Characterizing an Extended Fuzzy Logic System with Temporal Attributes for Real-Time Expert Systems

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Abstract: - The approximate reasoning is perceived as a derivation of new formulas with the corresponding temporal attributes, within a fuzzy theory defined by the fuzzy set of special axioms. In a management application, the reasoning is evolutionary because of unexpected events which may change the state of the expert system. In this kind of situations it is necessary to elaborate certain mechanisms in order to maintain the coherence of the obtained conclusions, to figure out their degree of reliability and the time domain for which these are true. These last aspects stand as possible further directions of development at a basic logic level. The purpose of this paper is to characterize an extended fuzzy logic system with temporal attributes, attained by incorporating the basic elements of a first-degree fuzzy logic and certain elements of temporal logic.

Key-Words: - Real-time Expert Systems, Fuzzy Reasoning, Temporal Attributes

1 Introduction

I presented, throughout Section 2, the formalization and logical justification of the reasoning specific to a Real-time Expert system based on Fuzzy Knowledge (version 2), called REFK2. I also highlighted, as aspects of reasoning about time, the attachment of certain temporal descriptors to the fuzzy statements, according to the interval-based (temporal) logic. There are three features of any formalization which are actually used to create the inferential subsystem within the REFK2 system: **i**) defining and reducing reality (the problem domain) to a linguistic model (the management model); **ii**) the possibility to represent the same reality in various aspects, according to the position from which one may look at this reality (the model is not unique, since it always depends on the intended purpose and on the type of representation and processing of knowledge); **iii**) abandoning the external world in order to carry out deductions, once a formalization of it has taken place (the inferential chains are based on the management model and on the evidence knowledge). After the presentation of syntax and semantics elements of the extended first-degree fuzzy logic with modal temporal operators, the concepts of rules of inference, demonstration for a fuzzy formula as well as elements of approximate reasoning theory (as an exploitation methodology of imprecise knowledge with respect to the states of the expert management system, described as multidimensional possibility distributions). I have also analyzed the features of the possibility reasoning and of fuzzy temporal reasoning in order to deepen the inferential properties of the REFK2 system (Section 3). The conclusions of the paper appear in Section 4.

2 Approximate Reasoning Modeling

The approximate reasoning refers to creating new rules of inference and translation. It is a mathematical instrument used for modeling the human reasoning based on imprecise knowledge. The theory suggested by Zadeh is based on intuitive rules and leads to operations with fuzzy relations [1, 6, 5], obtaining thus very useful applications. R. Lee, C. Chang and Zadeh went back to the concept of fuzzy set in the logic domain. This perspective has the advantage of demonstrating that the fuzzy logic is a generalization of bivalent logic, replacing the discrete feature of the latter with a continuous one.

If in the case of bivalent logic there are used methods that clear up every possibility of evaluation according to the interpretation function, when we refer to fuzzy logic this are no longer possible.

The formalism of the first-degree fuzzy logic represents the mathematical basis for the general theory of approximate reasoning [7]. A special feature of the human thinking is the effective use of natural language even within the process of logic reasoning. According to this observation, we can conclude that the mathematical model of the way in which man thinks (acts) in a management position and at a certain level of synthesizing decisions, could be based on the fuzzy logic [11, 4], combined with modal temporal features. I will tackle next the formalism of first-degree fuzzy logic, highlighting the structure of truth values, the extended syntax and the semantics of this formal logic system. I will underline in this way the connections between fuzzy logic and approximate reasoning, which is further analysed through the possibility reasoning, which is considered useful by the inference engine of the expert management system SECOMBCF [3].

2.1 The Syntax of the First-Degree Fuzzy Logic

The structure of truth values is a residual lattice written $\mathbf{L} = (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ where the 0 and 1 values are the smallest respectively the biggest elements, $\vee$ and $\wedge$ are the supremum, respectively infimum operators, $\otimes$ is the isotone product operator for both variables, $(L,\otimes,1)$ is a commutative monoid, and $\rightarrow$ is the residuation operator. Furthermore, $a \otimes b < c$ only if $a < b \rightarrow c$ $(\forall)$ $a,b,c \in L$. For $L = [0,1]$ the logical connectives are $\vee$ = **max**, $\wedge$= **min**, $a\otimes b = 0\vee (a+b-1)$ şi $a \rightarrow b = 1 \wedge (1-a+b)$. If we consider, for instance, $a\otimes b=$**min**(a,b), then the only corresponding residuation operator is the Gödel implication operator.

