Reaction-Diffusion Systems in Pattern Formation and Pattern Recognition Processes

ATSUSHI NOMURA
Faculty of Education
Yamaguchi University
Yoshida 1677-1, Yamaguchi 753-8513
JAPAN
anomura@yamaguchi-u.ac.jp

MAKOTO ICHIKAWA
Faculty of Letters
Chiba University
Yayoi-cho 1-33, Inage-ku, Chiba 263-8522
JAPAN

KOICHI OKADA
Center for the Promotion of Higher Education
Yamaguchi University
Yoshida 1677-1, Yamaguchi 753-8511
JAPAN

HIDETOSHI MIIKE
Graduate School of Science and Engineering
Yamaguchi University
Tokiwadai 2-16-1, Ube 755-8611
JAPAN

TATSUNARI SAKURAI
Graduate School of Science
Chiba University
Yayoi-cho 1-33, Inage-ku, Chiba 263-8522
JAPAN

Abstract: This paper presents a quick review of reaction-diffusion systems that self-organize spatio-temporal patterns in chemical and biological systems. A pair of reaction-diffusion equations having activator and inhibitor variables is a typical model of equations describing their pattern formation processes. Some of the reaction-diffusion systems can realize several functions of image processing such as edge detection and segmentation required for pattern recognition processes. Thus, reaction-diffusion systems which have been studied as pattern formation processes are also interesting topics as pattern recognition processes. The authors have recently done the research of realizing functions of edge detection, segmentation and stereo disparity detection by utilizing reaction-diffusion equations with large inhibitory diffusion. This paper additionally presents a brief introduction of our recent research topic, in particular, an edge detection algorithm with a discretely spaced system of the FitzHugh-Nagumo reaction-diffusion equations. Experimental results obtained for artificial and real images show the comparison between a previous standard algorithm and the reaction-diffusion algorithm designed for edge detection.

Key–Words: Reaction-diffusion, pattern formation, pattern recognition, long-range inhibition, image processing, computer vision, edge detection, non-linear reaction

1 Introduction

Reaction-diffusion systems self-organize variety of spatio-temporal patterns such as propagating circular, spiral and periodic waves. Those spatio-temporal patterns are observed in the Belousov-Zhabotinsky reaction system, a kind of a chemical reaction system. Particular points on the two-dimensional system of the chemical reaction exhibit non-linear reaction and diffusion processes on chemical species couple non-linear oscillators spatially. The non-linear reaction coupled with diffusion processes organizes the spatio-temporal patterns. Pattern formation processes are also found in biological systems [1].

A pair of reaction-diffusion equations having activator and inhibitor variables is a typical model in describing the pattern formation processes. Keener and Tyson proposed a pair of time-evolving partial differential equations having diffusion terms coupled with non-linear reaction terms [2]. By computing the reaction-diffusion equations numerically, we can obtain such the spatio-temporal patterns. Reaction-diffusion equations are well established as a mathematical model of a reaction-diffusion system.

Stationary pattern formation processes organized with rapid inhibitory diffusion are interesting phenomena. Turing proposed a scenario that organizes a stationary periodic pattern in a reaction-diffusion system, in which the inhibitor variable rapidly diffuses more than the activator variable does [3]. Although a diffusion process generally brings uniform distribution of a substance, the scenario presents a non-uniform and periodic pattern as a stationary state in a reaction-diffusion system. Gierer and Meinhardt motivated by the Turing's scenario proposed more realistic versions of reaction-diffusion equations with rapid inhibitory diffusion [4]. They additionally showed that the equations can simulate the pattern formation process observed in Hydra. The model of equations also successfully realized regeneration of the head of Hydra and grafting a head section to another terminal section of its body in computer simulation. Recent evidences found in biological systems support the Turing's scenario [5]; biologists have accepted the realistic version as a model of pattern formation processes observed in biological systems [6]. The key point of the scenario is the rapid inhibitory diffusion.

If turning our attention to pattern recognition processes in biological vision systems, we can find several interesting phenomena and their models. Lateral eyes of Limulus, which is a kind of club, exhibit the Mach bands effect, which is also found in the human visual system. Previous physiological experiments show that the Mach bands effect is due to the long-range inhibition in a lateral inhibition mechanism working on outputs of discretely spaced ommatidia, which are individual visual receptor units [7]. A model taking account of the lateral inhibition mechanism completely simulated the Mach bands effect in the lateral eyes of Limulus [8].

