Abstract—The paper focuses on the vibrations of elevating parts, the turret, and the body during a burst fire. Masses are elastically coupled together. The developed mechanical model has got 8 DOF. The calculated results of the dynamic model were verified with experimental data. Moreover, the excitation force as the input data was obtained analyzing of measured data in real weapon and afterwards substituted by the analytical expression preserving the impulse of the shot force in course of every functional cycle. The clearances were modelled by springs with different characteristics when they are excluded or not.

Keywords—Automatic weapon, Combat vehicle, Dynamic model, Excitation force, Weapon mounting.

I. INTRODUCTION

One of the main requirement posed on the current anti-aircraft systems and weapons mounted on the infantry fighting vehicles (IFV) is to achieve of the high hit probability of the targets.

The radar, computer and optoelectronics technology development enables to parametrize of targets with accuracy comparable with the weapon technical dispersion, see [3], [5]. For example the bearing and elevation angles determination is possible with 2 mrad errors and the target velocity even with 0.1%. The aiming of allowance is calculated with 0.5 mrad accuracy. The limiting factor deteriorative the fire accuracy are vibrations of mounting and basic structure of the weapon when the constant errors are the highest in elevation and they can achieve up to 10 mrad, see [12]. The errors vary in course of burst fire for every shot and the fall can get till 50%. The determination of these vibrations makes more difficult the nonlinearity being in principle type of backlashes, backstops, and dry frictions. The fundamental problem is the unfamiliarity with nonlinearity real values. In course of practice calculations is trouble with verification on the real system which leads to the effect that only one nonlinear element is studied. The calculations lead into the make up of the discrete models having inertial, elastic and damping properties mainly as lumped parameters and the continuum characteristics are considered less (FEM).

The dynamic model has been made up with the next presumptions:
the weapon system is being in the rest before firing,
the fire plane is coincident with the vertical plane passing through the longitudinal vehicle axis, it means the weapon has zero bearing,
the elevation angle is constant, i.e. the target is not tracked,
the limited rigidity have coupling between the hull and unsprung masses, between hull and turret and between turret and elevation parts,
during whole action the masses are changeless,
backslashes are expressed by means of the nonlinear reduced rigidities in the couplings,
all considered forces and moments act in one vertical plane and the system is symmetrical, i.e. the planar dynamic system is studied.

The force acting in the weapon and causing the motion of all weapon parts depends on the type of operation. During burst firing they are periodic in nature, see [1], [2]. The exciting force in the case study is indicated as \( F_E \) in Fig. 2.
The considered system has 8 DOF, see Fig. 1 and Fig. 2:

\[ \mathbf{q} = [y_{k_0}, \Theta, x_v, \gamma_v, x_{\text{turret}}, y_{\text{turret}}, y_{\text{elevating parts}}, \alpha_{\text{elevating parts}}], \]  

(1)

where

- \( y_{k_0} \) - vertical displacement of hull,
- \( \Theta \) - angular displacement of hull,
- \( x_v \) - longitudinal displacement of turret,
- \( \gamma_v \) - vertical displacement of turret,
- \( x_{\text{turret}} \) - angular displacement of turret,
- \( y_{\text{elevating parts}} \) - vertical displacement of elevating parts,
- \( \alpha_{\text{elevating parts}} \) - angular displacement of elevating parts.

Then the system can be described by the matrix equation:

\[ [\mathbf{M}][\ddot{\mathbf{q}}] + [\mathbf{K}][\mathbf{q}] = [\mathbf{Q}], \]  

(2)

where

\[ [\mathbf{M}] = [m_{k_0}, I_{k_0}, m_v, m_v, I_v, m_{\text{turret}}, m_{\text{elevating parts}}, I_{\text{elevating parts}}], \]  

(3)

is diagonal mass matrix determined from the system kinetic energy where

- \( m_{k_0} \) - mass of hull,
- \( I_{k_0} \) - hull mass moment of inertia with respect to the transverse axis passing through the gravity centre,
- \( m_v \) - mass of turret,
- \( I_v \) - turret mass moment of inertia with respect to the transverse axis passing through the gravity centre,
- \( m_{\text{elevating parts}} \) - mass of elevating parts,
- \( I_{\text{elevating parts}} \) - elevating parts mass moment of inertia with respect to the transverse axis passing through the gravity centre.