The syntax of the basic language of the extended first-degree fuzzy logic with modal temporal operators consists of: (x,y,...) *variables* and (c,d,r,...) *constants* seen as elements that describe the set of states of an expert management system $X^{SEC} = X^{SE} \cup X$, (f, g,...) *functional symbols* of n arity, a set of *symbols for the truth values* $\{a: a \in L\}$, *predicate symbols* of n arity, *a binary connective* $\Rightarrow$, a $\{o_i: i\in J\}$ set of *connectives of m arity*, a *symbol for the* $(\forall)$ *quantifier*, the o modal temporal operators (the following moment in time), □ (for all present or following moments in time), ◊ (for a present or following moment in time) and punctuation marks. The terms are classically introduced [9]. *The formulas* are defined as follows: **i)** an *a* symbol for a $(a\in L)$ truth value is an atomic formula; **ii)** if $t_1,...,t_n$ are terms and p a predicate symbol of n arity, then $p(t_1,...,t_n)$ is an atomic formula; **iii)** if A, B, $A_1,...,A_m$ are formulas and o_1 is a connective of m arity, then $A \Rightarrow B$, $o_1(A_1,...,A_m)$, $(\forall x)A$ are formulas. The $\rceil A$ formula is an abbreviation for $A \Rightarrow 0$. There are similarly defined the $(\vee)$ disjunction, the $(\wedge)$ conjunction, the $(\Leftrightarrow)$ equivalence, A&B, $(\exists x)$ A, $A^k = (A \,\&\, A \,\&\, ... \,\&\, A)\,/_{k \text{ times}}$; **iv)** The $x^{SE} \in X^{SE}$, $x\in X$ variables are formulas, and if g is a formula, then og, □g, ◊g are formulas, too; **v)** Any application of the above-mentioned rules for a certain number of times, determines a formula. Given a J_1 language of the extended first-degree fuzzy logic with modal temporal operators, the set of all terms will be noted M_{J1}, and the set of all formulas will be noted F_{J1}. If t is a term and A is a formula, then $A_x[t]$ is the formula obtained by substituting of the t term whenever the x variable appears freely in A. $g_s = (x_1\vee x_2 \vee\ldots\vee x_n)$ and $g_0=(x_{01}\vee x_{02}\vee\ldots\vee x_{0n})$ are given fuzzy formulas. The $g_0 \rightarrow ◊g_s$, $g_0 \rightarrow ◊□g_s$, $g_0 \rightarrow □◊g_s$ relations are formulas too, and they allow expressing certain qualitative management conditions.

2.2 The Semantics of the First-Degree Fuzzy Logic

The semantics of the first-degree fuzzy logic is defined as follows. A *structure* of the language of the fuzzy logic J_1, is characterised by $\overline{D} = (D, p_D, ..., f_D, ..., u, v,...)$ where D is a set, p_D D^n are relations of n arity assigned to each p n-ar predicate symbol and, f_D are n-are functions in D assigned to each functional symbol of n arity, whereas $u,v,... \in D$ are elements assigned to each u,v,... constant of the J_1 language. Take $\overline{D}$ a structure for the J_1 basic language. The interpretation function of the formulas in $\overline{D}$ is a $\widetilde{D}$: $F_{J1}\rightarrow L$ function, which assigns a truth value for any formula from F_{J1}, as it follows: **i)** $\widetilde{D}(a) = a$, $a\in L$; **ii)** Take $t_1,...,t_n$ terms without variables and p an n-ar predicate symbol. Then $\widetilde{D}(p(t_1,...,t_n))=p_D(\widetilde{D}(t_1),..., \widetilde{D}(t_n))$, where $\widetilde{D}(t_i)\in D$ is an interpretation of the $t_i\in M_{J1}$, i=1,...,n term; **iii)** $\widetilde{D}(A\Rightarrow B) = \widetilde{D}(A) \rightarrow \widetilde{D}(B)$, highlights the fact that A and B are closed formulas; **iv)** $\widetilde{D}(o_1(A_1, ..., A_n)) = o_1(\widetilde{D}(A_1), ..., \widetilde{D}(A_n))$; **v)** $\widetilde{D}((x)\ A(x)) = \wedge_{d\in D} \widetilde{D}(A_x[\mathbf{d}])$, where **d** is the name of the $d\in D$ element; **vi)** $\widetilde{D}(A(x_1,...,x_n)) = \wedge_{d1,...,dn} \widetilde{D}(A_{x1,...,xn}[d_1,...,d_n])$. Take $Y \underset{\sim}{\subset} F_{J1}$ a fuzzy set of formulas. The fuzzy set of semantic consequences of the Y fuzzy set (where $Y(B) \in L$ represents the truth degree of B in Y) is: $(C^{sem}Y)A = \wedge\{\widetilde{D}(A)$: $\overline{D}$ is a structure for J_1 and (for each $A\in F_{J1}$), $(Y(B)\leq \widetilde{D}(B))\}$.

