Image Denoising by Exploring the Context Information in the Wavelet Domain

Ajay Kumar Mandava, Emma E. Regentova and Markus Berli

Abstract—Traditional diffusivity based denoising models detect edges by the gradients of images, and thus are easily affected by noise. In this paper, we introduce a nonlinear diffusion denoising method based on the wavelet domain diffusivity model and context information. The shift-invariant property of the stationary wavelet transform makes it suitable for edge detection and derivation of texture information. In the proposed diffusion model, the modulus of gradient in a diffusivity function is substituted by the modulus of a wavelet detail coefficient. The diffusion of a wavelet coefficient is performed based on the information about the energy of the transform in a local neighborhood of coefficients across the scales. It is shown that the new model has better noise suppression and better perceptual quality power for high levels of noise. Objectively results are evaluated based on PSNR and Laplacian mean-square error (LMSE) metrics.

Keywords—Stationary wavelet, non-linear diffusion, context based diffusion

I. Introduction

THE diffusion process

$$u_t = (g(|u_x|)\, u_x)_x \tag{1}$$

can be obtained from a family u(x, t) of filtered versions of a signal f(x), where subscripts denote partial derivatives, and the simplification parameter with larger values corresponding to stronger filtering is the diffusion time t. The initial condition is given by f (.)

$$u(x, 0) = f(x) \tag{2}$$

The diffusivity g(s) is typically positive everywhere but rapidly and monotonically decreasing for s > 0. This ensures less blurring for strong edges than the noise and low-contrast details by the diffusion filter. Equation (1) covers a variety of filters, depending on the choice of the diffusivity function. Perona and Malik [1] introduced the following choice for g functions

$$g(|s|) = \frac{1}{1+\frac{|s|^2}{\lambda^2}} \tag{3}$$

where $|s|$ is the modulus of gradient and parameter λ is a constant.

Since its introduction in [1], diffusion models have been developed and applied to different areas of image processing including image denoising. Well known nonlinear diffusion methods are Perona-Malik's diffusion [1], Rudin-Osher-Fatemi's total variation [2], Mumford-Shah's functional [3], Weickert's tensor diffusion [4], Chambolle-Lions's [5], and Vogel-Omans's [6], total variation analyses, Chan-Vese's active contours [7], Fourth Order PDE [8] and other. More recently a number of interesting connections between wavelet shrinkage, regularization methods and PDEs have been established. Bao and Krim [9] addressed the problem of texture loss in diffusion scale-spaces by incorporating ideas from wavelet analysis and showed that using wavelet frames with wavelets of higher order than Haar leads to a good preservation of texture while removing noise and making the image more open to other processing. Malgouyres [10, 11] proposed a hybrid method that uses both wavelet packets and TV technique. Experiments showed that it can restore textured regions without introducing visible ringing artifacts. In [12] Chen developed three denoising schemes by combining PDE with wavelet. In the first proposed model, a diffusion function was introduced in the regularization term of the ROF model, and the modulus of gradient was substituted by the modulus of wavelet transform, the model could preserve edges better and had strong ability of resisting noise. But the major disadvantage of this model is its computational complexity. The other two models are improvements of the first one based on the features of noise in the wavelet domain and the multi-resolution analysis of wavelet transform, respectively. The second model overcomes the disadvantage of computational complexity in the first model and the last model was based on the character of the multi-resolution analysis of wavelet transform. Shih and Liao [13] addressed a single step of nonlinear diffusion that can be considered equivalent to a single shrinkage iteration of coefficients of Mallat's dyadic wavelet transform (MDWT). An iterated two-band filtering method solves the selective image smoothing problem and uses dyadic wavelet-based approximation to separate the high frequency coefficients from the low frequency ones in the decomposition process. A diffusivity function is used to retain useful data and suppress noise in the regularization process. A smoother version of the signal is computed by reconstructing the decomposed low frequency component and the regularized high frequency components. It has to be noted that no information about the contents is taken into account and thus both edges of extended objects and texture created edges can be affected by such a denoising approach.

