The stabilization of the rockets using a control system including a differentiator gyroscope, an accelerometer and a correction subsystem

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Abstract — The paper presents an angular stabilization system of the rockets in vertical plane using a differentiator gyroscope, an accelerometer and a correction subsystem. One has determined the transfer functions (in closed loop or in open loop) of the system in the complex variable or in discrete variable. All the eigenvalues of the system are placed in the left complex semi-plane (the proof of system’s stability). Using three different methods (least square method, instrumental variables’ method and neural networks method), one makes the identification of the system, obtaining (using a Matlab program and a Matlab/Simulink model) the frequency characteristics, responses to step or impulse input in the complex and discrete planes and time variations of the rocket’s overload and of the pitch angular velocity. With least square method (LSM) the output of the system and the output of the model were plotted. The identification is made very well – the two signals overlap. With the second identification method (instrumental variables method - MVI), one obtained the frequency characteristics for LSM and MVI on the same graphic. The identification may also be made using neural networks. Using this method, one obtained the individual responses of the control system and of the neural network before and after the training process.

Keywords — rocket, differentiator gyroscope, correction, identification, least square method, neural network.

I. INTRODUCTION

The stabilization systems for the anti-aircraft rockets, air-to-air rockets and ground-air rockets fulfill the functions of control over the load. Since most of these oscillations damping is weak ($\xi \leq 0.1$), it is difficult to control the overload. More the speed and flight altitude increases, the more difficult this mission is. Thus, the stabilization systems must correct the dynamic characteristics of the rockets. One also requires that the stabilization systems reduce the influence of external disturbances and internal noise. For this, bandwidth of the control and disturbance signals is chosen according to technical quality indicators [1].

II. DYNAMICS OF THE ROCKETS’ MOVEMENT

Next, one studies the stabilization systems’ dynamics of rockets with cross empennage. Mathematical model of rocket’s motion in the vertical plane is given by equations’ system (1), the coefficients being those of form (2).

$$\begin{cases}
\dot{\theta} = d_1 \alpha + d_2 \delta, \\
\dot{\omega}_z = d_3 \alpha - d_4 \delta - d_5 \omega_z, \\
\dot{\theta} = \omega_z, \alpha = \theta - \vartheta,
\end{cases} \quad (1)$$

where $\theta$ is the pitch angle of the rocket, $\omega_z$ – the pitch angular velocity, $\alpha$ – the incidence angle of the rocket, $\delta$ – the rocket’s command, $\vartheta$ – the slope of the trajectory; the other terms are coefficients with formula (2).

$$d_1 = \frac{\rho \frac{V^2}{2} Sc \alpha}{mV}, \quad d_2 = \frac{\rho \frac{V^2}{2} SlC \delta}{J_z},$$
$$d_3 = \frac{\rho \frac{V^2}{2} SlC \alpha}{J_z}, \quad d_4 = \frac{\rho \frac{V^2}{2} Sl^2 C \omega}{J_z};$$
$$d_5 = \frac{\rho \frac{V^2}{2} Sc \delta}{mV}. \quad (2)$$

To obtain the frequency characteristics of the system, step and impulse responses and identification of the system using three different methods (least square method, instrumental variables’ method and neural networks method), one uses the following coefficients

$$T_1 = \frac{1}{d_1} = T_V, \quad T_2 = \frac{1}{\sqrt{d_1 d_4 - d_3}} = T_V, \quad \xi = \frac{1}{2} \frac{d_1 + d_4}{\sqrt{d_1 d_4 - d_3}},$$
$$k_0 = \frac{d_1 d_2}{d_1 d_4 - d_3}, \quad k_d = \frac{d_2}{d_1 d_4 - d_3}, \quad k_w = k_0 V. \quad (3)$$

In the case of vertical flight of the rockets, the above equations set suffers little modifications.
\[ T_1 = \frac{1}{d_1} = T_T, \quad T_2 = \frac{1}{\sqrt{d_1 d_4 + d_3}}, \quad \xi = \frac{1}{2} \sqrt{d_1 + d_4}, \quad k_0 = \frac{d_4 d_4}{d_1 d_4 + d_3}, \quad k = \frac{d_4}{d_1 d_4 + d_3} \cdot k_w = k_0 V. \] (4)