When the mass matrix is diagonal it is favourable for calculations because the inversion of this matrix is simple. In order to the mass matrix will be diagonal several principles have to be kept such as deviation mass moments of inertia will be removed using of the suitable coordinate system, [18].

The \([\mathbf{K}]\) stiffness matrix elements are determined from the formulae (4), see [8], [11]:

\[ k_{ij} = \left( \frac{\partial^2 E_p}{\partial q_i \partial q_j} \right)_0, \]  

(4)

where

- \( E_p \) - system potential energy,
- \( n \) – number of DOF,
- \( 0 \) subscript – value in the balanced position.

The stiffness matrix is symmetrical due to the interchangeability of the corresponding derivation according to the \( q_i, q_j \) coordinates in (4).

For better understanding let us denote every general coordinate like this:

\[ y_{k_0}, \Theta, x_v, \gamma_v, x_{\text{turret}}, y_{\text{turret}} \]

\[ y_{\text{elevating parts}}, \alpha_{\text{elevating parts}} \]

The potential energy of the system is:

\[ E_p = 0.5k_1(y_{k_0} - \Theta a_1)^2 + 0.5k_2(y_{k_0} + \Theta a_1)^2 + \\
+ 0.5k_3(\Theta a_3 - y_{k_0} + y_v - \gamma_v a_5)^2 + \\
+ 0.5k_4(y_v + \gamma_v a_7 - \Theta a_4 - y_{k_0})^2 + 0.5k_5(\Theta a_5 + x_v + \gamma_v a_{17})^2 + \\
+ 0.5k_6(y_{k_0} \cos \alpha_k + \alpha_k a_{14} - x_v \sin \alpha_k - y_v - \gamma_v a_{10})^2 + \\
+ 0.5k_7(y_{k_0} \cos \alpha_k + \alpha_k a_{15} - x_k \sin \alpha_k - y_v - \gamma_v a_{11})^2 + \\
+ 0.5k_8(y_{k_0} \sin \alpha_k + x_k \cos \alpha_k + \alpha_k a_{16} + \gamma_v a_{13} - x_v)^2. \]  

(5)

The distances \( a_i \) are evident from the Fig. 2 with the exceptions these:

\[ a_1 \] - between \( C_H \) hull gravity centre and spring \( k_1 \),
\[ a_4 \] - between \( C_H \) hull gravity centre and spring \( k_4 \),
\[ a_5 \] - between \( C_V \) turret gravity centre and spring \( k_5 \),
\[ a_7 \] - between \( C_V \) turret gravity centre and spring \( k_7 \),
\[ a_{10} \] - between \( C_V \) turret gravity centre and spring \( k_{10} \),
\[ h \] - vertical distance between \( C_H \) hull gravity centre and \( C_u \) unsprung mass gravity centre.

The meaning of the elastic elements \( k_i \) is following:

- \( k_1 \) - reduced stiffness of the vehicle left part suspension,
- \( k_2 \) - reduced stiffness of the vehicle right part suspension,
- \( k_3, k_4, k_5 \) - reduced stiffness of the turret on the hull,
- \( k_6, k_7 \) - reduced stiffness of the trunnion,
- \( k_8 \) - this element is used for modelling of the stiffness of the elevating gear.

After derivation of the potential energy from (5) by means of (4) the stiffness matrix is given with the formulae (6).

The main nonlinearities considered in the system are: slackness in the ball raceway between turret and hull modelled by means \( k_i \) stiffness spring, slackness in the trunnion between turret and elevating parts modelled by means \( k_i \) stiffness spring and backlash in the elevating gear.
is represented by $k$, stiffness spring.

$$\mathbf{K} = \begin{pmatrix} k_{11} & k_{12} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{54} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{64} & k_{65} & k_{66} & k_{67} & k_{68} & k_{69} & k_{70} & k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{84} & k_{85} & k_{86} & k_{87} & k_{88} & k_{89} & k_{90} & k_{91} & k_{92} & k_{93} & k_{94} & k_{95} & k_{96} & k_{97} & k_{98} \end{pmatrix}. \quad (6)$$

The general forces, represented by $\mathbf{Q}$ matrix, encompass the others forces which are not included into matrix mentioned in parts of $\mathbf{M}$ and $\mathbf{K}$ matrixes. They are damping forces (tractive resistance force, forces (tractive resistance force, force straining backside part of track and force straining front side part of track) can be excluded from the solution. Otherwise the number of the degrees of freedom increases to nine.