If $(C^{sem}Y)A=1$, then for any fuzzy set of Y formulas the following relation occurs $\models$A and A is a tautology, and C^{sem} is the closing operator in *L*.

Lemma 1. A) $\models A\Rightarrow B$ only if $\widetilde{D}(A) \leq \widetilde{D}(B)$; **b)** $\models A\Leftrightarrow B$ only if $\widetilde{D}(A) = \widetilde{D}(B)$, for any $\overline{D}$ structure. This result is used in the derivation of tautologies. It is allowed the introduction of a set of fuzzy axioms to support the derivation of new formulas.

A *logical inference* is a $B_1,...,B_m$ sequence of formulas, each of them being either a logical and special axiom, or a formula derived from other formulas, using a rule of inference. The rules of logical inference can be schematically written under the form $A_1,...,A_n$ / B where $A_1,...,A_n$ are known formulas, and B is a derived fact ($A_1,...,A_n$, $B \in F_{J1}$). The rules of inference preserve the truth values after the inferential process.

3 The Analysis of the Inferential Process

The theory of approximate reasoning, as a methodology of exploitation of imprecise knowledge with respect to the state of the expert management system (noted with $\mathbf{x}^{SEC} \in X^{SEC}$ and represented as distributions of possibility), allows that, given certain logical inferences, strict characterizations of the values of linguistic variables to be obtained from the structure of the $\mathbf{x}^{SEC}$ state, compliant to the management purpose. The X^{SEC} set can be defined as a Cartesian product X^b **x** X^{int} **x** X, in which $\mathbf{x}^b = [x_1^b \; x_2^b \; ... \; x_{k_1}^b]^t \in X^b$. For instance, the x_1^b component marks, through its values, possible command events for the process, $x_i^b \in U^{(i)}$, i=2,...,k_1 where $U^{(i)}$ are the universes of discourse attached to the linguistic variables $X^{(i)}$ (chosen in order to characterise the $\mathbf{x}^{SE} \in X^b \mathbf{x} X^{int}$ state), X^{int} represents the set of internal states of the engine of inference, and X refers to the set of the states of the process.

Types of Knowledge Specific to a Real-Time Expert Fuzzy System

Creating certain efficient reasoning algorithms, within expert management systems, demands for a corresponding analysis of the type and signification of knowledge from the structure of the involved models. The elements presented in the next section of the paper refer mostly to the logical aspects regarding fuzzy inference, without paying too much attention to the semantics of the fuzzy rules. From this point of view, the implication and the multi-evaluated extensions can correctly express the problem of the semantics of the fuzzy rules, hardly investigated in literature. There are put forth three types of fuzzy rules „if..., then...” according to the [2,4] paper and these will be further presented in this paper: **i**) *Rules to qualify certainty*. These rules are expressed like “the more u∈A, the more sure v∈B”, which are translated by the relation (∀) u, $\mu_A(u) \le g_t(B)$, where $g_t(B)$ evaluates the degree of reliability of the statement v∈B when x=u. The g_t function can be any occurrence of the kind necessity, possibility, and probability; **ii**) *Gradual rules* (or rules to qualify truth) expressed by: “the more u∈A, the more v = f(u)∈ B”, i.e. there is a f : Supp(A) → Supp(B) function so as f(A) ⊆ B. This condition can be written down again as (∀)u∈A, $\mu_A(u) \le \mu_B(f(u))$, a relation as a definition of the fuzzy function in the paper [8]. This last relation can be relaxed by replacing the f function with the R fuzzy relation, and thus resulting the inequality: (∀)u,v, $T(\mu_A(u), \mu_R(u,v)) \le \mu_B(v)$, where T is a triangular norm. We can create this type of statements of the kind “the more u∈A and the more u is in relation with v, the more v∈B”. In this situation, the degree of truth of the antecedent restricts the degree of truth of the consequent; **iii**) *Rules to qualify possibility* expressed by: “the more u ∈ A the more possible v∈B”, which represents a partial description of the R relation between u and v. In this case, the inclusion AxB ⊆R takes place, and this further implies that $\mu_R(u,v) \ge \mathbf{min}(\mu_A(u),\mu_B(v))$. This type of rules is used in the fuzzy control process.