For each rocket's type one must obtain the variation in time of coefficients \( d_i, \) \( i = 1, 5 \). In fig.1 the time variation curves of these coefficients for an ERLIKON rocket are presented. The values of these coefficients for second 10 of the flight are

\[ d_1 = 1.125 \left[ \frac{1}{1} \right] s, \quad d_2 = 25 \left[ \frac{1}{1} \right] s^2, \quad d_3 = 14.285 \left[ \frac{1}{1} \right] s, \quad d_4 = 0.535 \left[ \frac{1}{1} \right] s. \] (5)

The coefficients \( d_i \) characterize the stability of the system and the stability reserves; if \( d_5 = 0 \), then \( n^0 = d_1 \). The maneuverability of the system may be expressed on a graded scale which permits the choose of optimal maneuverability [1].

III. ANGULAR STABILIZATION SYSTEM WITH DIFFERENTIATOR GYROSCOPE, ACCELEROMETER AND CORRECTION SUBSYSTEM

Stabilization system that uses differentiator gyroscope, although has superior dynamic performances, doesn't assure their constant in different flight regimes. That's why, this system is recommended only for the stabilization of the rockets' angular position. The mono-loop stabilization systems have some disadvantages which prevent their use for the overload's control. Much better are the bi-loop stabilization systems.

The block diagram of the rockets' angular stabilization system with differentiator gyroscope, accelerometer and correction subsystem is presented in fig.2 [1]. The input variable is the rocket's command \( u_r \), while the output of the system is the rocket's overload \( n_v \).

On the direct way of the system one has introduced an integrator gyroscope and on the feedback of the exterior contour – an acceleration transducer (accelerometer), a correction network with the transfer function

\[ H_c(s) = \frac{T_s^4 T_s^4 s + 1}{T_s T_s^4 s + 1}. \] (6)

and an amplifier with \( k_k \) amplification factor for the compensation of the voltage's failure at the output of the correction network (subsystem). The transfer function of the interior loop is calculated as follows

\[ H_o(s) = H_k(s) \cdot \frac{n_v(s)}{u_r(s)} = k_u \cdot \frac{H_c(s)}{1 + k_k \cdot \frac{T_s^4 T_s^4 s + 1}{T_s T_s^4 s + 1} \cdot k_\delta \cdot H_s(s)}. \] (7)

In equations (7) and (8) the values of the constants are
The numerator and denominator of the transfer function (10) are the one presented above. The coefficients that appear in the transfer function in open loop is calculated in rapport with the one presented above. After calculus, the transfer function in closed loop becomes

\[ H(s) = \frac{n(s)}{u(s)} = \frac{B_s s^2 + B_s s + B_0}{A_1 s^3 + A_2 s^2 + A_3 s + A_4} \quad (10) \]

the transfer function in open loop is calculated in rapport with the one presented above. The coefficients that appear in the numerator and denominator of the transfer function (10) are

\[ B_2 = k_u k_i k_a V T_1 T_2 T_3 T_4 T_5 ; B_1 = k_u k_i k_a V T_1 T_2 \cdot (T_4 + T_5) ; \]

\[ B_0 = k_u k_i k_a V T_1 T_2 ; \]

\[ A_4 = T_3 T_2 T_1 T_5 ; \]

\[ A_3 = T_2 T_3 T_1 + T_2 T_5 + T_1 T_5 + T_3 T_5 + T_2 T_1 ; \]

\[ A_2 = T_2 T_4 T_5 + 2 T_2 T_1 + T_3 T_1 + T_5 T_1 + T_3 T_5 + T_5 T_1 + T_1 T_3 ; \]

\[ A_1 = T_1 T_3 + T_5 T_1 + T_1 T_5 + T_1 T_3 + T_5 T_3 + T_3 T_5 + T_5 T_3 + T_5 T_3 + T_3 T_5 + T_3 T_5 + T_1 T_1 + T_3 T_3 ; \]

\[ A_0 = T_1 T_3 T_5 + 57.3 g k_i k_a V T_1 T_2 + k_a k_u k_i k_a V T_1 T_2 + k_a k_u k_i k_a V T_1 T_2 . \]

