Abstract — Microparticles transport and retention in porous media has received scientific attention due to its vast application in many disciplines like; transfusion medicine, environmental science, and petroleum reservoir engineering. At low Reynolds number flow, a balance between interactive surface forces and drag forces control particle movement and retention in porous media. Surface forces are electrostatic and van der Waals (vdW) forces. We used atomic force microscopy (AFM) to measure surface forces between particles and posts representing solid matrix of porous media in microfluidic models. We then for the first time combined the measured surface forces with the drag forces induced by the fluid flow on a moving or adhered particle to determine particle velocity or detachment criteria. We derived criteria formulation to determine particle adhesion, detachment, and rolling from force balances. We verified our theoretical formulations with extensive microparticle suspension flow experiments in microfluidic models representing porous media. We studied the effect of surface properties and injection flow rates on particle movement in interstitial space domain and adhesion and detachment from the solid matrix.

Keywords — Atomic force microscopy, drag forces, detachment and rolling criteria, surface forces.

I. INTRODUCTION

Transport and retention of microparticles in porous media is the controlling process in many daily used equipment and facilities in our lives. Filters, lungs, kidneys, underground potable water sources, underground oil reservoirs, blood flow in arteries and veins are among many other examples. Transport of microparticles in porous media is a complicated multiphysics process. The movement of particles in interstitial space domain is controlled by the drag forces on the particles and the interactive surface force [1] (Israelachvilli, 2006) between the outer layer of the particles and the solid matrix in porous media. Microparticle characteristics like shape, size, mass density, and surface properties; fluid properties like viscosity, density, pH, ionic strength, and porous media characteristics like pore topography, morphology, and pore wall surface forces, and operating conditions such as flow rates and temperature control the fate of a moving particle through interstitial space domain, whether it passes through or filter out. In this paper we present our theoretical analysis on microparticle adhesion and detachment on circular cylinder posts in microfluidic models representing porous media.

II. MICROPARTICLE VELOCITY IN INTERSTITIAL SPACE DOMAIN

We assume that interactive forces between the microparticles themselves are negligible and they move singly. This assumption is true for dilute suspensions where the gap distance between the particles is large. The posts are stationary circular cylinders with regular spacing patterns.

A. Microparticle Shape Factors

The suspended particles of interest are spherical and oblate spheroids. Finding the flow patterns around an oblate spheroid is much more complicated than for a sphere [2]-[4]. In order to keep the analysis simple, we use a correction factor for the non-sphericity of the spheroidal particles to find an equivalent sphere for the spheroidal particles. Perrin factor [5] has been used to find the equivalent sphere radius for spheroids.

\[
\gamma = \frac{\bar{a}}{a} = \frac{1 - \eta'^2}{\eta'^{(1/3)}} \tan^{-1}\left(\frac{(1 - \eta'^2)^{0.5}}{\eta'^{-1}}\right)^{0.5}
\]

(1)

\[
\eta' = \frac{a}{b} < 1
\]

(2)

where, \(\gamma\) is the average Perrin factor which is the arithmetic average of the \(\gamma\)'s in three (x, y, z) axes around a spheroid. a and b are the major axes of the oblate spheroid. a is the radius of an sphere with a volume equal to that for an oblate...
spheroid, i.e., \( a = (a^2 b)^{1/3} \). \( \bar{a} \) is the radius of an equivalent sphere to an oblate spheroid obtained from Eq. (1). Perrin factor for an oblate spheroid (\( a = 1 \mu m, b = 3 \mu m \)) is equal to 0.9355, and the equivalent radius \( \bar{a} \) is equal to 1.35 \( \mu m \).