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In this paper, we combine in a single framework the advantages of non-linear diffusion multiresolution decomposition and explore the context information. The diffusivity function is used as a weighting function to the wavelet coefficients of a stationary wavelet transform (SWT) which provides both scale invariance and context information, such as an object edge, noise or texture. The latter is derived from the local transform energy information. We compare the performance of the proposed method to the method in [13], which exploits the edge information derived from two wavelet sub-bands. We demonstrate the performance of the method on a known benchmark, that is the 8-bit Lena image. Additionally we show results of the method on a 16-bit X-ray Micro-Tomography (XMT) image of a soil sample.

The paper is organized as follows. Section II provides a theoretical background and introduces the new local context based diffusion in the stationary wavelet domain (SWCD). Section 3 shows the experimental results; the last section concludes the paper.

II. LOCAL CONTEXT BASED DIFFUSION IN STATIONARY WAVELET DOMAIN (SWCD)

In a decimated discrete wavelet transform (DWT) after high and low pass filtering, coefficients are down sampled. Although this prevents redundancy and allows for using the same pair of filters in different levels, this decimated transform lacks shift invariance. Thus, small shifts in the input signal can cause major variations in the distribution of energy of coefficients at deferent levels. Even with periodic signal extension, the DWT of a translated version of a signal X is not, in general, the translated version of the DWT of X. To restore the translation invariance one can average a slightly different DWT, called ε-decimated DWT, to define the stationary wavelet transform (SWT) [14]. SWT algorithm is simple and is close to the DWT one. More precisely, for level 1, all the decimated DWT for a given signal can be obtained by convolving the signal with the appropriate filters as in the DWT case but without down-sampling. The two-dimensional SWT leads to a decomposition of approximation coefficients at level j to four components: the approximation at level j+1, and the details in three orientations, i.e., horizontal, vertical, and diagonal). Considering the multi-sampling filter banks, SWT decomposition is shown in Eq.4.

$$
\begin{aligned}
A_{j,k_1,k_2} &= \sum_{n_1}\sum_{n_2} h_0^{\uparrow 2^j}(n_1-2k_1)\,h_0^{\uparrow 2^j}(n_2-2k_2)\,A_{j-1,n_1,n_2}\\
D^1_{j,k_1,k_2} &= \sum_{n_1}\sum_{n_2} h_0^{\uparrow 2^j}(n_1-2k_1)\,h_1^{\uparrow 2^j}(n_2-2k_2)\,A_{j-1,n_1,n_2}\\
D^2_{j,k_1,k_2} &= \sum_{n_1}\sum_{n_2} h_1^{\uparrow 2^j}(n_1-2k_1)\,h_0^{\uparrow 2^j}(n_2-2k_2)\,A_{j-1,n_1,n_2}\\
D^3_{j,k_1,k_2} &= \sum_{n_1}\sum_{n_2} h_1^{\uparrow 2^j}(n_1-2k_1)\,h_1^{\uparrow 2^j}(n_2-2k_2)\,A_{j-1,n_1,n_2}
\end{aligned}
\tag{4}
$$

Where $h_0^{\uparrow 2^j}, h_1^{\uparrow 2^j}$ respectively denote the (2^j -1) zeros padded between h_0, h_1. The inverse transform of SWT is shown in Eq.5.

$$
\begin{aligned}
A_{j-1,n_1,n_2} = \tfrac{1}{4}\sum_{i=0}^{3}\{&\textstyle\sum_{k_1}\sum_{k_2} g_0(n_1-2k_1-i)\,g_0(n_2-2k_2-i)\,A_{j,k_1,k_2}\\
&+\textstyle\sum_{k_1}\sum_{k_2} g_0(n_1-2k_1-i)\,g_1(n_2-2k_2-i)\,D^1_{j,k_1,k_2}\\
&+\textstyle\sum_{k_1}\sum_{k_2} g_1(n_1-2k_1-i)\,g_0(n_2-2k_2-i)\,D^2_{j,k_1,k_2}\\
&+\textstyle\sum_{k_1}\sum_{k_2} g_1(n_1-2k_1-i)\,g_1(n_2-2k_2-i)\,D^3_{j,k_1,k_2}\}
\end{aligned}
\tag{5}
$$

where A and D are approximation and detail coefficients respectively.

