# Memristor modeling based on its constitutive relation

Viera Biolkova, Zdenek Kolka, Zdenek Biolek, and Dalibor Biolek

***Abstract—*** Most hitherto published memristor models start from its state-space representation. Another approach is described in this paper, utilizing the unambiguous memristor characteristics, the so-called constitutive relation, i.e. the flux-charge relation. The constitutive relation is independent of the way the memristor interacts with its surroundings, and it determines the memristor uniquely like, for example, the voltage-current characteristic defines the resistor. The method is used for SPICE modeling of charge- and flux-controlled memristors.

***Keywords—***Constitutive relation, memristor, modeling, SPICE.

## I. Introduction

SINCE the memristor, an electrical component, introduced in the circuit theory in 1971 [1] and re-discovered by Hewlett-Packard (HP) in 2008 [2], is not and will not be in the near future available as a commercial component, its models are at least being developed which would enable computer simulations of the memristor behavior in its engineered applications. Currently, quite a number of such models exist which have been developed particularly for SPICE-type simulation programs [3]-[10]. Similar models appear for memcapacitor [11] and meminductor [12], and for circuits which transform memristor to memcapacitor and meminductor [13], [14]. Also a digital emulator based on a microcontroller and digital potentiometer was proposed in [15], and analog implementations of the memristor are described in [16] and [17].

In fact, however, the component manufactured by HP is not a memristor but rather a general memristive system [18] with somewhat more complicated behavior. We can mention a simple example when instead of the resistor, i.e. an ideal circuit component having only one feature – resistance – another component with several parasitic parameters is manufactured. Most SPICE models of the „memristor“ were developed just on the basis of the HP memristor. These models start from a simple mathematical description of the physical phenomena in the memristive system, having the form of first-order differential equation. Then the corresponding SPICE models are based on the state-space approach.

Analogously to the non-linear resistor, which is modeled unambiguously by its non-linear voltage-current (V-I) characteristic, the memristor, a memory version of the resistor, is modeled via a flux-charge relation, i.e. via a relationship between the time-domain integrals of voltage (TIV) and current (TII). According to [19], the V-I characteristic of the resistor and the TIV-TII characteristic of the memristor are called their constitutive relations (CR). The CR defines in a compact form and unambiguously a concrete circuit component. This means, for example, that CR is related to this component but not to the way this component interacts with its surroundings, and also that all other characteristics of the component can be derived from the CR (for example, both the differential and the DC resistance at a given operating point can be derived from the V-I characteristic of the resistor, the memristance and also the state-space model can be derived from the TIV-TII characteristic of the memristor, etc.).

The CR can now serve as a starting characteristic for computer modeling of the memristor. However, [20] is probably the only paper that describes a CR-based memristor model. In addition, a rather complicated model of HP „memristor“ is described there.

As a response to the above facts, this paper proposes simple SPICE models of charge- and flux-controlled memristors, with only CRs forming the input data of such modeling. That guarantees that the models will exactly exhibit all the memristor fingerprints, i.e. the memristance dependence on only the charge or flux, the identical zero-crossing points of the voltage and current waveforms, and the pinched hysteretic loops in the V-I characteristics, with hysteretic effects being attenuated for the higher repeating frequencies of exciting signal.

## II. Starting Points of CR-Based Modeling

When the memristor CR depicts the TIV (i.e. the flux $\varphi$) as a single-valued function of the TII (i.e. the charge $q$), or the

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V. Biolkova is with the Department of Radio Electronics, Brno University of Technology, Brno, Czech Republic (e-mail: biolkova@feec. vutbr.cz).

Z. Kolka is with the Department of Radio Electronics, Brno University of Technology, Brno, Czech Republic (e-mail: kolka@feec. vutbr.cz).

Z. Biolek was with the TESLA company, Roznov p.R., Czech Republic. He is now with the SSIER Roznov p.R., Czech Republic (e-mail: biolek@valachnet.cz).

D. Biolek is with the Department of Electrical Engineering, University of Defence, Brno, Czech Republic, and also from the Department of Microelectronics, Brnou University of Technology, Brno, Czech Republic (phone: +420-973442487; fax: +420-973443773; e-mail: dalibor.biolek@ unob.cz).

TII as a single-valued function of the TIV,

$$\varphi = \hat{\varphi}(q) \tag{1}$$

or

$$q = \hat{q}(\varphi) \tag{2}$$

where

$$\varphi(t) = TIV = \int_{-\infty}^{t} v(\alpha)d\alpha, \tag{3}$$

$$q(t) = TII = \int_{-\infty}^{t} i(\alpha)d\alpha, \tag{4}$$

then (1) and (2) are CRs of charge- and flux-controlled memristors. Examples of the CRs are shown in Fig. 1, with definitions of memristance $R_M$ and memductance $G_M$ as flux derivative with respect to charge, and charge derivative with respect to flux, respectively.