**B. Particle Velocity far from a Post**

The forces acting on moving particles in interstitial space domain are form drag \( \mathbf{F}_D \), frictional drag \( \mathbf{F}_f \), and surface forces such as electrostatic and van der Waals forces between the surface of the particles and posts. Body forces such as buoyancy or external forces such as electrical and magnetic fields are neglected in our analysis. Fig. 1 illustrates a moving microparticle near a circular post. Velocity field relative to a stationary coordinate are resolved tangentially and normally. We used Faxen’s law to calculate the force and torque on a rigid spherical particle that is moving in a flowing fluid [6]. According to Faxen’s law if a sphere to which fluid adheres is immersed in a fluid in motion at velocity \( \mathbf{V} \), and if the sphere center translates with velocity \( \mathbf{U} \) while sphere spins with angular velocity \( \omega \), then the force and torque on the sphere are;

\[
\mathbf{F} = 6\pi \mu a (\mathbf{V}_o - \mathbf{U}) + \mu \pi a^3 (\nabla \cdot \mathbf{V})_o \quad (3)
\]

\[
\mathbf{T} = 8\pi \mu a^3 (0.5[\nabla \times \mathbf{V}]_o - \mathbf{\omega}) \quad (4)
\]

where \( a \) is the radius of the sphere and \( \mu \) is the viscosity of the continuous phase fluid. The subscript “o” implies evaluation at the centre of the sphere.

**C. Surface Forces**

Surface forces act at very close distances. Repulsive surface forces are important when particles move at the vicinity of the posts. Repulsive forces are due to electrostatic double layer of the charges on the surface of the microparticles and posts. We measured surface forces between microparticles and posts in water using atomic force microscopy (AFM) [7]. Fig. 2(a) presents the steps in measuring surface forces. We attached a microparticle to the end part of a tip and moved the particle towards a surface (A: approach). Once the particle gets very close to the surface it started to respond to the interactive repulsive forces (B: repulsive interaction). We pushed the particle further down to touch the surface (C: Touched and pressed). Then we retracted the particle from the surface (C & D), and move it away from the surface (E). Interactive forces as a function of distance are plotted in Fig. 2(b). Fig. 3 shows a sample of the measured force spectroscopy data measured by AFM.

As shown in Fig. 3, at sufficiently far distances, interaction is negligible. The transition region when interaction is apparent but the surfaces have not come in contact is marked by a dotted ellipse. This region is where repulsive electrostatic forces interact. When the surfaces come in contact, van der Waals (vdW) forces [8] act as attractive forces. We used the retract force curve to measure attractive forces as marked by \( F' \)s is Fig. 3.

**D. Particle Velocity near a Post**

The resultant of the acting forces in \( r \)- and \( \theta \)-directions is given respectively as;

\[
\sum \mathbf{F}_r = \mathbf{F}_{Dr} - \mathbf{F}_s \quad (7)
\]

\[
\sum \mathbf{F}_\theta = \mathbf{F}_{D\theta} - \mathbf{F}_{sh} \quad (8)
\]

where, \( \mathbf{F}_{Dr} \) and \( \mathbf{F}_{D\theta} \) are the drag forces in \( r \) and \( \theta \) directions. \( \mathbf{F}_s \) is the repulsive force and \( \mathbf{F}_{sh} \) is the film shearing force acting on the particle. Squeezing resistance forces are inherently included in the repulsive forces measured by AFM. Repulsive forces act normal to the post and particle surfaces. Hence it was excluded from the \( \theta \)-direction force balance.
E. Shearing Resistance \((F_{sh})\) of the Fluid Between Particle and Post

Jeffrey and Onishi [9] suggested estimating the force associated with the shearing forces between the surfaces from:

\[
F_{sh} = (6\pi \alpha a U_0) \left[ \frac{-4\beta(2 + \beta + 2\beta^2)}{15(1 + \beta)^3} \ln \frac{1}{\varepsilon} + \frac{4(16 - 45\beta + 58\beta^2 - 45\beta^3 + 16\beta^4)}{375(1 + \beta)^4} \right] \cdot \varepsilon \cdot \ln \frac{1}{\varepsilon} + O(\varepsilon) \tag{9}
\]

