Simulation of Postural Sway by Elastically Tethered Random Walk

France Sevšek, Darja Rugelj

Abstract—Stabilometry is routinely used to assess postural steadiness of human body by measuring the movements of the centre of pressure (postural sway) of a standing subject with a force platform. Data acquisition time dependence of various postural sway parameters has been studied by analysing simulated data. A model was developed that described the centre of pressure movements as a random walker fixed to an elastic tether. Data series of various lengths that were simulated with different values of the elastic parameter were analysed by the same procedure as is normally used for the experimental ones. The total area of the simulated stabilogram and its ratio to the central area, as determined by the principal component analysis, were found to depend not only on the elastic parameter but also on the total path length and on the data acquisition time.

Keywords—force platform, Fourier analysis, random walk, stabilometry, sway area.

I. INTRODUCTION

HUMAN body has evolved as an efficient moving system. We can easily walk, run, climb or jump whereas still standing is usually only the starting position for the next movement. In normal subjects still standing is not a static but dynamic process where the body centre of mass is kept over the base of support by constant adjustments of various body segments which result in postural sway. As the human balance control system depends on feedback from the somatosensory, vestibular and visual systems, the measurement of the centre of pressure (COP) movements of a standing subject can give clues about the functioning these systems. This is the basis of stabilometry where a subject is asked to stand still on a special platform that is mounted on pressure sensors which transmit data via analogue to digital converter to a computer. In such a way, with suitable software, the time dependence of the trajectory of the COP can be determined. This is a well established method to assess balance properties of elderly and during rehabilitation. Anyhow, the interpretation of the results and the physiological meaning of the calculated parameters is still an open question. Besides the standard parameters, such as COP trajectory length, velocity, displacements and frequency distributions, the interpretations of measured stabilograms in terms of non-linear dynamics are also very promising [1]. Quite often it is also of interest to compare the areas within which the movement of the COP is confined [2]. In this case the principal component analysis (PCA) of the covariant matrix is mostly used where the eigenvalues are calculated from the covariant matrix [3]. To calculate the stabilogram area we have developed a new method [4] based on Fourier analysis of the data outline which gives not only better area estimate but also determines its shape.

Different data acquisition procedures have been proposed to quantify postural steadiness in order to assess differences between age groups [5], pathologies [6] and as outcome measurement after treatment protocols [7]. Time intervals for the data acquisition range from 10 s [7], 15 s [8], 20 s [6], 30 s [3] up to 120 s [9], [10]. As a reliable data acquisition protocol for COP measurements it was proposed to acquire 3 trials of 120 s each [9]. However, quite often the subjects under investigation are quite fragile and have diminished balance with difficulties to stay on the narrow base (feet together) for longer time. Besides, all subjects are expected to experience muscle fatigue in prolonged standing which introduces additional time dependence of the experimental data.

Thus, for given experimental conditions it is very important to select the most appropriate analysis of the COP trajectory, where the measurement time dependence of the calculated parameters is of utmost significance. This led us to study simulated data with well defined properties. In this work we present a data model that describes the COP movements as a random walker fixed to an elastic tether. Data thus simulated at different conditions were analysed by the same procedures as the experimental ones. The simulation procedure is similar to our previous one [11] from which it differs in the elastic energy expression.

II. METHODS

The stabilometric data were simulated by considering COP as a random walker that is tethered to a fixed central point by a tether of given length and elastic coefficient. The calculations were done by the random walk procedure, combined with Metropolis algorithm [12]. In each step the COP position was randomly moved where the maximal step length ($d_{max}$) was expressed in the unstressed tether length ($L_0$). The resulting position was then accepted with the probability $\exp(-(\Delta R^2 - \Delta R_0^2)/\alpha)$, where the elastic energy difference due to the move depended on the tether extension at the current ($\Delta R$) and previous position ($\Delta R_0$) with the elastic parameter $\alpha$ corresponding to twice the temperature divided by the elastic coefficient of the tether ($\alpha = 2T/\gamma$). After a large number of steps thus generated trajectory samples the configurations in accord with the canonical Boltzmann distribution. This model has only two independent parameters: the elastic coefficient
\( \alpha \) and the maximal possible trajectory length, defined as the maximal step length \( (d_{\text{max}}) \) multiplied by the number of simulated steps \( (N) \).