The interpretation of the semantics of fuzzy rules is important, since it allows the selection of certain ϕ - operators to match with the significance of the rule. In the case of gradual rules, Yager’s principle of minimum specificity is satisfying in order to obtain the distributions of possibility implied by these rules. For instance, the R relation from the gradual rules definition is a relational fuzzy equation with unknown μ_R [5]. Applying the minimum specificity principle leads us to the definition of the distribution of possibility $\pi_{x|y}(u,v)$, which expresses the semantics of the rule as a maximum solution in the $\mu_A(u) \le \mu_B(f(u))$ relation, i.e. $\pi_{x|y}(u,v) = \mathbf{sup}\{\alpha \mid T(\mu_A(u,\alpha) \le \mu_B(v), (\forall)\ u,v\}$. This result offers to the R – implications semantics of representation of gradual rules. The minimum specificity principle is not sufficient in order to solve the above-mentioned inequality, especially when B is fuzzy. This last problem demands the use of a qualification of α-certainty applied to B. The above-mentioned aspects entail possible domains of application of the various types of fuzzy rules, compared to their semantics for different kinds of reasoning: uncertain, interpolative, by analogy. In the case of the expert management system REFK2 prototype, the inferential subsystem based on fuzzy logic uses the scheme of inference generalised modus ponens. The knowledge-based reasoning represented as certain distributions of possibility, uses the notion of similarity defined as complement of distance.

If $\rho \in \mathbf{PSB}(X^{SEC})$ qualifies the given set of knowledge about all the states of the expert management system and $\varepsilon \in \mathbf{PSB}(X^{SEC})$ is the current observation on the $\mathbf{x}^{SEC}$ state, then ρ can become more specific compared to ε by their conjunctive combination, i.e. $\rho_\varepsilon =_{def} \mathbf{min}(\rho,\varepsilon)$. If ρ and ε are compatible, then $\rho_\varepsilon \in \mathbf{PSB}(X^{SEC})$ and for $(\forall)$ $\mathbf{x}^{SEC} \in X^{SEC}$, $\rho_\varepsilon(\mathbf{x}^{SEC}) \leq \rho(\mathbf{x}^{SEC})$, thus ρ_ε at least as specific as ρ and at least as informative as it concerning the imprecise characterization of $\mathbf{x}^{SEC}$. Tom make things simpler, we create a specialization of knowledge offered a priori by the expert about the state of the $\mathbf{x}^{SEC}$ system, with the help of the factual knowledge acquired. We focus the ρ knowledge referring to the $\mathbf{x}^{SEC}$ state with the help of ε. We can, thus, model the possibilistic expert systems and the corresponding reasoning, which allow us to characterize a $\mathbf{x}^{SEC} \in X^{SEC}$ state, based on certain imprecise information with respect to the $\mathbf{x}^{SEC}$ state, i.e. with the help of a $\mathbf{E} \subseteq X^{SEC}$ subset, for which $\mathbf{x}^{SEC} \in \mathbf{E}$. We consider that there can be components of the $\mathbf{x}^{SEC}$ state, defined as predicates, with firm truth values. In this case, too it is taken into account the condition that the truth values belong to the [0,1] interval and, thus, we can work unitarily only with the [0,1] interval. The expert management system administrates the knowledge specific to a state of the $\mathbf{x}^{SEC} \in X^{SEC}$ closed knot system, characterized at the k moment in time by $x_k^{SEC} = (\mathbf{x}_k,\ x_k^{SE})$. A specialization of the expert management system presented entails the absorption of a imprecise knowledge-based expert system in the management structure, just as in the case of the REFK2 system. The significance of this system derives from the fact that the imprecision will be represented by possibility distributions.

