}\cdot\left(y - y_a\right) + x_a \tag{5}$$

and

$$y(x) := \frac{y_b - y_a}{x_b - x_a}\cdot\left(x - x_a\right) + y_a \tag{6}$$

where (x_a, y_a) – are the coordinates of one end and (x_b, y_b) of the other end.

The process is repeated until approximately zero contact pressures are obtained along one edge of the active area.

The following cases may appear

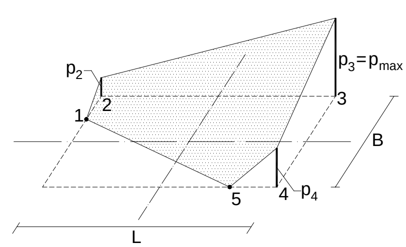

Fig. 1

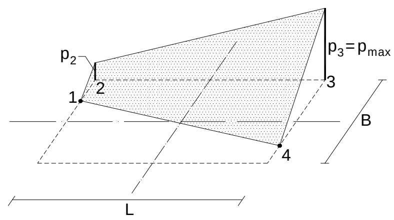

Fig. 2

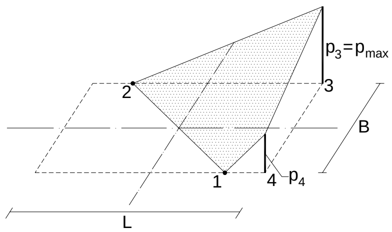

Fig. 3

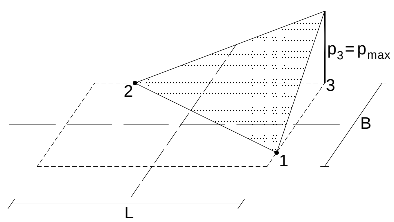

Fig. 4

Due to the applied bending theory relations the total volume of the pressure distribution diagram equalizes the resultant load *F*. The last check is done to see if the centroid of this volume is along direction of *F*. Mathematically this means that the load *F* is positioned in the opposite to zero contact pressures edge corner of the central core. The coordinates of the corner of the core in the principal system of axes of the active area are given by

$$x_s := -\frac{{i_y}^2}{x_{ng}} \tag{7}$$

and

$$y_s := -\frac{{i_x}^2}{y_{ng}} \tag{8}$$

If

$$x_s - e_{x\alpha} < \varepsilon \tag{9}$$

and

$$y_s - e_{y\alpha} < \varepsilon \tag{10}$$

the final solution of the problem has been reached.

The four possible pressure distributions for a rectangular footing under two-way eccentricity with active area where drawn up for initial positive e_x and e_y values. This numerical procedure is valid for any sign of applied bending moments. The active area and the maximum compression will rotate with respect to the centroid of the initial footing at bending moment sign changing.

3 Numerical examples

The algorithm of the proposed method has been written as an Mathcad application.

Example 1. Find the maximum soil pressure and the size of the active area in the case of an eccentrically loaded rectangular rigid foundation with the base footing of L=2,0m and B=1.6m subjected to the action of a centric compressive force N_z*=500 kN* and two positive bending moments M_x*=140 kNm* and M_y*=150 kNm* with linear pressure distribution.

Using the above described procedure we get an *active area of aria=2.6168* m^2 that is *81.77%* of the initial total area $aria_t$*=3.2* m^2. The shape of the active area corresponds with the first form given by Fig. 1.

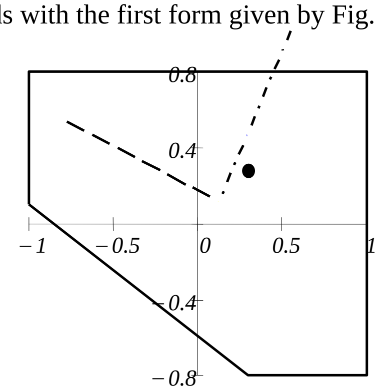

Fig. 5

The maximum pressure on the elastic soil is p_{max}*=502.86* m^2.

Example 2. Modifying in example 1 just the bending moment M_x increasing its value to M_x*=200 kNm* the shape of the active area corresponds now with the second form given by Fig. 2.

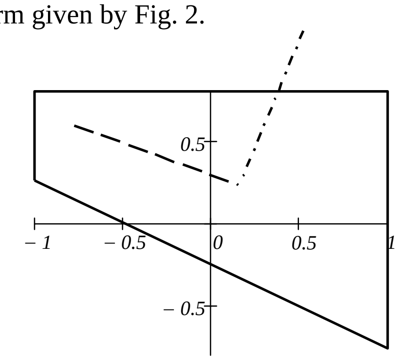

Fig. 6

Using the proposed algorithm we get an *active area of aria=2.095* m^2 that is just *65.55%* of the initial total area $aria_t$*=3.2* m^2.

The maximum pressure on the elastic soil is p_{max}*=657.38* m^2.

ISBN: 978-960-474-252-3

4 Conclusions

This paper presents a numerical method to determine the stress in an elastic soil and the size of the active area for a two-way eccentrically loaded rectangular footing, assuming a linear pressure distribution. The method consists in an iterative process which successively adjusts the position of the zero contact pressure edge to reach an active area fully in compression. The process is implemented in a computer program. The program directly evaluates the maximum pressure, the shape and size of the active area. Any load combinations may be considered. It has been found that zero contact pressure edge position of a footing does not depend on the magnitude of the eccentrically applied load, but on its position on the footing. The computer program represents a useful tool for Romanian designers as there is no specified calculus method for the linear pressure distribution in the presence of inactive areas in the Romanian codes.

References:

[1] R. Jarquio and V. Jarquio, Design of footing area with biaxial bending, *ASCE Journal of Geotechnical Engineering*, Vol. 109, No. 10, 1983, pp. 1337-1341

[2] D.M. Vitone and A.J.Valsangkar, Stresses from loads over rectangular areas, *ASCE Journal of Geotechnical Engineering Division*, Vol. 112, No.10, 1986, pp. 961-964

[3] H.M. Algin, Practical formula for dimensioning a rectangular footing, *Engineering Structures*, Vol. 29, No.6, 2007, pp. 1128-1134

[4] W.H. Highther and J.C. Anders, Dimensioning footings subjected to eccentric loads, *ASCE Journal of Geotechnical Engineering*, Vol. 111, No. 5, 1985, pp. 659-665

ISBN: 978-960-474-252-3