The actual calculation always started with a point somewhere inside the central region \( (R < L_0 = 1) \) and the first 50,000 points were rejected to allow the system to thermalize. After collecting the required number of data points they were analyzed with the same procedures that is normally used for the experimental data. The computations were performed on a PC-type computer, running under Linux operating system with the data simulation program written in Fortran using the portable pseudo-random number generator ran3 [13], based on a subtractive method that has very long period and no known defects.

Data analysis was done with a web-based software that had been specially developed for our stabilometric measurements. It is running under Linux (Fedora) operating system and consists of system procedures and data analysis programmes written in C, Fortran and PHP, and is publically accessible. For graph plotting our system relies on open code software Gnuplot. Such a centralised concept of data analysis greatly facilitates software maintenance and development, and enables users to do calculations from remote computers using different platforms.

The typical analysis of the stabilometry data started by selecting the desired time interval and by optional data smoothing by calculating moving average or Gaussian filtering. Gaussian function of chosen width \( (\sigma) \) is calculated in discrete points \( (x_i) \) as \( 1/(\sigma/(2\pi))^{1/2} \exp(-x_i^2/(2\sigma^2)) \) until it is smaller than a predefined value (e.g. 0.001). This discrete function is normalized and used for filtering the acquired data. It is expected that Gaussian function does not introduce new features in the filtered data [14]. From the resulting data standard statistical parameters are calculated, such as RMS and averages of the absolute values of COP displacements and velocities. Afterwards we proceed by plotting time and frequency distribution diagrams, and finish by determining the outline of the measured data, calculating its Fourier coefficients, area, fractal dimensions and other related parameters.

The procedure to determine the outline of the COP movement area has been described elsewhere [4]. In brief: data points are converted into polar coordinates by calculating their distances \( R_i \) from the area centre and the respective polar angles \( \phi_i \). The full angle is then divided into chosen number of intervals, usually between 50 and 100, depending on the number of data points and required precision. In each angular interval the point that is furthest from the centre is determined. To these outline points an analytical expression is fitted [15]:

\[
R(\phi) = R_0 + \sum_{m=1}^{m_{\text{max}}} [A_m \cos(m\phi) + B_m \sin(m\phi)], \quad (1)
\]

where \( A_m \) and \( B_m \) are the appropriate Fourier coefficients and \( m_{\text{max}} \) the maximal chosen number of coefficients to describe the outline.

Fitting is done by solving the resulting system of linear equations by the method of LU decomposition [16] which decomposes the matrix into the product of a lower and an upper triangular one from which the solutions can be calculated by a simple substitution. As the resulting Fourier coefficients \( A_1 \) and \( B_1 \) are generally not zero the centre of the contour is moved accordingly and all the coefficients are recalculated as described above. This procedure is repeated until the coefficients \( A_1 \) and \( B_1 \) are sufficiently small.

From the obtained Fourier coefficients of the outline the postural sway area was simply calculated from the Fourier coefficients as:

\[
A = \int_{0}^{2\pi} R(\phi) dR d\phi = \pi R_0^2 + \pi \sum_{m=1}^{m_{\text{max}}} [A_m^2 + B_m^2]. \quad (2)
\]

III. RESULTS AND DISCUSSION

The influence of the experimental time on the parameters of stabilograms was studied by analysing the simulated data. As described above, series of 90,000 points were calculated at various values of the elastic parameter \( \alpha \). The radius of the inner circle, which corresponds to the length of the unstretched tether \( (L_0) \), was set to 1. The same value was also used for the maximal step length \( (d_{\text{max}}) \). The average acceptance ratio for the moves was thus between 0.23 at \( \alpha = 0.00001 \) and 0.12 at \( \alpha = 3 \). Different segments of thus simulated data were then analysed by the same procedure as is normally used for our experimental data and the average values of the resulting parameters were calculated.

