Abstract— Advanced engineering ceramic materials such as silicon carbides and silicon nitride have been used in many engineering applications. Cutting of such materials with abrasive waterjet is becoming the most recent cutting technique for its inherent advantages. In the present study, an elastic-plastic erosion model was adopted to develop an abrasive waterjet model for cutting brittle materials. As a result, a cutting model based on fracture mechanics was derived and introduced. The suggested model predicts the maximum depth of cut of the target material as a function of the fracture toughness and hardness, as well as process parameters. The maximum depth of cut predicted by the suggested model was compared with published experimental results for three types of ceramics. The effect of process parameters on the maximum depth of cut for a given ceramic material is also studied and compared with experimental work. The comparison reveals that there is a good agreement between the model predictions and experimental results, where the difference between the predicted and experimental values of the maximum depth of cut was found to take an average value of 10%.

Keywords—Abrasive Waterjet, Cutting ceramics, Waterjet cutting, Waterjet modeling.

I. INTRODUCTION

Abrasive waterjet cutting operates by the impingement of a high-velocity abrasive-laden water jet against the work-piece. It produces no heat, and therefore no heat-affected zone, to degrade metals or other materials. The finished edge obtained by the process often eliminates the need for postmachining to improve surface finish.

A coherent waterjet is formed by forcing high-pressure abrasive-laden water through a tiny sapphire orifice (figure 1). The momentum transfer between the water and the abrasives creates a focused high-velocity stream of particles that exits the nozzle at more than twice the speed of sound and cuts as it passes through the workpiece. Cuts can be initiated at any point on the workpiece and can be made in any direction of contour, linear, or tangential. The narrow kerf produced by the stream results in neither delamination nor thermal or nonthermal stresses along the cutting path.

In addition to applications in the machining of superalloys; armor plate; titanium; and high-nickel, chromium, and molybdenum alloys, abrasive waterjet machining can also be used to cut concrete, rock, glass, ceramics, composites, and plastics. The ability of the abrasive waterjet to cut most metals without any thermal or mechanical distortion places this innovative process on the leading edge of material cutting technology and has accelerated its development.

Advanced ceramics have been increasingly used in optical, electronic, mechanical and biological industries due to their inherent superior high temperature strength, hardness, wear and corrosion resistance. Since ceramics are extremely hard, high cost and dimension accuracy are two main problems encountered in the cutting process. Abrasive waterjet cutting technique has emerged as a promising machining method for ceramics and hard materials, in general.

Fig. 1 Typical nozzle configuration for mixing abrasive with waterjet in an abrasive waterjet cutting head

Machining performance, including depth of cut and cut quality is a major technological challenge to the AWJ machining technology. Machinability of ceramics by abrasive waterjets has been studied by Hocheng and Chang [1]. They tried to correlate between the quality of aluminum oxide and silicon nitride ceramics in slotting and the major machining parameters of the abrasive waterjet. Their experimental
results for slot cutting were evaluated in terms of material removal rate, kerf shape and surface roughness. They concluded that a sufficient supply of hydraulic energy as well as fine-mesh abrasives at moderate traverse speed produce a smooth kerf surface. Ramulu and Arola [2] have undertaken an experimental investigation to determine the influence of the cutting parameters on the surface roughness and kerf taper of a graphite / epoxy laminate, machined by an abrasive waterjet system. The feasibility of using abrasive waterjets for precision drilling of small diameter holes in a ceramic-coated component has also been studied by [3]. The results obtained indicate that the hole quality can be controlled by controlling the jet dwell time and feed rate. Chen et al. [4] have studied experimentally the effect of jet impact angle on the cutting quality. They also applied new oscillation technique for the cutting head to better cutting ceramic materials. Liu et al. [5] carried out a computational fluid dynamics study to understand the jet and particle dynamic characteristics so as to optimize the jetting and process parameters for enhancing the cutting performance. Wang and Liu [6] developed a jet characteristics model that enabled to evaluate the particle velocity distribution along and across an AWJ. Srinivasa et al. [7] have studied the influence of impingement angle and feed rate on the kerf geometry of silicon carbide. As a result, they established a good basis for developing strategies for controlled 3D AWJ machining of complex shapes. Other research works were done to improve the cut surface, observe the effect of process parameters on the cut surface and depth of cut, [8]-[11]. 

In order to effectively control and optimize the AWJ cutting process, predictive models for the depth of cut, are required. Yang et al. [12] used a neural network approach in modeling the surface roughness, while Saxena and Paul [13] developed numerical models for the various kerf characteristics. A number of mathematical models for the material removal rate and depth of cut have been reported, including those using solid particle erosive theories ([14], [15]), an energy conservation approach ([4], [16]), fracture mechanics ([17], [18]), dimensional analysis [19].