The damping forces are implicated in forces having form

$$F_{a_k} = b_k \dot{q}_i \quad (7)$$

where $b_k$ - damping coefficient pertaining to the $i$th degree of freedom.

The equation are solved after arrangements in the next form

$$[\mathbf{M}]^{-1}([\mathbf{Q}]-[\mathbf{K}][\mathbf{q}]). \quad (9)$$

The equations describing the hull movement are:

$$m_{ko} \ddot{y}_{ko} + k_{11} y_{ko} + k_{12} \dot{\theta} + k_{14} y_v + k_{15} y_{y_v} = - \dot{y}_{ko} b_{yo}, \quad (10)$$

$$I_{ko} \ddot{\theta} + k_{21} y_{ko} + k_{22} \dot{\theta} + k_{23} x_v + k_{24} y_v + k_{25} y_{y_v} = - b_\theta \dot{\theta}. \quad (11)$$

The turret movements are expressed with three differential equations:

$$m_{ko} \ddot{y}_v + k_{31} \dot{\theta} + k_{33} x_v + k_{34} y_v + k_{35} y_{y_v} + k_{36} x_{E_v} + k_{37} y_v + k_{38} \alpha_E = - \dot{y}_v b_{y_v}, \quad (12)$$

$$m_{ko} \dot{y}_v + k_{41} \dot{\theta} + k_{42} \dot{\theta} + k_{43} y_v + k_{44} y_{y_v} + k_{45} x_{E_v} + k_{46} y_{y_v} + k_{47} \alpha_E = - \dot{y}_v b_{y_v}, \quad (13)$$

$$I_{ko} \ddot{\theta} + k_{51} y_{ko} + k_{52} \dot{\theta} + k_{53} x_v + k_{54} y_v + k_{55} y_{y_v} + k_{56} x_{E_v} + k_{57} \alpha_E = - \dot{\theta} b_{\theta}, \quad (14)$$

$$m_{ko} \ddot{x}_E + k_{61} x_v + k_{62} \dot{\theta} + k_{63} y_v + k_{64} \alpha_E = - x_{E_v} b_{x_E}. \quad (15)$$

$$I_{ko} \ddot{\theta} + k_{71} y_{ko} + k_{72} \dot{\theta} + k_{73} x_v + k_{74} y_v + k_{75} \alpha_E = - \dot{\theta} b_{\theta}, \quad (16)$$

$$I_{ko} \ddot{\theta} + k_{81} y_{ko} + k_{82} \dot{\theta} + k_{83} x_v + k_{84} y_v + k_{85} \alpha_E = - \dot{\theta} b_{\theta}, \quad (17)$$

### III. CALCULATION RESULTS

The system of the differential equations (10) – (17) has been solved using the Runge-Kutta of the 4th order integration method. The suitable integration step has been chosen as 0.0001s for the specific purpose. It corresponds to known condition between the minimal integration step and the maximal considered frequency $f_{max}$ of the undamped system, as it is recommended in [6] or [10] as well

$$\Delta t_{min} = \frac{1}{\pi f_{max}}. \quad (18)$$

In case of the spring stiffness change during impacts the integration step has to be cut down several times. It is necessary to get suitable results.

The main parameters of the parts under vibrations are given in the Tables I, II and III.