The class of the possibilistic expert systems can entail the temporal reasoning also. In this situation, the rules background is not consisted of $R_i^{M_j} \subseteq U^{Mj} \times V^{Lj}$, j=1,...,m, $M_j \in M_0$ relations anymore, but of $\rho_{\mathbf{i}}^{\mathbf{M_j}} \in \mathbf{PSB}(U^{Mj} \times V^{Lj})$ multidimensional possibility distributions instead, to which we attach temporal descriptors like $\mathbf{DT}_\alpha$, which can be punctual (α=p) or interval(α=i). These temporal descriptors can be modeled with the help of certain distributions of possibility, so as to attach the fuzzy statements of temporal features [10]. The attachment of the temporal fuzzy descriptors is specific to artificial intelligence techniques, but from the point of view of automation, this idea is equal to the fuzzyfication of the moments of time within the discrete events systems theory, a class of systems which the expert management system developed in this paper is part of. So as to elaborate an actual model for an expert system, in which to make possible the development of the temporal possibilistic inference, we will refer to the structure of the expert system based on the fuzzy inference. The temporal descriptors are operators that characterize the temporal properties of a P fuzzy statement and these can be:

i) *punctual* $\mathbf{DT}_p\left(P, \int_T \frac{\mu(t)}{t}, u\right)$; **ii**) *of interval type* $\mathbf{DT}_i\left(P, \int_T \frac{\mu_1(t)}{t}, \int_T \frac{\mu_2(t)}{t}, u\right)$, in which P is a fuzzy sentence, $\int_T \frac{\mu(t)}{t}$ is a T-number that describes the point on the axis of time at which statement P takes place, and μ represents the membership function of the moment of time associated to P. Similarly, the T-numbers $\int_T \frac{\mu_1(t)}{t}$, $\int_T \frac{\mu_2(t)}{t}$, that represent the moments of emergence and extinction of the event described by the P statement are also interpreted. The available information about a certain moment in time will be represented within the model by a possibility distribution. In this way we can define the $\mathbf{DT}_p$ operator as it follows:

Definition 1. The punctual time descriptor is characterized by $\mathbf{DT}_p : \mathcal{F}(U^{(i)}) \times \mathcal{F}(T) \to [0,1]^2$, so as $\mathbf{DT}_p(X^{(i)},t) = (\alpha_1, \alpha_2)$, with $\alpha_1 \in [0,1]$, $\alpha_2 \in [0,1]$, α_1 representing the value of the possibility degree π_i attached to the P_i (P_i = "$X^{(i)}$ is F_i", $X^{(i)}$ statement is the linguistic variable defined on the $U^{(i)}$ universe, while F_i is the value of the $X^{(i)}$ linguistic variable defined on $\mathcal{F}(U^{(i)})$); α_2 represents the value of the membership degree at the T_i fuzzy moment of time, for which the P_i statement is true, $\mu_{Ti}(t) \in \mathcal{F}(T)$, i=1,...,n.

For instance, take P_i ="The temperature of the cauldron increases for approximately three minutes". If we use the $\mathbf{DT}_p$ operator, we can represent the P_i statement as it follows: $\mathbf{DT}_p\left(P_i, \int_T \frac{\mu_1(t)}{t}, u\right)$, for which $\mu_{Ti}(t)$ can be a trapezoidal distribution of possibility like: $(g_1, d_1, \varphi_1, \delta_1)$, with $g_1 = d_1 = 3$, $\varphi_1 = 1$, $\delta_1 = 1$.

The $\mathbf{DT}_i$ time descriptor can derive from the $\mathbf{DT}_p$ descriptor, by associating a P_i event, two moments of time, T_a and T_d (the moment of emergence and, respectively, the moment of extinction of the P_i event). The fuzzy time interval can be represented with the help of the T_a and T_d instances.

Definition 2. If there is a pair of fuzzy instances T_a and T_d from $\mathcal{F}(T)$, the time interval that appears after T_a and before T_d can be defined as a fuzzy set $T(T_a,T_d)$, so that $\mathbf{DT}_i : \mathcal{F}(U^{(i)})\mathbf{x}\mathcal{F}(T)\rightarrow[0,1]^2$, in which $U^{(i)}$ is the universe of discourse associated to the $X^{(i)}$ linguistic variable from the structure of P_i. For $(\forall)t\in T(T_a,T_d)$ the following relation occurs:

$$\mu_{T(Ta,Td)}(t)=\rho * \mu_{T[Ta,Td]}(t)+(1-\rho) * \mu_{T]Ta,Td[}(t) = \\ =\rho * \mathbf{sup}_{s\le t\le s'} \mathbf{min}(\mu_{Ta}(s), \mu_{Td}(s'))+(1-\rho) \\ * \mathbf{inf}_{s\le t\le s'} \mathbf{min}(1-\mu_{Ta}(s'),1-\mu_{Td}(s)), \rho\in[0,1] \quad (1)$$

