Control of Chaos in Nonlinear Gyros Via Optimal Backstepping Method

A.R. Khalilzadeh and M. Taleb Ziabari

Abstract—This paper has presented chaos Control in the Gyroscope using the backstepping approach. Backstepping method has positive parameters. Generally, parameters are chosen arbitrarily, the values of these parameters are important because the incorrect values lead to instability. Genetic algorithm finds best values for parameters. GA finds the optimal value for the fitness function. This selected fitness function is for minimizing the least square error. Fitness function forces the system error to decay to zero rapidly that it causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach to its minimum value.

Keywords—Gyro chaotic, Lyapunov, Backstepping, Genetic Algorithm.

I. INTRODUCTION

In recent years, much attention has been paid to the investigation of chaos synchronization. Since the discovery of chaos synchronization [1], there have been tremendous interests in studying the synchronization of chaotic systems, see [2, 3] and the references therein for a survey of recent development. As chaotic signals could be used to transmit information from a master system to a slave system in a secure and robust manner, chaos synchronization has been intensively studied in communications research [4-8]. Recently, specialists from nonlinear control theory turned their attention to the study of chaos synchronization and its potential applications in communications. [9] Presented a speed gradient method for adaptive synchronization of chaotic systems. [10] Casted the problem of chaos synchronization as a special case of observer design. [11] Proposed a robust nonlinear $H\infty$ synchronization method for chaotic Lur’s systems with applications to secure communications. [12] Considered the problem of controlled synchronization of nonlinear systems using a passivity based design method. [13] Presented an adaptive observer-based synchronization scheme, where an adaptive observer for estimating the unknown parameters of the master system was designed, which corresponds to the parameter modulation for message transmission. Due to these developments, chaos synchronization as well as chaos communications have attracted revived interests in the nonlinear control community. The idea of chaos synchronization was utilized to build communication systems to ensure the security of information transmitted [14-17].

In particular, backstepping design and active control have been recognized as two powerful design methods to control and synchronize chaos. It has been reported [18-20] that backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems.

The goal of this paper are two fold; in the first is designed the backstepping controller for Control of Chaos in Nonlinear Gyro system. The method is a systematic design approach and consists in a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the design of the control. Second, optimize the controller with Genetic Algorithm. Genetic algorithm optimizes the controller to gain optimal and proper values for the parameters. GA minimizes the fitness function to find minimum current value. On the other hand fitness function finds minimum value to minimize least square errors.

The paper is organized as follows. Section 2 describes Gyro chaotic system. Section 3 presents backstepping method. Section 4 design backstepping controller for synchronization. Section 5 describes GA and algorithm used here. Section 6 results of this work would be compared with the chaos control results of Backstepping method [18]. Section 7 provides conclusion of this work.

II. GYRO CHAOTIC SYSTEM

The model system which we study is the gyroscope which has attributes of great utility to navigational, aeronautical and space engineering, and have been widely studied. Generally, gyros are understood to be devices which rely on inertial...
measurement to determine changes in the orientation of an object. Gyros are recently finding application in automotive systems for Smart Braking System, in which different brake forces are applied to the rear tyres to correct for skids.

Recently, presented the dynamic behaviour of a symmetric gyro with linear-plus-cubic damping, which is subjected to a harmonic excitation and the Lyapunov direct method was used to obtain the sufficient conditions of the stability of the equilibrium points of the system.

The equation governing the motion of the gyro after necessary transformation is given by[6]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g(x_1) - \alpha x_2 - bx_2^3 + \beta \sin x_1 + f \sin \omega t \sin x_1$$

where $$g(x_1) = -\alpha^2 \left(1 - \cos x_1\right)^2$$ and $$\alpha^2 = 100, \beta = 1, a = 0.5, b = 0.05, \omega = 2, f = 35.5$$ and initial condition $$(x_1, x_2) = (1, -1)$$. The nonlinear gyro given by equation (1) exhibits varieties of dynamical behaviour including chaotic motion - displayed in Fig.1 and Fig.2.