where,

\[
\begin{align*}
\beta &= \frac{\pi R}{2a} \quad (10) \\
\varepsilon &= \frac{d}{a} \quad (11)
\end{align*}
\]

and \(d\) is the gap between the particle and post. Fig. 4 presents the results of instantaneous shearing forces \((F_{sh})\) between an approaching microparticle \((a = 1.35\mu m)\) toward a post \((3\mu m\) in diameter) at different gap spacing. The shearing force is maximum at \(\theta=90^\circ\) and is zero at \(\theta=0^\circ\) and \(180^\circ\), simply because the \(U_\theta\) is maximum at \(\theta=90^\circ\) and zero at \(\theta=0^\circ\) and \(180^\circ\). The coordinate of \(\theta\)-direction is illustrated in Fig. 1. Eq. (9) is singular at zero gap distances. We assumed a minimum gap distance of 0.1 \(\mu m\) in our calculations, i.e. \(F_{sh}=0\) at \(r = R+0.1\), where \(R\) is the radius of a post. The value of 0.1 \(\mu m\) is approximately the distance that repulsive forces start to interact as suggested by AFM measurements marked by the dotted ellipses in Fig. 3.

Fig. 2: Different stages of approaching and retracting a particle towards and away from a surface in atomic force microscopy (AFM) experiments to measure interactive forces as a function of distance.

Fig. 3: A typical approach and retract force curves between a microparticle and a post measured by AFM. The section of the trace curves marked by a dotted ellipse is the repulsive forces due to the electrostatic forces and lubrication resistance to displace the fluid between the surfaces of the particle and post.

Fig. 4: Typical shearing forces imposed by the fluid at close contact to the moving particle. This plot is for a particle interacting with a post with radius \(R\) \((V_\infty=10^{-5}m/s, a=1.35\mu m, \ R = 1.5\mu m)\)

A balance between the combination electrostatic and fluid squeezing repulsive forces and drag forces control the velocity of a moving particle near a post. The rotational movement of the particles is negligible. Therefore, the rotational and coupling elements in the mobility dyadic \((M)\) can be set equal to zero. We use constant and isotropic mobility factor \((M)\) [10]-[11] in our analysis to relate force to velocity as presented in Eq. (12).

\[
M = 6 \cdot \pi \cdot \mu \cdot a \quad (12)
\]
And the force balance suggests that;

\[ U_0^t = (F_{Dr} - F_{s}) \cdot M \quad (13) \]
\[ U_0^t = (F_{D0} - F_{s}) \cdot M \quad (14) \]

where \( U_0^t = U_0(t, \theta) \) and \( U_0^t = U_t(t, \theta) \) are the components of the particle velocity vector associated with the overall force balance on a particle.

III. PARTICLE ADHESION AND DETACHMENT

Once a particle comes in contact with the surface of a post, it may adhere to the surface. After adhesion, drag forces act either to press on or to detach the particle from the post, depending on the location of particle adhesion to the circular cylindrical post as illustrated in Fig. 5. The overall balance between the adhesive surface forces and the drag forces determines the fate of a particle to detach, shear off, roll, or remain adhered. \( F'_{Dr} \) and \( F'_{D0} \) are the components of the drag forces on an adhered particle to a post. \( F'_s \) is the AFM-measured adhesive force between the particle and post—maximum adhesion force in retract force curve as marked in Fig 3.

A. vdW Surface Forces and Frictional Forces

dvan der Waals (vdW) surface forces are attractive forces between the surface of the microparticle and posts. We measured vdW forces using AFM as shown as \( F'_s \) in Fig. 3. Frictional forces act at the contact surface of the particle and post. Contact static friction force \( F_{sf} \) is defined as,

\[ F_{sf} = \mu_s \cdot (F_{Dr} + F'_s) \quad (15) \]