Differentiating both sides of Eqs. (1) and (2) yields the voltage-current relations

$$\frac{d\varphi}{dt} = v(t) = \frac{d\hat{\varphi}(q)}{dq}\frac{dq}{dt} = R_M(q).i(t), \tag{5}$$

$$\frac{dq}{dt} = i(t) = \frac{d\hat{q}(\varphi)}{d\varphi}\frac{d\varphi}{dt} = G_M(\varphi).v(t). \tag{6}$$

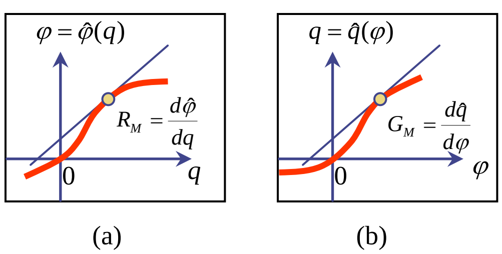


Fig. 1 Examples of the constitutive relations of (a) charge-, (b) flux-controlled memristor

Consider CRs (1) and (2) in the form of the Taylor series

$$\varphi = \hat{\varphi}(q) = \sum_{k=1}^{\infty} r_k q^k, \tag{7}$$

$$q = \hat{q}(\varphi) = \sum_{k=1}^{\infty} g_k \varphi^k, \tag{8}$$

with real $r_k$ and $g_k$ coefficients. Then Eqs. (5) and (6) describe conventional Ohm's law, with the memristance of the charge-controlled memristor, and the memductance of the flux-controlled memristor being charge- and flux-dependent according to the formulae

$$R_m(q) = \sum_{k=1}^{\infty} k.r_k q^{k-1} = r_1 + \sum_{k=2}^{\infty} k.r_k q^{k-1}, \tag{9}$$

$$G_m(\varphi) = \sum_{k=1}^{\infty} k.g_k \varphi^{k-1} = g_1 + \sum_{k=2}^{\infty} k.g_k \varphi^{k-1}. \tag{10}$$

Eqs. (9) and (10) confirm that when the CRs are linear, the memristance or memductance is then independent of the circuit variables, and the memristor behaves as a linear resistor. The memristive effect is described by the remaining terms of the Taylor series.

The above analysis yields a simple procedure of CR-based SPICE modeling of the memristor: For the charge-controlled memristor, the time-domain integration of the current should be performed in order to get the charge. From the charge, the instantaneous value of the memristance can be computed via Eq. (9). The voltage drop on the memristor then results from the classical Ohm's law. For the flux-controlled memristor, the flux is computed as a time-domain integral of the voltage, then the memductance is computed via Eq. (10), and the current is finally given by the product of the memductance and the voltage.

## III. Memristor SPICE Models

Figures 2 (a) and (b) contain block-oriented models of charge- and flux-controlled memristors. The left-side general models are concretized on the right for Eqs. (9) and (10), which take into account the Taylor expansions of the CRs. Note that SPICE does not enable direct modeling of variable resistors. That is why voltage controlled sources (for charge-controlled memristor) and current controlled sources (for flux-controlled memristor) are used on the right side of Fig. 2.

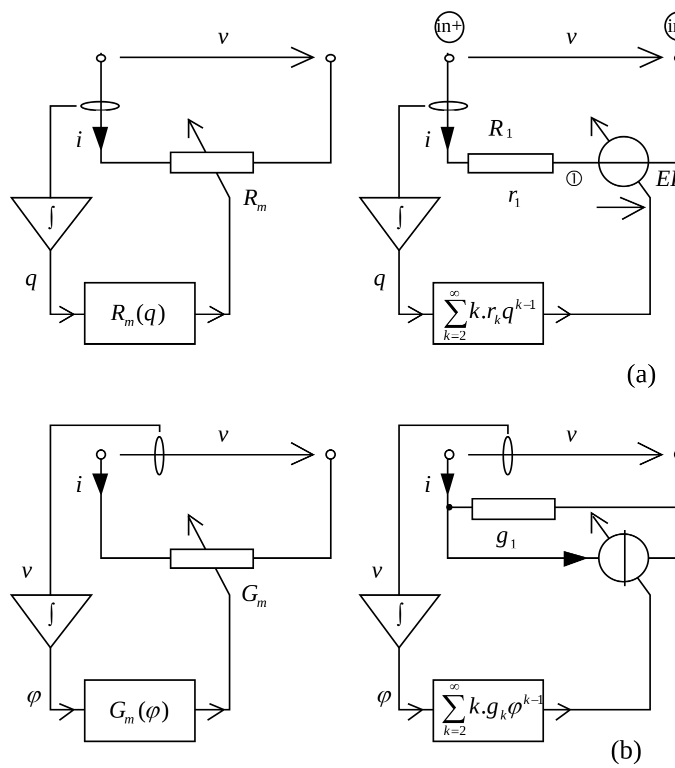


Fig. 2 Proposed models of (a) charge-, b) flux- controlled memristor

The $r_1$ and $g_1$ elements in the model remove the so-called potential conflicts of two sources. For example, if the charge-controlled memristor was modeled via an ideal voltage source, it could not be excited by an external voltage source.