![Fig.1 shows the phase portrait of gyro scope system.](image1)

![Fig.2 shows the states trajectory variation for gyro scope system.](image2)

III. BACKSTEPPING DESIGN FOR STRICT-FEEDBACK SYSTEM

Consider the strict-feedback nonlinear system as follow:

$$\begin{align*}
\dot{x}_i &= f_i(x_1, ..., x_n) + g_i(x_1, ..., x_n)x_{i+1} \quad ; 1 \leq i \leq n-1 \\
\dot{x}_n &= f_n(x) + g_n(x)u
\end{align*}$$

(2)

Where $$x = [x_1, ..., x_n]^T$$, $$f_i(0)$$ and $$g_i(0)$$ are smooth functions with $$f_i(0) = 0$$ and $$g_i(0) \neq 0$$.

Step 1.

Considering the first subsystem of Eq.(2), we take $$x_2$$ as a virtual control input and choose

$$x_2 = \frac{1}{g_1(x_1)}[u_1 - f_1(x_1)]$$

(3)

The first subsystem is changed to $$\dot{x}_1 = u_1$$.

Choosing $$u_1 = -k_1 x_1$$ with $$k_1 > 0$$, the origin of the first subsystem $$x_1 = 0$$ is asymptotically stable, and the corresponding Lyapunov function is $$V_1(x_1) = x_1^2 / 2$$, Eq. (3) is changed to

$$x_2 = \Phi_1(x_1) = \frac{1}{g_1(x_1)}[-k_1 x_1 - f_1(x_1)]$$

(4)

Step 2.

Considering $$(x_1, x_2)$$, take $$x_3$$ as a virtual control input and choose

$$x_3 = \frac{1}{g_2(x_1, x_2)}[u_2 - f_2(x_1, x_2)]$$

(5)

The $$(x_1, x_2)$$ subsystem is changed to:

$$\begin{align*}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= u_2
\end{align*}$$

(6)

Which is in the form of integrator backstepping, so the control law $$u_2$$ is as follow:

$$u_2 = -\frac{\partial V_1}{\partial x_1}g_1(x_1) - k_2 \left[ x_2 - \Phi_1(x_1) \right] + \frac{\partial \Phi_1}{\partial x_1} \left[ f_1(x_1) + g_1(x_1)x_2 \right]$$

(7)

Where $$k_2 > 0$$. This control law asymptotically stabilizes $$(x_1, x_2) = (0, 0)$$ and Lyapunov function is as (8).
Substituting (7) into (5) gives
\[ x_3 = \Phi_2(x_1, x_2) = \frac{1}{g_2} \frac{\partial V_1}{\partial x_1} g_1 - k_2 (x_2 - \Phi_1) + \frac{\partial \Phi_1}{\partial x_1} (f_1 + g_1 x_2) - f_2 \]  
(9)

The remaining step can be deduced by analogy. Until step \( n \), we shall determine the actual control law \( u = \Phi_n(x) \), which can asymptotically stabilize Eq.(2). The backstepping method expanded for class of nonlinear MIMO systems [16].

IV. DESIGN CONTROLLER WITH BACKSTEPPING

In the following we will use backstepping approach to design a controller for supressing chaos in gyroscope system. To achieve this, we introduce a control signal \( u(t) \) to (1) and rewrite the system in strict-feedback form as follows:
\[ \dot{x}_1 = x_2 \]  
(10)
\[ \dot{x}_2 = g_2(x_2) - a x_2 - b x_2^3 + b \sin x_2 + f \sin wt \sin x_1 + u(t) \]

The backstepping is used to bring the states \( x_1, x_2 \) to the desired references via the torque \( u \) calculated with four steps.

Step 1.

Considering the first subsystem of Eq.(2).
\[ \dot{x}_1 = x_2 \]  
(11)

Construct the joint Lyapunov function.
\[ V_0(x_1) = \frac{1}{2} x_1^2 \]  
(12)

Take \( x_2 \) as a virtual control input and choose.
\[ x_2 = \Phi_0(x_1) = -k_1 x_1 \]  
(13)

Step 2.