Erosion of ceramic materials has been generally viewed as a brittle fracture process, which occurs mainly by chipping. More modern view of ceramic erosion is based on the assumption that plastic deformation plays a crucial role in the chipping process (e.g. [20]-[22]). The morphology of fractures formed in ceramic materials during impact can be divided into two classes depending on whether the impacting particle is blunt or sharp ([23], [24]). The distinction between blunt and sharp particle impact is a distinction that depends on the role of plastic deformation in the impact process. The particle velocity that characterizes the transition between the formation of Hertzian cracks and the formation of radial cracks depends on the hardness, fracture toughness, and surface structure of the target material.

II. THE WATERJET CUTTING MODEL

In the present section, an AWJ cutting model is developed and presented. The model is based on an erosion model for brittle materials [25] and given by:

\[
\delta V = c v^{22/9} t^{1/3} \rho^{11/9} K_c^{-4/3} H^{1/9}
\]  

(1)

Where \(\delta V\) is the volume removal rate by an individual particle, \(c\) is the proportionality constant, \(v\) is the particle impact velocity, \(r\) is the particle radius, \(\rho\) is the particle density, \(K_c\) is the fracture toughness, and \(H\) is the vickers hardness of the target material. It is worth mentioning at this point that the erosion model, given by equation (1), suggests that the material volume removal depends, among others, on the fracture toughness of the target material, \(K_c\). The model is based on the assumption that the lateral crack size is proportional to the radial crack size, and that the depth of the lateral crack is proportional to the maximum particle penetration. Equation (1) can now be used to reach the following removal rate:

\[
\dot{V} = C \frac{c}{\pi} \left( \frac{m_s}{v_o} v_c^{22/9} r^{2/3} \rho^{2/9} K_c^{-4/3} H^{1/9} \right) f_1(\alpha_c)
\]  

(2)

where

\[
f_1(\alpha_c) = \frac{1}{5} \left( \sin \alpha_c \cos^4 \alpha_c - \frac{4}{15} \sin^3 \alpha_c \right) + \frac{4}{5} \sin \alpha_c
\]

In (2), it was assumed that the cutting front is parabolic, \(x = k y^2\), and the particle velocity \(v\) varies according to \(v = v_s \cos^2 \alpha\). The abrasive efficiency factor, \(C\), was also introduced. Details on these assumptions can be found in [26] and [17]. It was further shown in [26] that the material removal rate may be given by:

\[
\dot{V} = u d_j h f_2(\alpha_c)
\]  

(3)

where

\[
h = \frac{\tan \alpha_c}{2 k}
\]

and

\[
f_2(\alpha_c) = \frac{1}{2} \tan \alpha_c \left[ \tan \alpha_c \cos \alpha_c + \ln \left( \tan \left( \frac{\pi}{4} + \frac{\alpha_c}{2} \right) \right) \right]
\]

Now, equating (3) to (2), one gets:

\[
h = B g(\alpha_c) \left( \frac{1}{u d_j} \right) \left( \frac{m_s}{v_o} v_c^{22/9} r^{2/3} \rho^{2/9} K_c^{-4/3} H^{1/9} \right)
\]  

(4)

Where the following substitutions were made:

\[
\frac{1}{f_2(\alpha_c)} \text{ for } C , \ B \text{ for } \frac{c}{\pi}, \text{ and } g(\alpha_c) \text{ for } \frac{f_1(\alpha_c)}{f_2(\alpha_c)}
\]
Equation (4) now represents the proposed cutting model for brittle materials. Authors are responsible for obtaining any security clearances.

III. MODEL PREDICTIONS

The cutting model given by (4) relates the maximum depth of cut, \( h \), to the target material properties and the major process parameters. The only unknowns in the model are the constant \( B \) and the function \( g(\alpha_e) \). The exit angle, \( \alpha_e \), as illustrated in [17], would be in the range of 70° to 90°. The value of \( g(\alpha_e) \) for an average value of \( \alpha_e = 80° \) is calculated and included in table (1). The experimental results of the maximum depth of cut of Zeng and Kim [27], were used to calculate the model constant \( B \). The value of this constant is calculated and given also in table (1). It is interesting to find that the constant \( B \) is practically independent of the material type and process parameters. Therefore, the constant reported in table (1) is actually an average value of all the constants which have been obtained from using three ceramic materials (AD 85, AD 94, and AD 99.5) and 15 different process parameters.