From the above two equations, we can verify that SWT includes redundant information and shift- invariance suitable for accurate edge detection and denoising in the wavelet domain. Smooth regions in image are represented mainly by approximation coefficients. Level 1 and Level 2 detail subbands convey the noise and the fine-grain texture information, respectively. The higher scales carry the information of edges of extended objects which are to be preserved. Therefore, the image can be denoised by diffusing the detail coefficients in level 1 and level 2. One can also perform it selectively depends on the importance of the information to be preserved. Based on this consideration the following is implemented.

The algorithm proceeds as follows:

(1) Perform SWT on the input image. We use Haar wavelet basis and a scale 3.

(2) The detail coefficients (vertical ($cD^{(v)}$), horizontal ($cD^{(h)}$) and diagonal ($cD^{(d)}$)) obtained in step 1 are processed by diffusivity function (3a). In the diffusivity function $|s(x,y)|$ is the edge estimate at pixel (x,y), usually approximated by gradient magnitude operator, and is the edge threshold parameter. The edge estimate is given by

$$|s_i(x,y)| = |cD_i^{(k)}(x,y)| \tag{6}$$

Where i = 1 or 2 or 3 and k = v or h or d.

Vertical Coefficients:

$$cD_i^{(v)} = cD_i^{(v)} * g(|s_i|)$$

Horizontal Coefficients:

$$cD_i^{(h)} = cD_i^{(h)} * g(|s_i|)$$

Diagonal Coefficients:

$$cD_i^{(d)} = cD_i^{(d)} * g(|s_i|) \tag{7}$$

Conditions for detail coefficients diffusions

a) If $E^k_{1,m\,x\,n} < E^k_{2,m\,x\,n} < E^k_{3,m\,x\,n}$, then this is an actual object edge, we diffuse less in scale 2 and more in scale 1.

b) If $E^k_{3,m\,x\,n} < E^k_{2,m\,x\,n} < E^k_{1,m\,x\,n}$, then this is noise, we diffuse more in scale1 and scale2.

c) If $E^k_{1,m\,x\,n} < E^k_{3,m\,x\,n} < E^k_{2,m\,x\,n}$, this is texture, we diffuse less in scale2 and scale 1.

where k = v (vertical) or d (diagonal) or h (horizontal)

$$E^V_{i,m\,x\,n} = \sum_{i=1}^{w} |\,cD_i^{(v)}\,|^2$$

$$E^D_{i,m\,x\,n} = \sum_{i=1}^{w} |\,cD_i^{(d)}\,|^2$$

$$E^H_{i,m\,x\,n} = \sum_{i=1}^{w} |\,cD_i^{(h)}\,|^2$$

where w is m x n neighborhood in scale j

(3) Wavelet Reconstruction: The denoised image is obtained by implementing the inverse SWT using the

approximation coefficients and detail coefficients obtained in step 2.

III. EXPERIMENT

To study the performance of SWCD we choose Gaussian additive noise of zero mean, i.e., $\mu=0$ and variance $\sigma^2= 0.02, 0.04, 0.06, 0.08$.

The evaluation is performed based on objective criteria which are

1. Peak Signal to Noise Ratio (PSNR)

$$\mathrm{PSNR} = 10\log\frac{255^2}{\mathrm{MSE}} \qquad (8)$$

2. Laplacian Mean Square Error (LMSE)

$$\mathrm{LMSE} = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}[L(x(m,n))-L(\hat{x}(m,n))]^2}{\sum_{m=1}^{M}\sum_{n=1}^{N}[L(x(m,n))]^2} \qquad (9)$$

Where L is a Laplacian operator.