In the above-mentioned relation, $T[T_a,T_d]$ indicates the fuzzy set from the axis of time which is possible after T_a and before T_d. Similarly, the $T]T_a,T_d[$ interval indicates the fuzzy set of points from T which are necessarily after T_a and before T_b. $T(T_a,T_d)$ represents the average between $T[T_a,T_d]$ and $T]T_a,T_d[$. The **DT** temporal descriptor can highlight three types of temporal relations [10]:

i) *Absolute or real-time description*. The "The temperature increases from around 5 o'clock until around 6 o'clock" statement is represented by: $\mathbf{DT}_i$ $(P,(g_1,d_1,\varphi_1,\delta_1),(g_2,d_2,\varphi_2,\delta_2), u)$, in which P is the fuzzy sentence "The temperature increases", $(g_1,d_1,\varphi_1,\delta_1)$ and $(g_2,d_2, \varphi_2,\delta_2)$ are T-number with the property that $g_1= d_1=5$, $g_2=d_2=6$, u=1hour, $\varphi_1=\varphi_2=0.25h$, $\delta_1=\delta_2 = 0.25h$;

ii) *Relative description.* This relation signifies the fact that time can be determined by adding a fuzzy time interval at a temporal point of reference. The $\mathbf{DT}\left(P_1,\left(\frac{1}{0}\right),\min\right)$ representation and $\mathbf{DT}(P_2,(g_1,d_1,\varphi_1,\delta_1),\min)$ shows that the fuzzy statement P_1 took place approximately g_1 minutes earlier than P_2 $(g_1 = g_2)$;

iii) *Before/After Description***.** This means that the relation between the time mentioned by the temporal descriptor and its temporal point of reference can be illustrated by the before and after relations. For instance, the $\mathbf{DT}\left(P_1,\left(\frac{1}{0}\right),\min\right)$ and $\mathbf{DT}\left(P_2,\left(\frac{1}{t>0}\right),\min\right)$ representations, show the fact that the emergence of the fuzzy statement P_1 took place before the fuzzy statement P_2. In this way, a fuzzy rule like R_i, i=1,...,n, can be explained as it follows: if TD_1 (P_1) and TD_2 (P_2) and ... and TD_k (P_k) then TD_j (P_j) $(|L_j|=1)$ with a possible coefficient of reliability $W_j\in[0,1]$. The corresponding index subset has the k_0 cardinal. We can take into account a $TD'_1(P_1)$,..., $TD'_{k0}(P_{k0})$ set of fact also, and to each fact there is attached a α_j, j = 1,...,k_0 $(k_0=|M_k|)$ coefficient of reliability.