Considering all system
\[ \dot{x}_1 = x_2 \]  
\[ \dot{x}_2 = g_2(x_2) - a x_2 - b x_2^3 + \beta \sin x_2 + f \sin wt \sin x_1 + u(t) \]  
(14)

Take \( u \) as an actual control input and choose:
\[ u = \Phi_n(x_1, x_2) = -(1 + k_2) x_1 \]  
\[ -(k_1 + k_2) x_2 - g(x_1) \]  
\[ +a x_2 + b x_2^3 - \beta \sin x_1 - f \sin wt \sin x_1 \]  
(15)

And take the Lyapunov function as
\[ V_n(x_1, x_2) = V_0 + \frac{1}{2} (x_2 - \Phi_0)^2 \]  
(16)

V. GENETIC ALGORITHM

The most of optimization algorithms are based on the gradient of the cost function, so for the ill choice of the initial point or the interval search, these algorithms can be misled on the locally optimum and can’t achieve the globally optimum. For this problem, a class of optimization algorithms, like genetic algorithms, is developed to avoid this constraint.

In its most general usage, genetic algorithms refer to a family of computational models inspired by evolution. These algorithms start with many initial points in order to cover all search interval and encode a potential solution to a specific problem on a simple chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implantation of genetic algorithms begins with a population of chromosomes randomly bred. We evaluate each chromosome by using objective function called Fitness function. In order to apply the genetic reproductive operations called crossover and mutation, we select, randomly, two individuals called parents and we apply the crossover operation, if its probability reaches, between parents by exchanging some of their bits to produce two children . A mutation is the second operator applied on the single children by inverting its bit if the probability reaches. After this stage we obtain two population : a parent population and a children population, the individual who has a goodness solution is preserved [17].

The genetic algorithms are used to search the optimal parameters \( k_j \) in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response. The fitness function used is
\[ f = \frac{1}{n} \sum_{i=1}^{n} (x_i - x_{ti})^2 \]  
(17)

\( x_i \) is system state and \( x_{ti} \) is favorit mood for \( x_i \). Based the system purpose for placing the states at
zero value; \( x_d = 0 \). By the training, can be obtained optimal parameters as \( k_1 = 2.861, k_2 = 1.493 \).

### TABLE I.

**GENETIC ALGORITHM PARAMETERS**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size population</td>
<td>100</td>
</tr>
<tr>
<td>Maximum of generation</td>
<td>300</td>
</tr>
<tr>
<td>Prob. crossover</td>
<td>75</td>
</tr>
<tr>
<td>Prob. mutation</td>
<td>0.001</td>
</tr>
<tr>
<td>( k_1 ) search interval( d )</td>
<td>[0.1 1.0]</td>
</tr>
</tbody>
</table>

### VI. DISCUSSION

In ref.[18] the Backstepping method was used to control the chaotic gyro that Fig.3 to Fig.5 represent the simulation results. But in this study is used Optimal Backstepping method. Now we would compare the results of the proposed method and the results in ref.[18]. The result of ref.[18] are showed in Fig.3 to Fig.5. The states trajectory variation for chaotic gyro are showed in Fig.3 and Fig.4. The control signal trajectory variation is showed in Fig.5.

![Fig.3 shows the \( x_1 \) state trajectory variation for gyroscope system.](image)

![Fig.4 shows the \( x_2 \) state trajectory variation for gyroscope system.](image)

By comparing the Figs., the following results can be obtained.

- In the Optimal Backstepping Method in relation to the Backstepping Method, the system states are stabilized by a more limited control signal. Consequently, it is less possible that the control signal to be saturated.
- In the Optimal Backstepping Method in relation to the Backstepping Method [18], Controlwill be accomplished in a much shorter time.

Considering the results obtained from simulations, the much more efficiency of Optimal Backstepping Method in relation to the Backstepping Method will be demonstrated.

### VII. CONCLUSIONS

This paper has presented a new hybrid backstepping approach with genetic algorithm that is demonstrated to have more optimal behavior when compared with previous methods. This approach is used for chaos Controlin the Gyroscope by using backstepping method for controlling of Gyroscope chaos. This controller has positive parameters. If the parameters are chosen incorrect, the system does not response well and it's possible the system be instable.

The genetic algorithm optimals the controller to gives the best result for minimizing fitness function. Fitness function is choosed for minimizing Least Square State Space. This function decrease the overshoot, setting time and minimizing Error. Also by selecting different fitness function can have other appropriate results.

### REFERENCES