The latter metric is based on the edge measurement. In Fig. 1, we show a part of the original Lena image, the original with the added Gaussian noise with $\mu=0$ and $\sigma^2 = 0.04$, and images denoised using method as given in [13], and the herein proposed SWCD method with $\lambda = 20$ and 10 at scales 1 and 2 respectively for condition a), $\lambda = 100$ and 50 at scale1 and scale2 respectively for condition b) and $\lambda = 10$ and 5 at scales 1 and 2 respectively for condition c) for the window of 3x3 pixels. In Fig.2 we show the original Lena image with a marked area chosen in a relatively smooth part of the image and enlarged marked area with the noise, and results of diffusion. We notice that the method proposed in [13] produces large artifacts not seen in Fig.2e which displays the result of the proposed approach. Table 1 provides the results for the reference method [13] and implementations of the SWCD with different window sizes. The best results are obtained for the 3x3 neighborhood pixel window that is a smaller neighborhood statistics. In Fig.3 we show the results of denoising performed on a segment of an original 16 bit XMT image (Source: Advanced Light Source at Lawrence Berkeley National Laboratory). A Gaussian noise with mean $\mu=0$ and variance, $\sigma^2 = 0.02$ is added to the original. The method in [13] produces artifacts on both sharp edges between air and soil structures and short edges which constitute texture features of components and does that not uniformly. In contrast, the proposed method shows a good smoothening results along the extended edges of a pore structures and the edges of the texture. Results with SWCD are particularly encouraging for high levels of noise.

IV. CONCLUSION

The paper presented a non-linear diffusion in the wavelet domain. Unlike the method proposed by Shi in [13] the diffusion is controlled adaptively by the local context. The redundant representation of SWT adds the shift invariance which is shown to be effective for denoising; and the analysis of energy in a local neighborhood across scales provides the context information for the choice of the magnitude of diffusion. Based on the evaluation results, the SWCD shows a high performance based on both objective measures for medium to high levels of noise. Additionally the method does not produce visual artifacts and the perceptual quality as a result is higher.

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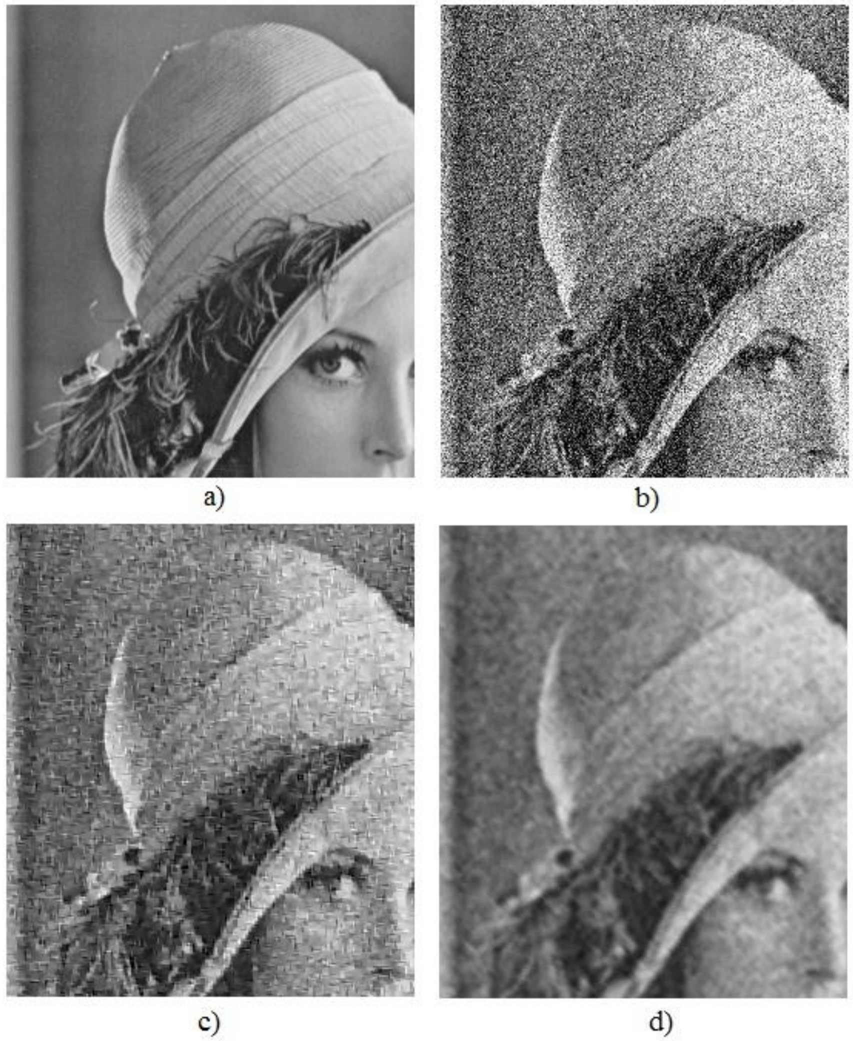

Fig.1. Results for Gaussian noise σ^2= 0.04; a)Original Image b)Noisy Image
c) Diffusion based on the method in [13] d) diffusion with SWCD (3X3)

Table I: (a) PSNR; (b) and LMSE for denoising of Gaussian noise (σ^2= 0.02, 0.04, 0.06, 0.08) in Fig. 1a.