As an example, a stabilogram of simulated 3000 points is shown in Fig. 1. This length corresponds the usual one minute measurement with sampling frequency of 50 Hz. These data were calculated with the elastic parameter \( \alpha = 1 \) whereas for drawing the smooth outline 10 Fourier coefficients were used. The rugged outline in Fig. 1 connects the points to which the fitting was done.
As the total path length was linearly dependent on the number of analysed points at given value of the elastic parameter, it was the independent variable of choice. Thus, in Fig. 2 the stabilogram area is presented as a function of the total path length. The plotted areas are the average values of the ones calculated from the Fourier coefficients of the outline using (2). It is seen that with increasing the path length, which corresponds to longer measurement time, the area of the stabilogram increases. This increase is more pronounced the larger the value of the elastic parameter \(\alpha\). When the elastic parameter is very small the movement is mainly confined to the central circle of the radius \(L_0 = 1\) whereas larger elastic parameter means that the simulated COP spends more time further from the central region. Here longer simulation times and thus path lengths result also in larger stabilogram areas as more moves with small probability have chance to happen.

In stabilometry it is usual to calculate the area by the principal component analysis (PCA) of the covariant matrix [3]. Here the eigenvalues \(\sigma_0^2\) are calculated from the covariant matrix \(\sigma_{xy}^2\):

\[
\sigma_{xy}^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}),
\]

where \(\bar{x}\) and \(\bar{y}\) are the mean values and the summation is done over all \(N\) measured points.

The two eigenvalues are thus

\[
\sigma_0^2 = \left(\sigma_{xx}^2 + \sigma_{yy}^2 \pm \sqrt{(\sigma_{xx}^2 - \sigma_{yy}^2)^2 + 4(\sigma_{xy}^2)^2}\right)/2.
\]

The sway area is then reproduced by an ellipse with the two principal axes 1.96 \(\sigma_0\) which would, in ideal case of Rayleigh distribution, enclose 85.35 % of all points.

For our data it was found, as expected, that the PCA area depends on the elastic parameter, but is virtually independent of the path length.

From our previous work [2] the ratio between the total area as determined by the Fourier analysis of the outline and the one given by the PCA procedure seemed to be the most promising discriminating parameter between the stabilograms that differed mainly by the small number of COP excursions from the central region. We thus named it the index of sudden excursions (ISE). From our simulated data it was found that this index depended on the elastic parameter and was also very sensitive to the actual path length (Fig. 3).

IV. CONCLUSIONS

A large number of stabilograms have been simulated by an elastically tethered random walk model and analysed by our standard procedure for the experimental data. This model proved to be quite simple but could, by varying the elastic parameter, result data series that were similar to the experimental ones.

It was shown that the total areas of the simulated stabilograms depended not only on the elastic parameter but also on the total path length and thus on the experimental time. The same was true also for the calculated values of the index of sudden excursions (ISE) which was defined as the ratio of the total stabilogram area to the one calculated by the principle component analysis. On the other hand for our model the stabilogram areas calculated by the standard PCA method were virtually independent of the path length.

As shown from our simulated data the total area and ISE proved to be very sensitive to the actual properties of the stabilogram. They both reflect isolated large excursions of the COP outside the central region of the stabilogram which occur with low probability. Besides the calculated Fourier coefficients of the stabilogram outline give the information about its shape. As it was also shown that both the total area and ISE depend on the data acquisition time they may be relied on only for comparing the stabilograms measured under the
same conditions and the data acquisition time must be always reported together with these parameters.

The use of elastic tethered random walk model for description of the actual stabilometric measurements and the clinical meaning of the elastic parameter still remain to be studied.

All the described computer programs are available from the authors upon request.

REFERENCES