where \( \mu_s \) is the contact static friction coefficient. The contact static friction force is proportional to the applied load normal to the surface of the post. The normal load here is the balance between the attractive forces and the radial component of the applied drag forces. The relation between \( \mu_s \) and normal load is linear to a threshold, after the threshold point, the particle starts to move, and the corresponding contact friction is called kinetic friction (\( \mu_c \)). Kinetic friction is usually less than the threshold static friction coefficient, and is constant for a wide range of the velocities. The particle shears off and detaches if the \( \theta \)-component of the drag forces exceeds \( F_{sf} \) otherwise it rolls or remains adhered. Rolling friction coefficient (\( \mu_r \)) is proportional to the normal load similar to the kinetic friction, and is much smaller than the kinetic friction coefficient. If the \( \theta \)-component of the applied drag forces exceeds the rolling friction, particle starts rolling on the surface of the post.

B. Drag Forces Acting on an Adhered Particle to a Post

The drag forces on an adhered particle to a post have two components: form drag \( (F'_{p}) \) and friction drag \( (F'_f) \) [12]. The pressure drag is obtained by integrating the pressure field around a particle as;

\[ F'_{p} = \int_0^\pi P(t = a) \cdot \cos \theta \cdot dA \quad (16) \]

where, \( dA = 2\pi \cdot a^2 \cdot \sin \theta \cdot d\theta \) for a sphere. The pressure field around a sphere is needed to solve the above integral. Langlois [12] solved the Navier-Stokes equation using stream function to obtain the pressure field for axi-symmetric flows around a sphere. Using the pressure field obtained by Langlois [12] (1964), the form drag on a particle attached to a post derived as;

\[ F'_{p} = \int_0^\pi P_\infty \cdot \left( \frac{3\mu \cdot a \cdot V}{2a^3} \right) \cdot \cos 0 \cdot 2\pi \cdot a^2 \cdot \sin 0 \cdot d\theta \quad (17) \]
We considered damping effects induced by an array of posts on the fluid velocity in our analysis.

The frictional drag force is the integral of the shear stresses on the surface of a particle;

\[ F'_t = \int_A \tau_{t0}(r = a) \cdot dA \]  \hspace{1cm} (18)

where, \( \tau_{r0} = \mu \left( \frac{1}{r} \frac{\partial V_r^s}{\partial \theta} + \frac{\partial V_\theta^s}{\partial r} \right) \) for a spherical particle.

Using velocity fields (by considering the damping effect) around a sphere attached to a post we obtain:

\[ \tau_{t0} = -\mu V_\infty \sin \theta \left( \frac{3a^3}{2r^3} \right) \]  \hspace{1cm} (19)

where \( V_\infty \) is the flow velocity corrected by damping effect. Alternatively, instead of applying damping coefficient to \( V_\infty \), an average velocity around the post can be used. The average velocity is the average of the fluid velocity from the surface of the post \((r = R)\) to a distance equal to the diameter of the particle \((r = R+2a)\). The average velocities in \( r \)-direction and \( \theta \)-direction are respectively;

\[ \bar{V}_r = \frac{\int_{r=R}^{r=R+2a} V_r \cdot dr}{\int_{r=R}^{r=R+2a} dr} = \frac{\int_{r=R}^{r=R+2a} V_\infty \cdot \cos \theta \left( 1 - \frac{R^2}{r^2} \right) \cdot dr}{2a} = \frac{2a}{(R+2a)} V_\infty \cdot \cos \theta \]  \hspace{1cm} (20)

\[ \bar{V}_\theta = \frac{\int_{r=R}^{r=R+2a} V_\theta \cdot dr}{\int_{r=R}^{r=R+2a} dr} = \frac{\int_{r=R}^{r=R+2a} -V_\infty \cdot \sin \theta \left( 1 + \frac{R^2}{r^2} \right) \cdot dr}{2a} = \frac{(2a+R)}{(R+2a)} V_\infty \cdot \sin \theta \]  \hspace{1cm} (21)

\( \bar{V}_r \) and \( \bar{V}_\theta \) are the average velocity components of the flowing fluid around an adhered particle to a post.