#### TABLE I. HULL PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{ko}$</td>
<td>hull mass</td>
<td>12.538 kg</td>
</tr>
<tr>
<td>$I_{ko}$</td>
<td>hull mass moment of inertia</td>
<td>22.544 kg.m²</td>
</tr>
<tr>
<td>$k_1$</td>
<td>suspension stiffness on the front side of hull</td>
<td>4.56 x 10⁴ N/m</td>
</tr>
<tr>
<td>$k_2$</td>
<td>suspension stiffness on the rear side of hull</td>
<td>4.56 x 10⁴ N/m</td>
</tr>
<tr>
<td>$a_1$</td>
<td>distance of the front hull spring from to gravity centre</td>
<td>1.174 m</td>
</tr>
<tr>
<td>$a_2$</td>
<td>distance of the rear hull spring from to gravity centre</td>
<td>1.174 m</td>
</tr>
<tr>
<td>$b_{y_v}$</td>
<td>dumping coefficient of angular motion</td>
<td>7.41 x 10⁴ N.m.rad⁻¹</td>
</tr>
<tr>
<td>$b_{x_E}$</td>
<td>dumping coefficient of linear motion</td>
<td>4.3 x 10⁵ N.s/m</td>
</tr>
</tbody>
</table>

The parameters in Table I have been determined from the technical documentation ($m_{ko}, k_1, k_2$) where suspension stiffnesses consist of the torsion bar springs and their values have been linearized in the operating point. The reduced distances $a_1, a_2$ have been calculated from the six sprung wheels by the torsion bars on the both sides. The damping coefficients $b_{y_v}, b_{x_E}$ have been settled by measurement of the
real weapon during the fire and the movement.

The parameters in Table II and Table III have been set by the same way.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>TURRET PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>( m_v )</td>
<td>turret mass</td>
</tr>
<tr>
<td>( I_v )</td>
<td>turret mass moment of inertia</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>spring stiffness on the front side of turret</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>spring stiffness on the rear side of turret</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>spring stiffness in the horizontal direction of turret</td>
</tr>
<tr>
<td>( b_{\gamma} )</td>
<td>dumping coefficient of angular motion</td>
</tr>
<tr>
<td>( b_{x_1}, b_{y_1} )</td>
<td>dumping coefficients of linear motion</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>ELEVATION PART PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>( m_E )</td>
<td>elevating parts mass</td>
</tr>
<tr>
<td>( I_E )</td>
<td>elevating parts mass moment of inertia</td>
</tr>
<tr>
<td>( k_6 )</td>
<td>spring stiffness of trunnion in vertical direction</td>
</tr>
<tr>
<td>( k_7 )</td>
<td>spring stiffness of elevating gear element</td>
</tr>
<tr>
<td>( k_8 )</td>
<td>spring stiffness of trunnion in horizontal direction</td>
</tr>
<tr>
<td>( b_{\alpha} )</td>
<td>dumping coefficient of angular motion</td>
</tr>
<tr>
<td>( b_{x_1}, b_{y_1} )</td>
<td>dumping coefficients of linear motion</td>
</tr>
</tbody>
</table>

The exciting force which was determined from the measuring is portrayed in Fig. 3. It is necessary to remind that the resultant force in Fig. 3 is put together from two 30 mm coupled cannons burst firing. They have the advanced primer ignition. It has been determined measuring by means of strain gauges where weapon casing are fixed to the cradle. The sampling rate has been 615 Hz which matches to the time between samples 1.625 ms. The force is being determined according to the operation of the automatic weapon. In [1] there is reminded that the force transmitted to the mount has a maximum value for the first shot fired. For the second shot there is a reduction in the firing force which is still further reduced for the third shot. It can be seen from the functional diagram in [1] that the maximum force applied to the mount occurs at the instant that the barrel is arrested and the breech carrier begins to act on the buffer. The spectral density variation of the firing force acting on the mount shows that the basic frequency is given by the rate of fire. Other frequencies are higher harmonics. The impulse of the force acting on the mount is the same as the impulse of the firing force, which allows quick and simple calculations of the force applied to the mount to be made.

In this case study has been considered the gas operated automatic cannon, see [3], having recoiling barrel and triggering when barrel moves into front position before every shot (the system is known as soft recoil system). In addition the buffers damping impacts of the breech block carrier and the barrel in the rear and front positions have been used, see [4]. The calculation of this weapon tends to be somewhat complicated and then the use of the measured data seems to be more suitable. Afterwards it is possible to proceed to the simplification of the exciting force when the main condition – the whole impulse of the exciting force during one shot equals to the impulse of the shot force causing by the gas pressure in the barrel – will be filled.

