Moisture transfer phenomena in porous materials. Numerical modeling

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Abstract - The purpose of the work is the determination of unsteady variation of the moisture content in the walls. The mathematical model is based on humidity transport equations in porous media. The humidity transport takes place as a result of the presence of both moisture content and vapor pressure differences.

Using the proposed mathematical model, a computer program was developed in EES (Engineering Equation Solver). Numerical simulations were made for an office room, considering interior walls containing a gypsum board layer. The results showed that humidity absorption-desorption in an interior wall depends on: wall structure, materials properties, inlet air parameters, initial relative air humidity in the wall and room occupancy.

Keywords – Absorption-desorption, humidity, moisture transfer, numerical modeling.

I. INTRODUCTION

The present paper proposes a new mathematical model necessary in the study of humidity absorption or desorption in interior walls, a model that is also useful for a detailed humidity balance. According to this model, simulations were made for an office room.

Moisture in porous building materials can lead to several kind of damages like salt crystallization, mould growth [1] or frost damage.

Such studies are important for maintaining hygro-thermal comfort at the interior, and also for reducing energetic costs related for obtaining the interior comfort in general.

II. MOISTURE TRANSPORT PHENOMENA. PARAMETERS

A. Material’s moisture content

Moisture content (u) of a certain material is defined as the quantity of water absorbed by the mass of that material.

If the quantity of water absorbed is related to the material’s volume, the moisture content becomes:

\[ w = u \cdot \rho \] \hspace{1cm} (1)

All materials with open porosity could have moisture content between 0 and \( w_m \) (maximum moisture content when all the pores are filled with water). This maximum level of moisture content can be reached only by artificial treatment. In practical cases this level cannot be reached.

The relation between the moisture content, relative humidity and temperature represents the hygroscopic curve of the material. It is characteristic for each material and can be generated for the moistening process-desorption curve and for the drying process-desorption curve. Fig. 1 represents the hysteresis curve for gypsum board [2], [10].

The absorption and desorption are the result of two phenomena: the water absorption in the pores (mono or multi-molecular layers) and capillary condensation in micro-pores for relative humidity more than 40%. The influence of the temperature is less important than the relative humidity impact.

Fig. 1: Hygroscopic curve (absorption and desorption isotherms) for gypsum board
The affinity between water and porous materials defines the hygroscopicity notion and the difference between absorption and desorption is called hygroscopic hysteresis.

To draw the hygroscopic curve of a material, different analytical relations can be used; one of the expressions is used for the studied case, for gypsum board [3]:

$$u = \frac{0.00336 \cdot \phi}{(1 + 10^{-8} \cdot \phi) \cdot (1 - 0.901 \cdot \phi)} \text{ [kg/kg]} \quad (2)$$

The evolution of moisture content, function of relative humidity, is represented in Fig. 2. Theoretical curve was verified to be in agreement with experimental values provided by N.I.S.T. (National Institute of Standards and Technology, Gaithersburg).

![Fig. 2: Moisture content dependency on relative humidity for gypsum board](image)

- **B. Water vapor diffusivity \((D_w)\)**
  
  This parameter is obtained by the formula [4]:

  $$D_w = \frac{\delta_p \cdot P_{sat}}{\rho \cdot \frac{\partial u}{\partial \phi}} = \frac{\delta_p \cdot P_{sat}}{\rho \cdot \xi} \quad [m^2/s] \quad (3)$$

- **C. Specific capacity of water vapors \((\xi)\)**
  
  Specific capacity represents the absorption curve’s slope of the material:

  $$\xi(t, \phi) = \frac{\partial u(t, \phi)}{\partial \phi} = \frac{\partial u(\phi)}{\partial \phi} \quad [kg/kg] \quad (4)$$

- **D. Water vapor’s permeability \((\delta_p)\)**
  
  For homogeneous materials, the permeability is obtained as a product between the material’s permeance \((M)\) and the layer thickness of the material \((\Delta x)\).

  $$\delta_p = M \cdot \Delta x \quad [kg/(m \cdot s \cdot Pa)] \quad (5)$$

  The permeance is a function of relative humidity [5], having the expression:

  $$M = \exp(A_0 + A_1 \cdot \phi + A_2 \cdot \phi^2) \quad [kg/(m^2 \cdot s \cdot Pa)] \quad (6)$$

  \(A_0, A_1, A_2\) are coefficients calculated by mathematical regression.

  The permeance tends to be constant and doesn’t depend on relative humidity. This process is showing the importance of water-vapor diffusion in the pores for the humidity transport.