Variance(σ^2)	**0.02**		**0.04**		**0.06**		**0.08**	
	PSNR	LMSE	PSNR	LMSE	PSNR	LMSE	PSNR	LMSE
Noisy Image	17.20	63.51	14.45	119.75	13.00	166.91	12.03	207.64
Method in [13]	26.59	1.83	21.99	6.01	19.49	11.44	17.98	16.61
SWCD_3x3	27.07	1.47	25.55	1.85	24.41	2.17	23.58	2.47
SWCD_5x5	26.75	2.49	25.19	3.74	23.96	5.03	23.12	5.84
SWCD_7x7	26.47	3.28	24.76	5.30	23.56	6.93	22.65	8.60

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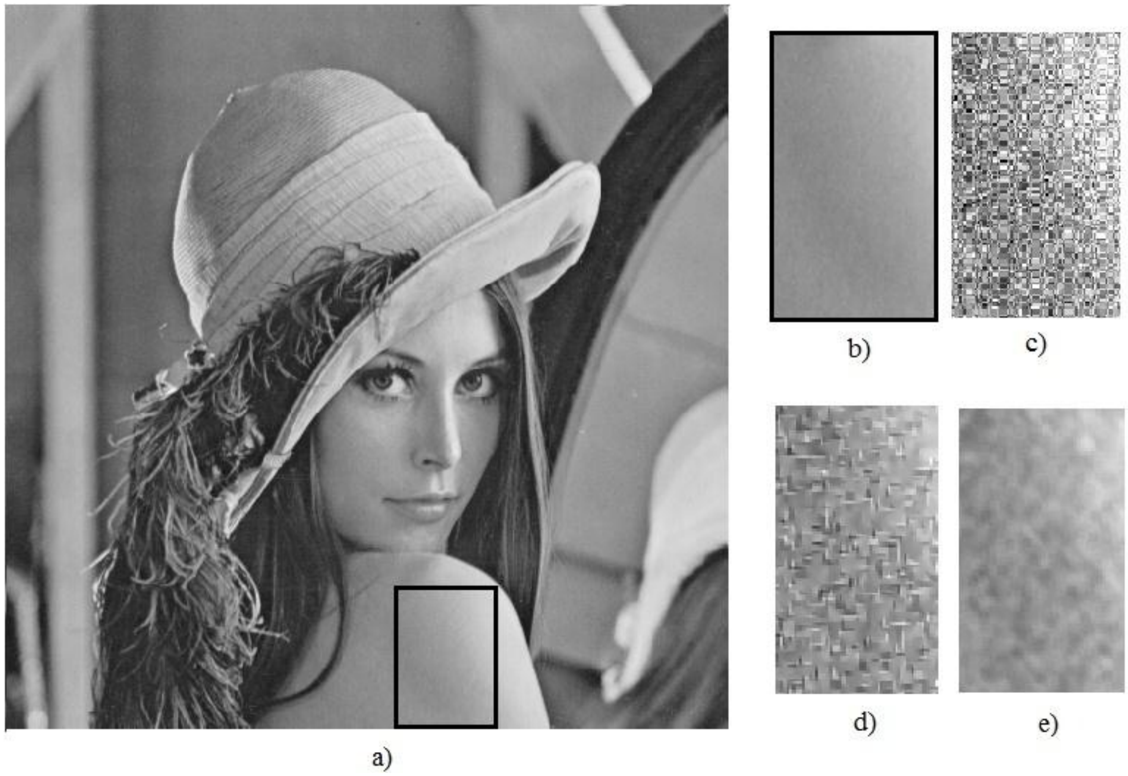