Then the \( F_E \) force has been approximated by the analytical formula preserving the impulse of the shot force, [13], [14] and pictured in the Fig. 4 as well:

\[
F_E = F_{SS} \left( 1 + \sin(\omega_k t + \psi) \right)
\]

where

\[
F_{SS} = \frac{I_H}{t_{FC}}
\]

where

\[I_H\] - impulse of shot force depending on the ballistic properties of the round,

\[t_{FC}\] - time of the functional cycle.

At the same time the main excitation frequency is

\[
f_E = \frac{1}{t_{FC}}
\]

\[
\omega_k = 2\pi f_E
\]

and

![Fig. 3 Measured excitation force](image-url)
\( \psi \) - phase shift enabling to solve problem when several weapons are mounted (multiple mounting of weapons) and firing with different initial time and with different characteristics as well.

The resultant excitation force is given as follows

\[
F_E = \sum_{j=1}^{n} F_{E_j},
\]

where \( j \) – number of firing weapons.

When one weapon fires then \( \psi = 90^\circ \).

\( T_n \) – the rise time given with the tangent line in the point where the output (hull angular displacement) achieves the half of the steady state value.

In our case there is approximately 1.56 Hz (hull eigenfrequency \( f_0 = 1.1 \) Hz).

The angular displacement/time history of the hull after 10 shots represents the Fig. 5. After three seconds when vibrations of the weapon system are damped the hull is in the basic position.

The influence of the clearance (in our case study 5 mm) in the turret ball path on the hull vibration does not considerable difference. On the other hand at the beginning of the turret motion several impacts of the hull and the turret occur as it is pictured in Fig. 6 where angular vibrations of the turret with respect to the hull are. After the transient period when the clearance is taken-up the turret moves with regular frequency.

It is possible to draw an inference from checking calculations that the hull behaves as a low frequency filter uniformly transmitting signals up to the specific cut off frequency. The range is known as a band pass where for the cut off frequency is valid the formulae, see [8]:

\[
f_n = \frac{1}{2T_n},
\]

where
The elevating parts vibration (absolute motion), see Fig. 8, during ten shots gives very important outcome that the main influence on the aiming errors is cased with the hull as it is possible to observe comparing with the Fig. 5. The relative variances of the elevating parts vibrations and the turret or the elevating parts and the hull are less than 1 mrad.

![Fig. 8 Elevating parts angular displacement](image)

### IV. CONCLUSION

The dynamic model in Fig. 2 enables to determine the effect of the parameters used there mainly their changes in course of the operation and in the service.

The results given in the figures reflect a good coincidence with the real piece which was explored according to presented theory. The theory was verified on the two coupled 30 mm automatic cannon. The success of the analysis depends on the experimental verification of the other parameters as are clearances in seating etc, see [7].

It has been proved that the main influence on the aiming errors have the hull vibrations. It makes possible to simplify the dynamic model to the two or three DOF, see [16], [17].

The sensitivity analysis will follow in the future when the changes of the hull, turret, elevating parts parameters will be considered.

In future it is supposed to study the influence of the change operation of the elevation and azimuth drive, the influence of the move on the ground when firing and the changes of the rate of fire. Using the equation (19) it is possible to solve the different excitation of the system when the operation system of the automatic weapon is changed.

The concluding step will work out the three-dimensional dynamic model involving among others the bearing angle and weapon mounting on the wheel combat vehicles, see [15].

The published theory has been applied in the Czech research institutes and in the University of Defence in Brno as an additonal study material for students.

### REFERENCES


**Jiri Balla** born in Poprad (Czechoslovakia), 6th June 1954. MSc degree in mechanical engineering at Military academy in Brno 1978. PhD degree in field weapons and protection against them at Military academy in Brno 1986. Assoc Prof of Military academy in Brno 1998 in field military technology, weapons and ammunition. Professor of Defence University in Brno 2006 in same field as Assoc Prof. Current the author’s major field of study is dynamics of weapon barrel systems. He worked in military units as ordnance officer. After PhD studies he was a teacher as lecturer and associate professor. He was visiting fellow at Royal Military and Science (RMCS) in Shivenham (UK) 1996,1997, 1998. Currently he is a professor at University of Defence in Brno at Weapons and ammunition department.

The main books:

Prof. Balla is member of Czech Association of Mechanical Engineers (CzeAME).