### III. HUMIDITY TRANSPORT IN POROUS MATERIALS: THEORETICAL BASES.

The main support for modeling was the humidity transport equation in porous materials, to determine the dynamic variation of moisture content for the studied material. The analysis focused on an interior wall having a gypsum board layer as its component. Humidity transport is possible through the water and/or water vapors transport.

In the presented model [11], the following hypotheses are taken into consideration:

- vapor diffusion is determined by the vapor pressure gradient
- vapor pressure difference can be established according to the humidity content, by means of absorption (isothermal) curves of humidity
- Fick’s diffusion law is applied
- materials analyzed have homogeneous properties
- the air flow through the considered structure is neglected
- the density of the dry material remains constant during the absorption process.

Humidity transport takes place as a consequence of potential differences: concentration difference \((u)\) and vapor pressure difference \((p_v)\).

The general humidity transport equation is:

$$\rho_{mat} \frac{\partial u}{\partial t} = -\nabla \cdot m_m \quad (7)$$

The humidity transport equation is particularized for materials with large pores and for capillary materials. The moisture flow for macro-porous materials (where the capillary effect is low) is:

$$m_m = -\delta_p \cdot \nabla p_v \quad (8)$$

The moisture flow in capillary materials is rendered by:

$$m_m = -\rho_{mat} D_w \nabla u - \delta_p \nabla p_v \quad (9)$$

Thus, the general humidity transport equation [6] for capillary materials becomes:

$$\rho_{mat} \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x} \left[-D_w \cdot \rho_{mat} \frac{\partial u}{\partial x} - \delta_p \frac{\partial p_v}{\partial x} \right] \quad (10)$$

### IV. HUMIDITY ABSORPTION MODELLING FOR AN INTERIOR WALL

Humidity transport and accumulation phenomena modeling in interior walls had, as a starting point, the
general humidity propagation equation for porous materials [6]. This equation was integrated by using the finite difference method, for a material layer of thickness $\Delta x$. The resulting equation is:

$$\rho_{mat} \cdot A = \left[ -\frac{D_v \cdot \rho_{mat} \cdot B - \Delta \rho \cdot C}{\Delta x} \right]$$  \hspace{1cm} (11)

The following notations were used:

$$A = \frac{\Delta u}{\Delta t}, \quad B = \frac{\Delta u}{\Delta x}, \quad C = \frac{\Delta \rho_v}{\Delta x}$$  \hspace{1cm} (12)

The differences of potential occurring in the humidity transport were stressed out as follows:

- the moisture content difference in the wall structure:

$$\Delta u = u_{wall,new} - u_{wall,old}$$  \hspace{1cm} (13)

where: $u_{wall,new} = \text{moisture content in the wall, at the actual moment}$

$u_{wall,old} = \text{moisture content in the wall, at the preceding moment}$.

- the vapor pressure difference:

$$\Delta \rho_v = p_{v,room} - p_{v,wall,new}$$  \hspace{1cm} (14)

where: $p_{v,room} = \text{partial water vapor pressure in the room}$

$$p_{v,wall,new} = \text{wall vapor pressure, at the actual moment}$.$

Relative humidity is expressed according to partial vapor pressure and to maximal vapor pressure, $p_{sat}$.

$$RH_{room} = \frac{p_{v,room}}{p_{sat}} \quad \text{and} \quad RH_{wall,new} = \frac{p_{v,wall,new}}{p_{sat}}$$  \hspace{1cm} (15)

where: $RH_{room} = \text{relative humidity of room air}$

$RH_{wall,new} = \text{relative humidity of wall air, at the actual moment}$.

By means of the humidity transport equation, the moisture content in a wall structure could be determined, at different moments [7]. The model was rendered in the computer program EES (Engineering Equation Solver).

The water flow that can be accumulated in the mass of room walls is calculated by the equation:

$$\dot{M}_{wall} = m_{wall} \cdot \frac{\Delta u}{\Delta t}$$  \hspace{1cm} (16)

The term expressed by $\frac{\Delta u}{\Delta t}$ is the moisture content variation in the wall, kg water/kg material.

This equation was used in the room humidity balance:

$$\dot{m}_{puls} \cdot w_{puls} - \dot{m}_{ex} \cdot w_{ex} + \dot{M}_{oc} - \dot{M}_{wall} = 0$$  \hspace{1cm} (17)

Where: $\dot{M}_{oc} = Nbr \cdot g_0$  \hspace{1cm} (18)

By solving the equation system (11), (16), and (17) the relative humidity of inside air and of wall air can be determined, as well as the variation of these parameters in the given time intervals [8], [9].