Fig.3 The frequency characteristics for the system from fig.2

![Fig.3 The frequency characteristics for the system from fig.2](image)

For the above system one obtains, using a Matlab program and a Simulink model, the frequency characteristics (fig.3), indicial functions in the complex plane and in discrete plane, responses to impulse input in the complex and discrete planes and time variations of the rocket’s overload and of the pitch angular velocity. Also, one identifies the systems using three different methods (least square method, instrumental variables’ method and neural networks method). For each of these methods, some graphics were obtained. For the system from fig.2, the indicial functions and responses to impulse input in the complex and discrete planes are presented in fig.4 (the first two graphics correspond to the complex plane, while the last two correspond to the discrete plane).

The program calculates the matrices that describe the state equations of the system in the complex or discrete plane

\[ [-1 - 4.5 - 1638 - 7188 - 13980] \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ A = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ C = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ A = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

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\[ A = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ D = [0] ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ A = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ C = 10^3 ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ D = [0] ; \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

![Fig.4 The indicial functions and responses to impulse input in the complex and discrete planes for the system from fig.2](image)

\[ p_1 = -1000; \]

\[ p_2 = -19.5 + 32.5i; \]

\[ p_3 = -19.5 - 32.5i; \]

\[ p_4 = -2.4 + 2i; \]

\[ p_5 = -32.4 - 2i, \]

the zeros, the transfer functions in complex description or in discrete description, the stability margins and so on.

For the stabilization system one plots (using the Simulink model from fig.5) the time variation of the system’s output – rocket’s overload (\( \dot{u} \)) and time variation of the pitch angular velocity (\( \dot{\theta} \)) - fig.6. From these graphic characteristics and from the analysis of the system’s eigenvalues (poles) one notices that the system is a stable one with very dynamic properties.
IV. IDENTIFICATION OF THE SYSTEM USING THE LEAST SQUARE METHOD (LSM)

A state estimator must assure the controllability of the system whose parameters are estimated, whatever the adaptive structure [2], [3]. The least square method doesn’t always give models characterized by controllability. That’s why in some cases it must be modified. The system A and the estimated model \( \hat{A} \) are described by the equations

\[
\begin{align*}
L(z^{-1})y(t) &= \sum_{i=1}^{n} \alpha_i z^{-i} \hat{b}_i(t) + \hat{C}(z^{-1}) \hat{e}(t) + d, \\
\hat{L}(z^{-1})\hat{y}(t) &= \sum_{i=1}^{n} \beta_i z^{-i} \hat{b}_i(t) + \hat{\hat{C}}(z^{-1}) \hat{\hat{e}}(t) + d,
\end{align*}
\]

(14)

where \( z^{-1} \) – the delay operator, \( L(z^{-1}) \) and \( M(z^{-1}) \) are polynomials containing the coefficients of the discrete transfer function, \( \hat{e}(t) \) is the noise applied to the model and polynomials \( \hat{L}, \hat{M} \) contain the estimated coefficients of \( L \) and \( M \).

The LSM algorithm (least square algorithm) modification is based upon the discrete transfer function modification through origin pole \( (z = 0) \) compensation. The modified LSM algorithm (LSMM) builds a convergent vector \( v(t) \) and with it the vector of the estimated parameters [4]

\[
\hat{b}'(k) = \hat{b}(k) + P(k)v(k).
\]

Thus, the coefficient \( \hat{b}' \) is almost non-null.

The control law may be chosen of general form

\[
u(k) = R(z^{-1}, \hat{b}')u(k) + S(z^{-1}, \hat{b}')y(k),
\]

(16)

with the polynomials

\[
R(z^{-1}, \hat{b}') = \sum_{i=1}^{n} z^{-i}r_i(\hat{b}'), \quad S(z^{-1}, \hat{b}') = \sum_{i=0}^{B} z^{-i}s_i(\hat{b}').
\]

(17)

The closed loop system is described by equation [4]

\[
W(k + 1) = D(z^{-1}, \hat{b}')W(k) + \begin{bmatrix} \hat{e}(k + 1) \\ 0 \end{bmatrix},
\]

(18)

where

\[
D(z^{-1}, \hat{b}') = \begin{bmatrix} \hat{L}(z^{-1}, \hat{b}') & \hat{M}(z^{-1}, \hat{b}') \\ zS(z^{-1}, \hat{b}') & zR(z^{-1}, \hat{b}') \end{bmatrix},
\]

(19)

and

\[
\hat{e}(k + 1) = x^T(k)\hat{b}(k) - b^T(k) + n(k + 1),
\]

(20)

\( n(k + 1) \) is a white noise.