Due to the change of values of the variables in a dynamic way within the technological field, the evolution of the $x^{(i)}$ (t) specimens can engender dynamic corresponding symptoms in the fact base. Moreover, the basic rules describe the dependences between the numerical values of the symptoms by means of fuzzy sentences P_j, $j\in M_k$, $M_k\in M_0$, and the temporal relations between these symptoms by using the $\mathbf{DT}_j$ associated temporal descriptors. In the case of the REFK2 system, this kind of knowledge does not interfere, since the temporal aspect appears only as real time, and not as the reasoning concerning time also. In fact, this last aspect of time is adequate in the artificial intelligence systems with applications in diagnosis. An important feature of time in expert systems in order to manage processes is represented by their real-time behavior, capable to guarantee a satisfying response time. By introducing certain real-time restrictions inside an expert system, we provide the system with features like: **i**) reasoning is evolutionary and non-monotonous because of the dynamic aspect of the application; **ii**) unexpected events can change the state of the expert management system. There is a series of additional problems if we take into account the temporal characteristics, associated both to the model and to the evidence system that reflect the state of the process at a certain moment in time. These problems can be summarized as it follows: **i**) The filtering of a temporal fuzzy rule demands that, beside the numerical filtering, to adequately solve the temporal filtering also, i.e. the temporal attributes attached to the motives within the structure of the fuzzy sentences $\mathcal{R}_i$, i=1,...,n, must filter the temporal attributes formed by the corresponding dynamic symptoms from the fact base in a fuzzy sense. It is also necessary to determine a method of numerical temporal filtration, so as to evaluate the degree of filtration between $\mathcal{R}$ and X^b; **ii**) the way in which the conclusion can be inferred (i.e. the result of the inference and the corresponding degree of reliability) and which is the domain of time associated to it. The model associated to the filtering stage from the structure of a temporal fuzzy reasoning system based on intervals, must satisfy a series of conditions presented in above example. From the filtering point of view, we can obtain various situations, by choosing a corresponding window of filtration in the $U^{(i)}$ **x** T bi-dimensional space. Figure 1 presents a similar type of filtration. The filtering window can be a point or a rectangle, depending on how the temporal attribute attached to the P_i sentence is: punctual or interval. In the fact base, the evolution of values afferent to the $X^{(1)}$ and $X^{(2)}$variables, generates certain manifestations or specific situations, which are determined for the $\mathbf{x}^b$ state of the expert system. Meanwhile, the rules in which the $X^{(1)}$ and $X^{(2)}$ variables appear (implicitly attached to the P_1 and P_2 sentences), highlights the presence of some temporal descriptors that define the temporal relation between P_1 and P_2.

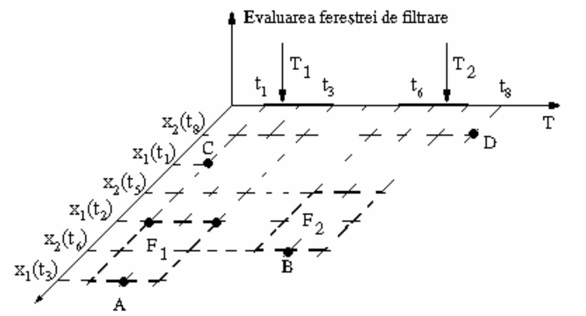

Figure 1. *The bi-dimensional filtering space*

In this way, we can give top *priority to numerical filtering* by choosing $x_1(t_3)$ and $x_2(t_6)$ to be filtered with P_1 and, respectively P_2. We obtain a good result of the numerical filtering, but the temporal filtering offers weak results instead. We can give *priority to the temporal filtering* as compared to the numerical filtering. The results may favorably change for the temporal filtering compared to the first case (for example, we choose $x_1(t_3)$ and $x_2(t_8)$ or $x_1(t_1)$ and $x_2(t_6)$ in order to filter with P_1, respectively P_2). It is obvious the fact that there are other choice possibilities also in the $U^{(i)}$ x T space of the window filtering position. The unsolved problems from a practical point of view represent the means by which the width of the filtering windows is determined (F_1, F_2), their best possible positioning within the $U^{(i)}$xT (i=1,2) space, the summary inside the filtering windows of the evolutions afferent to the $X^{(1)}$ and $X^{(2)}$ variables in values that can be further undergo a numerical filtering with P_1 and P_2, by assessing the consistency of the filtering phase on the whole. The advanced stages in order to obtain the reasoning strategy are: determining the time domain, temporal and numerical filtering. Once these properties have been mentioned, we may continue the development of the advanced model for the class of possibilistic expert systems, as it follows:

i) *Determining the time domain*. We assume that the P_i fuzzy sentence that describes the $X^{(i)}$ linguistic variable takes place in an interval specified through its temporal descriptor. We have to determine the $[t^i_{b_j}, t^i_{e_j}]$ time domain of $X^{(i)}$ corresponding to the temporal characteristics of P_i, i.e. the width of the filtering window and its position in the $U^{(i)}$xT space, giving top priority to temporal filtering. There are various methods to determine the time domain. We present the method based on relative time description only. We consider that P_r is the fuzzy sentence that takes place at the reference moment afferent to P_i, and $X^{(r)}$ is the linguistic variable from the structure of P_r. If the temporal reference point is described by a before/after number, then time t_r corresponding to $X^{(r)}$ (for the temporal reference point) can be calculated as it follows:

$$t_r = \begin{cases} \mathbf{inf}\{t \mid \pi_r(x_r(t)) > \gamma_r, t \in T\} & \text{if } t_r \text{ is the emergence time of the event described by } P_r \\ \mathbf{sup}\{t \mid \pi_r(x_r(t)) > \gamma_r, t \in T\} & \text{if } t_r \text{ is the extinction time of the event} \\ (\gamma_r & \text{represents the temporal filtering threshold)} \end{cases} \quad (2)$$