V. RESULTS

The simulation was made in the following conditions [15]:
- the considered room has a surface of 15.12 m$^2$ and three interior walls having a layer of gypsum board with the thickness $\Delta x = 0.018$ m.
- the gypsum board layer is initially dry: $RH_{wall} = 20\%$
- the ventilation airflow rate is of 40 m$^3$/h/person.
- two cases were analyzed: the office is empty (Nbr = 0, which happens at night), and the office is occupied (Nbr = 2, which happens during day-time).
- the specific humidity flow rate is $g_0 = 33$ g/h/pers. (0.92*10$^{-5}$ kg/s), in winter, for a $t_{in} = 20\degree C$ and occupants resting.

Note: The initial wall relative humidity (20%) is rarely met in actual fact. This value was chosen in order to ensure the limit conditions for the study [12].

![Fig. 3: Relative humidity values evolution for an empty space](image-url)
The way in which the water flow absorbed into the walls mass varies can be traced in Fig. 5. For a dry wall, the absorption phenomenon is more intense at the beginning, while, after 5-6 hours, the wall reaches the saturation point. In the case of an occupied office, the water quantity absorbed by the wall is higher than for an empty space [14].

An overall image of the dynamic behavior of a wall, according to the evolution of inlet air relative humidity, is presented in Fig. 6.

The office was considered occupied, and the initial wall relative humidity is of 40%. Three situations can be singled out in the diagram:

- when $\text{RH}_{\text{puls}} = 90\%$, the inside air relative humidity, and that of the wall air tend to increase towards the balance state, reached after 4-5 hours. The phenomena of humidity absorption into wall structure is similar to the previously defined situation.
- when $\text{RH}_{\text{puls}} = 40\%$, the relative humidity of the inside air increases a little, and reaches the balance state after 1 hour of evolution.
- when $\text{RH}_{\text{puls}} = 20\%$, the conditions achieved in the room are no longer favorable for the occupants’ hygro-thermal state of comfort. The inside air relative humidity and that of the wall air reach a balance state after 1 ½ - 2 hours of evolution. In this final case, a process of humidity desorption in the mass of the walls takes place. The interior walls will give away part of the water amount accumulated into their structure by the room air.

VI. CONCLUSIONS

Humidity absorption – desorption in an interior wall depends on several factors: the wall structure and the properties of the materials used [13], the inlet air parameters, the initial relative air humidity in the wall and the room occupancy.

When the office is occupied, the inside air and wall air relative humidity reaches a balanced state after 4-5 hours, while, if the room is empty, the balance takes place after 4 hours. If the wall initial relative humidity is of 40%, and the inlet air has a level of 90% relative humidity, the room balance is reached after 5 hours. Also, if the inlet air has a 20% relative humidity, the interior walls will give off the accumulated humidity in an interval of 1 ½ - 2 hours.

This study can be useful in determination of the humidifying degree necessary during the cold season inside the buildings in order to obtain the interior comfort for the occupants.

NOMENCLATURE

- $\rho_{\text{mat}}$ = density of the dry porous material, kg/m$^3$
- $u$ = moisture content of material, kg water/kg material
- $t$ = time, s
- $x$ = distance, m
- $m_m$ = moisture flow, kg/(m$^2$ s)
- $\delta_p$ = water vapor permeability, kg/(m s Pa)
- $\xi$ = specific capacity of water vapors, kg/kg
- $p_v$ = vapor pressure in the porous material, Pa
- $p_{\text{sat}}$ = saturation pressure of water vapor, Pa
- $D_w$ = water vapor diffusivity, m$^2$/s
- $\text{RH}_{\text{room}}$ = relative humidity of room air, %
- $\text{RH}_{\text{wall}}$ = relative humidity of wall air, %
- $\text{RH}_{\text{puls}}$ = relative humidity of inlet air, %
- $m_{\text{wall}}$ = mass of the walls, kg
- $M_{\text{wall}}$ = moisture flow adsorbed by the walls, kg water/s
\[ \dot{m}_{\text{puls}} = \dot{m}_{\text{ex}} = \text{ventilation airflow rate, kg/s} \]

\[ w_{\text{puls}} = w_{\text{ex}} = \text{inlet air humidity ratio, kg water/kg air} \]

\[ \dot{M}_{\text{oC}} = \text{humidity flow from persons, kg/s} \]

\[ \text{Nbr} = \text{number of persons in the room} \]

\[ g_{\text{o}} = \text{specific humidity flow rate, kg/(s pers)} \]

\[ t_{\text{puls}} = \text{supply air temperature, °C} \]

**REFERENCES**