In the Matlab program the input \( u \) and the perturbation \( e \) of the leaded system are chosen as random type. For the \( \hat{b} \) parameters of model \( \hat{A} \) estimation one uses ARX operator from Matlab, which has the following syntax \( th=ARX(z,nn) \), where \( z = [y \ u] \) – matrix that contains the output vector
\( y(t) \) and the input vector \( \{u\} \); \( nn = [na \ nb \ nc] \) - defines the denominator order \( na \), numerator order \( nb \) and the model’s delay \( nc \); \( \theta \) returns the estimated parameters in \( \text{theta} \) format (the elements of the vector \( \hat{b} \)).

The program plots the characteristics \( y(t) \) and \( \hat{y}(t) \) presented in fig.7; \( y(t) \) is the output of the control system \( (A) \), while \( \hat{y}(t) \) is the output of the estimated model \( \hat{A} \). As one can see in the above figure that the identification is made very well - the two signals overlap \( (\hat{y} \rightarrow y) \).

V. IDENTIFICATION OF THE SYSTEM USING THE INSTRUMENTAL VARIABLES’ METHOD (MVI)

This method is a generalization of LSM. It gives the estimated parameters only for the determinist part of the model \( \hat{A} \). The equation equivalent to the second equation (14) is

\[
y(k) = x^T(k) \hat{b} + \hat{e}(k). \tag{21}
\]

By multiplication of this equation with \( W(k) \) - instrumental variable vector (whose elements haven’t physic significations, they are only necessary “instruments” for the \( \hat{b} \) estimation), one obtains the equation of estimator \( \hat{b} \)

\[
\hat{b} = \left[ \sum_{k=1}^{N} W(k)x^T(k) \right]^{-1} \left[ \sum_{k=1}^{N} W(k)y(k) \right], \tag{22}
\]

where \( N \) is the measurements number; the vector \( W \) may be chosen in different ways. Let vector \( W \) be [5]

\[
W(k) = F(z^{-1}) [u(k-1) u(k-2) \ldots u(k-n_w)]^T, \tag{23}
\]

where \( n_w = m + n \); if \( \hat{L}(z^{-1}) \) and \( \hat{M}(z^{-1}) \) are the \( \hat{L}(z^{-1}) \) and \( \hat{M}(z^{-1}) \) polynomials estimations, one chooses

\[
F(z^{-1}) = \hat{L}^{-1}(z^{-1}). \tag{24}
\]

The input \( u \) and perturbation \( e \) of the leading system are random type too. For the parameters estimation of the vector \( \hat{b} \) one uses, in Matlab, instead of operator \( \text{ARX} \), the operator \( \text{iv4} \). In fig.8 the frequency characteristics for the system (using LSM - continuous line, blue color and MVI – dashed line, red color) are plotted.

VI. IDENTIFICATION OF THE SYSTEM USING THE NEURAL NETWORKS’ METHOD

Flying parameters’ modification and atmospheric disturbances lead to difficulties in stability derives calculus and to flying objects’ models stabilization. That’s why one may use identification methods or state estimate methods [6], [7], [8], [9], [10], [11]. The identification method presented in this paper is based on a neural network’s use. As one can see in fig.9 [6], for off-line identification, a feed-forward neural network is used; the network is trained by minimizing the quadratic quality indicator \( J(k) = \frac{1}{2} e^2(k), e(k) \) being the training error.