Note that Fig. 2 does not contain the initial states of the integrators, i.e. the initial charge or flux which should be additional input data of the model. They can be recalculated to initial memristance or memductance via Eqs. (9) or (10).

As an example, consider a charge-controlled memristor with the CR

$$\varphi = r_1 q + r_3 q^3,\ r_1=5\ \mathrm{VA}^{-1},\ r_3= 10\ \mathrm{kVA}^{-3}\mathrm{s}^{-2} \tag{11}$$

Differentiating (11) with respect to $q$, one can prove that the memristance is under any circumstances i.e. for an arbitrary charge, positive, and that the given memristor is theoretically implementable as a passive component:

$$R_m = \frac{d\varphi}{dq} = r_1 + 3 r_3 q^2 \tag{12}$$

The PSPICE subcircuit of this memristor, compiled on the basis of Fig. 2 (a), is given below:

```
.subckt QC_memristor in+ in- params: qinit
.param r1 5 r3 10k
EQ Q 0 value={qinit+SDT(i(ER))}
R1 in+ 1 {r1}
ER 1 in- value={(3*r3*V(Q)^2)*i(ER)}
.ends QC_memristor
```

The voltage of controlled source „EQ“, connected between node Q and the ground, is equal to the time-domain integral of memristor current, i.e. the charge. The value of this charge, i.e. V(Q), is used for computing the voltage of the controlled source “ER”. The integral is computed via PSPICE function SDT.

The above model was used for the simulation of a memristor excited by the harmonic voltage source with an amplitude of 1.3 V and a repeating frequency of 1 Hz. The results of transient analysis in Fig. 3 show all the basic fingerprints of the memristor, as mentioned in Section 1. It can be also demonstrated that increasing the frequency causes an attenuation of the hysteretic effects in the V-I characteristic.

## IV. Conclusions

A methodology described here enables simple modeling and computer simulation of the memristor which is described by its charge-flux constitutive relation. The only necessary condition for such modeling is to have a mathematical representation of the memristance as a function of charge for the charge-controlled memristor, and of the memductance as a function of flux for the flux-controlled memristor.

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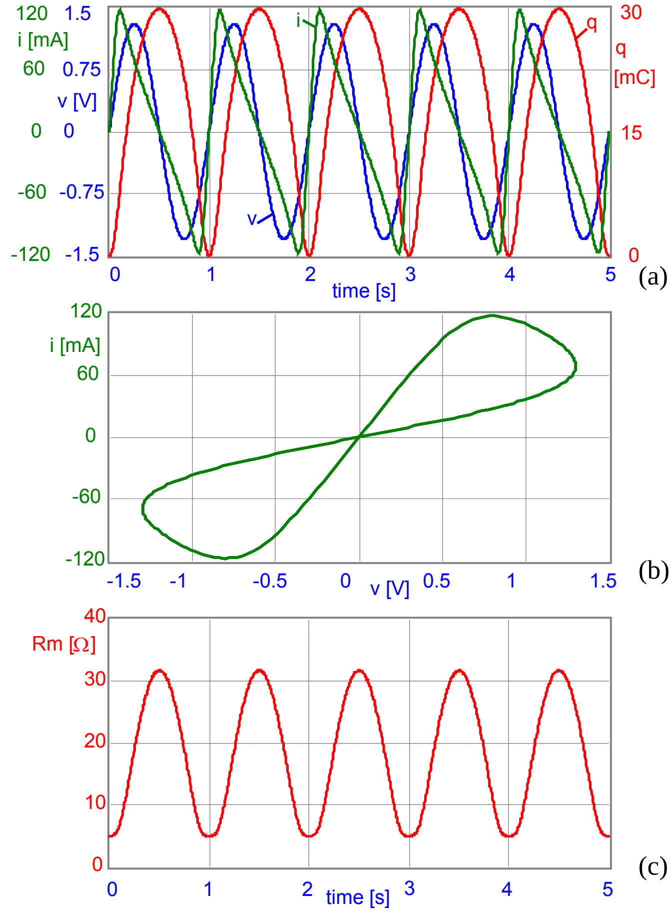


Fig. 3 Simulation results for the memristor with CR (11): (a) waveforms of voltage, current, and charge, (b) pinched hysteretic loop in V-I characteristic, (c) time evolution of the memristance

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