We integrated pressure and shear stress along the particle surface to derive form and frictional drag on an adhered particle to a post as;

\[ F'_{p\theta} = -\frac{4\pi \mu a^2}{(2a+R)} V_\infty \cdot \cos \theta \]  \hspace{1cm} (22)

\[ F'_{f\theta} = -\frac{8\pi \mu a^2}{(2a+R)} V_\infty \cdot \cos \theta \]  \hspace{1cm} (23)

\[ F'_p = \frac{-4\pi \mu a^2}{(2a+R)} V_\infty \cdot \sin \theta \]  \hspace{1cm} (24)

\[ F'_f = \frac{-8\pi \mu a^2}{(2a+R)} V_\infty \cdot \sin \theta \]  \hspace{1cm} (25)

Figs. 6 and 7 present the results of \( r \)- and \( \theta \)-component drag forces on an adhered particle to a post, respectively.

Fig. 6: \( r \)-component drag forces applied on an adhered particle to a post by the flowing fluid. \((V_\infty=5(10)^{-5}m/s, R=1.5\mu m, a=3\mu m)\)

Fig. 7: \( \theta \)-component drag forces applied on an adhered particle to a post by the flowing fluid. \((V_\infty=5(10)^{-5}m/s, R=1.5\mu m, a=3\mu m)\)

Drag forces (\( r \)-component) are maximum at the stagnation points (180° and 0°). The direction of the \( r \)-component of drag forces is toward the post surface at the upstream (180°>0°>90°), which keeps the particle attached. At the downstream (90°>0°>0°), the direction of drag force (\( r \)-components) is outward and tends to detach the particle from the post. The \( \theta \)-component drag force is zero at 180° and 0°, and is maximum at 90°. The \( \theta \)-component of the drag forces tends to shear off or roll the adhered particle at all locations on a post.
IV. DETACHMENT, SHEARING OFF, AND ROLLING CRITERIA

The competition between the attractive forces and drag forces on an adhered particle to a post determines whether the particle detaches, shears off, rolls or remains adhered [13]. We define appropriate criteria for these scenarios in next section.

A. Detachment Criterion

Detachment occurs if the radial (outward from the post surface) drag forces are larger than adhesive forces which suggest the criterion in Eq. (26);

$$-F'_s \pm F'_{pr} \pm F'_{fr} > 0$$  \hspace{1cm} (26)$$

Since the direction of the forces is assumed positive outward from the post surface, adhesive forces are negative. This equation suggests that the strength of the adhesive forces and the injection flow rate can be controlled in a way to minimize particle detachment for specific systems.

B. Shearing off Criterion

The \( \theta \)-component of the drag forces can shear off an adhered particle from a post if the criterion below is satisfied.

$$\left| \frac{-F'_s \pm F'_{pr} \pm F'_{fr}}{\pm F'_{ro} \pm F'_{bo}} \right| \cdot \mu_s < 1$$  \hspace{1cm} (27)$$

Shears off possibility is higher at the downstream \( (0 < \theta < 90) \) or rear section of a post because drag forces are positive.

C. Rolling Criterion

Particle rolling or tethering on the surface of a post is the next possibility of events on an adhered particle to a post. The rolling criterion is similar to Eq. (27) with \( \mu_s \) replaced by \( \mu_r \).

$$\left| \frac{-F'_s \pm F'_{pr} \pm F'_{fr}}{\pm F'_{ro} \pm F'_{bo}} \right| \cdot \mu_r < 1$$  \hspace{1cm} (28)$$

where \( \mu_r \) is the rolling friction coefficient. Particles preferably roll rather than shearing off from the surface of the posts, since \( \mu_r \ll \mu_s \).

Our analysis are in line with the with microfluidic micromodel experimental results, where adhered particles to the posts did remained adhered for a certain injection rate and then start to roll and finally detach by increasing injection flow rate.

ACKNOWLEDGMENT

We thank Mr. M. Moravvej for drawing Fig. 2.

REFERENCES