The dynamics of the rockets’ movement may be described by equation

\[
y(k) = f\{y(k-1) y(k-2) \ldots y(k-n_y) u(k-q) \ldots u(k-q-n_u+1)\}, \tag{25}
\]

with \( y = \theta \) – the pitch angle, \( u = u_c \) – rocket’s command, \( q \) – dead time; \( n_y \) and \( n_u \) express the system’s order.
If nothing is known about the control system \((n_y, n_u, q, f\) and \(n_h\) – the number of hidden layer neurons), by identification one determines these parameters. So that, starting from minimal neural network’s architecture (numbers \(n_u, n_h, n_y\) and \(q\)) and imposing a value for the error \(e(k)\) and a maxim number of training epochs, the neural networks begins the training process. If the error \(e(k)\) doesn’t tend to the desired value then \(n_u, n_y\) and \(n_h\) are modified [6].

For identification process’s simulation of the rockets’ dynamics with neural network one may use the discrete transfer function associated to the system. A neural network with one hidden layer is chosen. This network is characterized by \(5, 3, 1= h, u, n\) and \(q=[6]\). One chooses calculus steps \((p)\), which is equal with vector \(y\)’s components number (the values at respective moments of the control system). The matrix of neural network \(P\) is obtained (it has the dimension \((n_u+n_y)\times(p-3)\)). Also, matrix \(T\) (of desired output of the network, which represents control system’s output values matrix) is the matrix of the system output’s values at time moments corresponding calculus steps; \(\text{dim}(T) = n_x \times (p - 3)\), \(n_x\) being output neurons’ number (in this example \(n_x = 1\)).

In fig.10 one presents the output of the system from fig.2 (blue color) and the output of the NN (red color) before training. After the training process, the two signals overlap (fig.11).

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By neural network’s training pseudo – neurons weights matrix \(W_1\) and hidden layer neurons weights vector \(W_2\) are obtained. Also, vectors \(B_1\) and \(B_2\), which contains polarization coefficients’ values (bias) for neurons from hidden layer and for output neuron, respectively, are obtained. For this stabilization system they are

\[
W_1 = \begin{bmatrix}
-6.4859 & 5.1131 & 7.2158 & 28.3934 \\
7.2276 & -3.7265 & 1.8016 & 34.9041 \\
-5.8420 & 10.9790 & -7.7418 & -8.5953 \\
-9.3483 & -1.0506 & 5.3482 & 28.8782 \\
-12.1872 & 2.0662 & 8.4692 & 0.1911
\end{bmatrix} \quad B_1 = \begin{bmatrix} 1.2729 \end{bmatrix} ; \quad (26)
\]

\[
W_2 = \begin{bmatrix}
0.0925 & 0.2145 & -0.5448 & 0.044 & -0.2575 \\
1.3135 & 0.1911 & 8.4692 & 2.0662 & 1.6618 \\
8.5953 & 7.7418 & -10.9790 & 8.4692 & 5.3482 \\
34.9041 & 2.0662 & 8.4692 & 0.1911 & 5.3482 \\
22 & 11 & 1 & 1 & 1
\end{bmatrix} \quad B_2 = \begin{bmatrix} 0.0217 \end{bmatrix}.
\]

VII. CONCLUSIONS

The paper presents a stabilization system for the rockets’ movement in vertical pane using a differentiator gyroscope, an accelerometer and a correction subsystem. One has determined the transfer functions (in closed loop and in open loop) of the system; a study of stability is made. All the eigenvalues of the systems are placed in the left complex semi-plane. This is a proof of system’s stability. The system responds very fast to a step input – the duration of the transient regimes is about 2 seconds. Using three different methods (least square method, instrumental variables’ method and neural networks method), one makes the identification of the system. One obtains, using a Matlab program and a Simulink model, the frequency characteristics, indicial functions in the complex plane and in discrete plane, responses to impulse input in the complex and discrete planes, the poles, the zeros, the stability margins and
so on. With the least square method (LSM) the output of the system and the output of the model for the two systems are plotted (fig. 7). As one can see in this figure, the identification is made very well - the two signals overlap \( \hat{y} \rightarrow y \).

With the second identification method (instrumental variables method - MVI), one obtained the frequency characteristics for LSM and MVI on the same graphic.

The identification may also be made using neural networks. Using this method, one obtained the indicial response of the system and of the neural network before and after NN's training. Before training the two signals were different, but after training (about 45 epochs) these signals overlap too. One also obtained the weights and the biases of the neural network. The dependence between the error of the training process and the training epochs number for the system is plotted (the error tends to its imposed value \( 10^{-6} \)).

REFERENCES