### TABLE 1

<table>
<thead>
<tr>
<th>MODEL</th>
<th>( h = B g(\alpha_e) \left( \frac{1}{u d_{i}} \right) \left( \frac{V_{o}}{v_{a}} \right)^{2.29} r^{2.19} \rho^{-1.9} K_{e}^{-4.3} H^{1.9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(\alpha_e) )</td>
<td>0.05571 at ( \alpha_e = 80° )</td>
</tr>
<tr>
<td>( B )</td>
<td>1.003 x 10^{-2}, 1 x 10^{-3} st. dev. and 2.2 x 10^{-4} st. error</td>
</tr>
</tbody>
</table>

Having determined the constant \( B \) and the function \( g(\alpha_e) \), the present cutting model can now be used to determine the maximum depth of cut. It is worth mentioning here that in an earlier work by the author, [17], a waterjet cutting model has been developed which was based on a different erosion model than the one used here. A comparison between the two models will be made as we demonstrate the predictive capability of the present model.

The predicted depth of cut of the present model for 7 different ceramic materials are shown in figure (2), refer to table (2) for the ceramic materials used and their properties. For the sake of comparison, prediction results of the previous model (1998) are also included in figure (2). The figure clearly displays a good agreement between the predictions of both models. The two models are found to predict the same maximum depth of cut for all materials used (except for silicon carbide) within an average value of 3.9%.

Figure (3) represents the variation of the depth of cut with the traverse speed for two water pressures, namely; 100 and 400 MPa. It can be seen that both models predict the same maximum depth of cut within an average of 13% difference for the 100 MPa pressure, and 18% difference for the 400 MPa. However, it is to be noted that the present model predicts values of depth of cut higher than those predicted by the previous model for the 100 MPa pressure. But, the opposite can be seen for the 400 MPa pressure, where the previous model predicts a larger depth of cut.

![Graph showing depth of cut for different ceramic materials](image.png)

**Fig. 2** Depth of cut for different ceramic materials

(450 g/min abrasive flow rate; 1.25 mm nozzle diameter; 70 mm/min traverse speed; 200 MPa water pressure.)

### TABLE 2

<table>
<thead>
<tr>
<th>PROPERTIES OF CERAMIC MATERIALS</th>
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<tbody>
<tr>
<td>Ceramic material</td>
</tr>
<tr>
<td>------------------</td>
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<tr>
<td></td>
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<tr>
<td>Density (kg/m$^3$)</td>
</tr>
<tr>
<td>Hardness (GPa)</td>
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<tr>
<td>Compressive stress (MPa)</td>
</tr>
<tr>
<td>Fracture toughness (MPa$\sqrt{m}$)</td>
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<tr>
<td>Thermal conductivity (W/m K)</td>
</tr>
<tr>
<td>Coeff. of thermal expansion (1/°C)</td>
</tr>
<tr>
<td>Specific heat (J/kg K)</td>
</tr>
</tbody>
</table>
IV. COMPARISON WITH EXPERIMENTAL WORK

Comparison of the present model with the experimental results of Zeng and Kim [27] for two process parameters (traverse speed and abrasive flow rate) are shown in figures (5) and (6) for the AD99.5 ceramic. The range of process parameters used in these figures is the same as that used in [27].

The effect of jet traverse speed on the maximum depth of cut is shown in figure (5), where the experimental results of Zeng and Kim are included for comparison. The figure is seen to display a good agreement between the present model predictions and the experimental data for the maximum depth of cut, particularly for the range of traverse speeds between 25 mm/min and 125 mm/min. Table (3) reports the percentage difference between the predicted and the experimental values of the depth of cut.

### Table 3

<table>
<thead>
<tr>
<th>Traverse speed (mm/min)</th>
<th>22</th>
<th>48</th>
<th>70</th>
<th>95</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>%ge difference</td>
<td>0%</td>
<td>12%</td>
<td>5.5%</td>
<td>16%</td>
<td>23%</td>
</tr>
</tbody>
</table>

Comparison of the model with experiments at different abrasive flow rates is presented in figure (6). The model is seen to predict the experimental results reasonably well, within a maximum of 8.3%. Percentage differences between the predicted and experimental results are given in table (4) for the different abrasive flow rates.
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to brittle materials.

In the present work, a cutting model for brittle materials has been introduced and its predictions are discussed and compared with experimental results. The cutting model is based on an available erosion model which contain the property of fracture toughness, among other properties, of the target material. It is established that an erosion model containing the property of the fracture toughness is suitable for brittle materials such as ceramics.

The proposed model offers simple and closed form equation that can be used to predict the maximum depth of cut for brittle materials.

The cutting model is found to predict the experimental maximum depth of cut within an average value around 10%; averaging over all process parameters. It is also found that the predicted values of the maximum depth of cut correlate with the experimental results with a correlation coefficient of $\approx 0.95$.

V. CONCLUSIONS

REFERENCES