Fig.2 Subjective quality; a)Original Image with a marked area b) enlarged marked area of the original; c) Marked area affected by noise $\sigma^2= 0.04$; d) diffusion in the marked area by method [13]; e) diffusion in the marked area with SWCD (3X3)

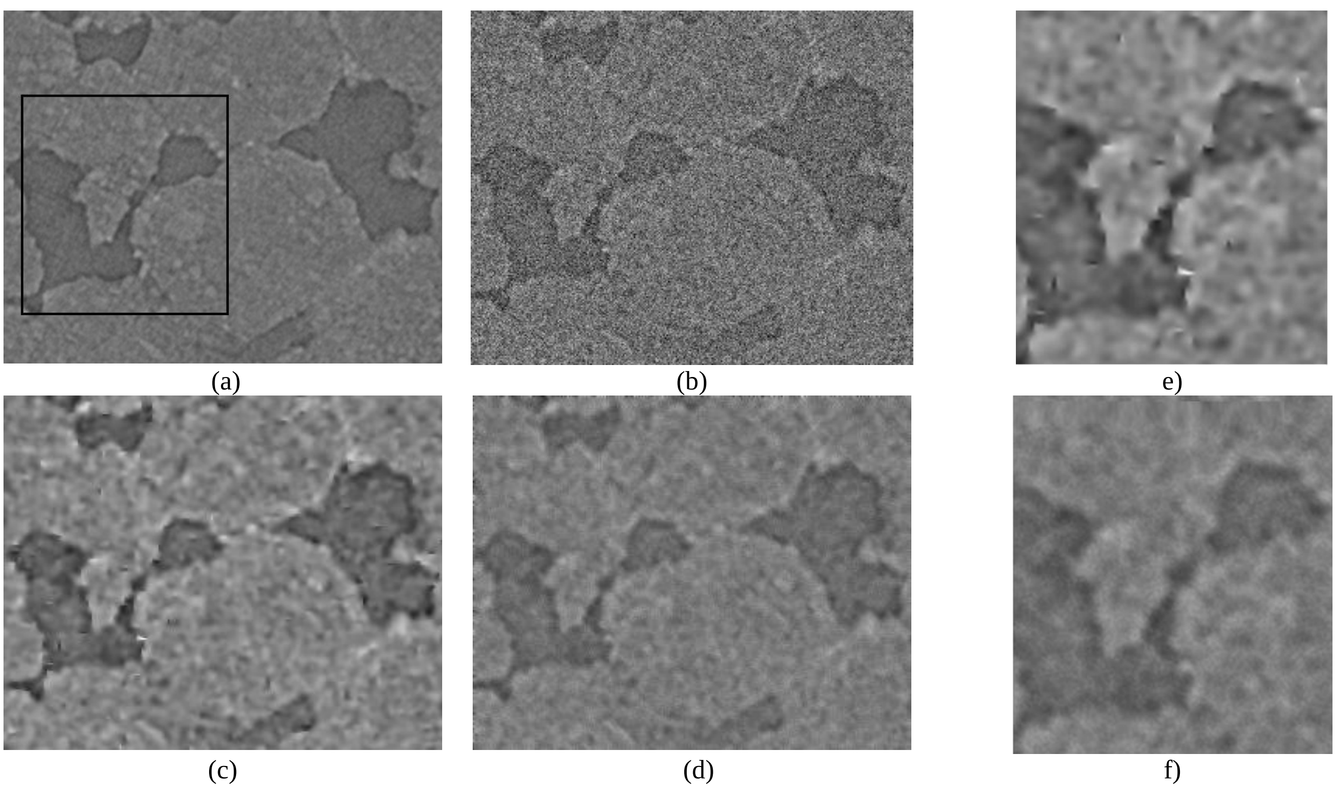

Fig 3 Demonstration of the method on a 16-bit XMT image of soil (ALS, Lawrence Berkeley National Laboratory, Berkeley, CA); a) a region with soil aggregates (light) and air-filled pores (dark); b) noise a) (14.13dB); result by [13] (PSNR = 21.96dB); d) result by SWCD(26.83dB); e) marked area diffused according to the method by [13];f) marked area diffused by SWCD .

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