The time domain $[t^i_{b_j}, t^i_{e_j}]$ corresponding to P_i can be obtained by adding fuzzy intervals $\int_T \frac{\mu_j(t)}{T}$ to the value of t_r.

ii) *Temporal filtering* is realized by comparing the relation between the time domains of the variables determined in **i**) with the time domains specified by the corresponding time descriptors. A reliability coefficient is defined inside the temporal filtering process;

iii) *Numerical filtering* takes place only if a certain event $e_0 \in E_0$ has emerged, or only in the presence of some $e_r \in E_r$ events. We consider that any of these events is described by a P_i event. Due to temporal filtering we know if the P_i event emerged, is about to emerge etc., in other words, we know its degree of emergence. Even if the time domain $[t^i_{b_j}, t^i_{e_j}]$ corresponding to some specimen values was determined, the problem of synthesizing a single value from the $x_i(t)$ specimens set situated in inside the time domain from **i**) appears, which must eventually filter with the P_i fuzzy sentence. This synthesis takes place closely related to the semantics of the P_i sentence and compared to the used synthesis method. Typical to these methods is the estimation of the $[t^i_{s_1}, t^i_{f_1}]$ and $[t^i_{s_2}, t^i_{f_2}]$ time domains, that signify the time intervals in which the values of the $X^{(i)}$ variable can be synthesized in a single value. The possible maximum time of the emergence of the P_i

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event will also interfere, and also the possibility distribution attached to the P_i event.

4 Conclusion

In the present paper I analyzed the formal aspects of the reasoning corresponding to an expert management system of the technological processes that includes imprecise knowledge and time variables. With this aim, I extended a first-degree logic fuzzy system with temporal modal operators that allow the justification of the synthesis of certain linguistic process management models. The process of modeling the approximate reasoning assumes the definition of certain fuzzy sets of evaluated closed formulas, which are actually fuzzy subsets of certain sets of special axioms. The description of some models that include also attributes like temporal descriptors, we highlight the fact that the specification and synthesis of fuzzy management models is marked, from a logical point of view first of all, by the presence of the possible and the necessary. The temporal precedence relations can appear especially in diagnosis applications, where the introduction of time is made from the exterior and these types of applications allow symptoms classification. The formulas from the extended first-degree fuzzy logic domain with temporal modal operators can be used in order to model various management strategies. For instance, take $g_s = (x_1 \vee x_2 \vee \ldots \vee x_n)$, where $x_i \in X_s \subseteq X$ and take $g_0 = (x_{01} \vee x_{02} \vee \ldots \vee x_{0n})$ in which x_{0i} are initial states for the state variables of the $(1 \le i \le n)$ process. Take $X^b \subset X^{SE}$ and $g_b = (x_{b1} \vee x_{b2} \vee \ldots \vee x_{bn})$ in which $x_{bi} \in X^b$. The $g_0 \rightarrow \Diamond g_s$ formula can be seen as an admission condition. The formulas: **i**) $g_0 \rightarrow \Diamond \Box g_s$ (if the process starts from one of its initial states, then, after a certain number of moments of time its state will always be found in X_s); **ii**) $g_0 \rightarrow \Box \Diamond g_s$ (if the process starts from one of its initial states, then it will be in X_s for an infinity of times); **iii**) $\Box g_b \rightarrow \Box g_s$ (if the inputs of the process are always in a X^b set, then the states of the process will always remain in X_s set), characterizes properties which can be thought of as the equal of the stability demands. The temporal logic is a particular type of modal logic and provides a formal framework which allows the description of the way in which certain systemic properties can be specified, and it is useful in a more profound understanding of the state of the systems. It is very important to know these facts when we refer to the expert management systems of technological processes, in order to analyze the time evolution of the states and events sequences, to implement and verify the system itself. We can more adequately specify the behavior of the management system within the temporal logic formalism, since this kind of specifications have a greater expressivity in comparison to the classical logic specifications. The temporal logic properties cover many of the dynamic behavior aspects of the knowledge-based management systems. That is why we consider that the logic formalism presented above is important for the creation of the REFK2 system, since it is an attempt of including both fuzzy and temporal attributes.

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