While many researchers were interested in reaction-diffusion systems, Kuhnert et al. reported that a photo-sensitive Belousov-Zhabotinsky reaction system can enhance edges and extract segments on intensity distribution [9]. More recently, Sakurai et al. proposed a method of controlling chemical wave propagation by utilizing laser light illumination and succeeded in designing a path of the propagation [10]. These previous results have completely linked reaction-diffusion systems with image processing and computer vision research. Adamatzky et al. named a class of computer algorithms utilizing reaction-diffusion systems ”reaction-diffusion algorithm” [11].

From an engineering point of view, the Chua's circuit is an interesting topic for reaction-diffusion systems [12]. It generates a non-linear oscillation and a resistively coupled Chua's circuit system realizes a reaction-diffusion system. The Chua's circuit in two-dimensional version self-organizes spatio-temporal patterns of circular and spiral waves and the Turing pattern [13]. Thus, a reaction-diffusion system can also be implemented on a circuit system.

The authors and their co-workers have proposed several reaction-diffusion algorithms for image processing and computer vision research. A discretely spaced system of the FitzHugh-Nagumo reaction-diffusion equations detects edges and segments from an image intensity distribution provided as an initial condition of the equations [14, 15]. Kurata et al. analyzed the system and obtained a condition for stable results of edge detection and segmentation [16]. Nomura et al. proposed an algorithm of detecting a stereo disparity map from a pair of stereo images. The stereo algorithm utilizes multiple reaction-diffusion systems exclusively linked [17]. We have imposed the rapid inhibitory diffusion on these reaction-diffusion algorithms.

This paper firstly describes a quick review of reaction-diffusion systems and then describes a simple idea for reaction-diffusion algorithms. In addition, this paper presents a recently proposed edge detection algorithm and its experimental results in comparison to a standard algorithm.

2 Reaction-Diffusion System

A reaction-diffusion system is generally described with a set of time-evolving partial differential equations. Each equation consists of a diffusion equation coupled with a reaction term; the reaction term usually describes a non-linear phenomenon observed in nature. Most typical form of a reaction-diffusion system is described with a pair of reaction-diffusion equations having an activator variable u and an inhibitor variable v, as follows:

$$\partial_t u = D_u \nabla^2 u + f(u, v), \quad (1)$$

$$\partial_t v = D_v \nabla^2 v + g(u, v), \quad (2)$$

in which, $f(u, v)$ and $g(u, v)$ are reaction terms; the variables u and v are defined in one-dimensional space x or two-dimensional space (x, y) and in time t; $\partial_t = \partial/\partial t$ and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$; D_u and D_v are diffusion coefficients.

The Belousov-Zhabotinsky reaction exhibits non-linear reaction of chemical species; the Oregonator model describes the non-linear reaction. Since the chemical species induce chemical reaction and simultaneously diffuse, the chemical reaction system on two-dimensional space self-organizes spatio-temporal patterns of such as propagating circular and spiral waves. Keener and Tyson proposed a model of reaction-diffusion equations with the Oregonator model [2].

We can find a wide variety of reaction-diffusion systems in the fields of physics, chemistry and biology [1]. One of the most popular reaction-diffusion equations is a pair of the FitzHugh-Nagumo reaction-diffusion equations [18, 19]. The pair qualitatively

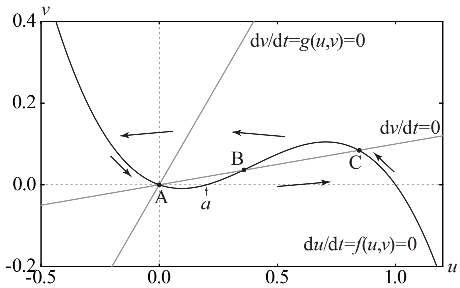

Figure 1: Phase plot of the FitzHugh-Nagumo ordinary differential equations: $\mathrm{d}u/\mathrm{d}t = f(u,v)$ and $\mathrm{d}v/\mathrm{d}t = g(u,v)$ [see Eqs. (3) and (4) for the reaction terms $f(u,v)$ and $g(u,v)$]. The points A and C are stable steady states and B is an unstable steady state. Depending on the parameters a and b, the system of the equations becomes a uni-stable system or a bi-stable system. An excited state refers to the point C and its neighboring area and a resting state refers to the point A and its neighboring area. Under the positive small constant $0 < \varepsilon \ll 1$, depending on $\mathrm{d}u/\mathrm{d}t$ and $\mathrm{d}v/\mathrm{d}t$, a solution (u,v) traces the trajectory denoted by arrows as time proceeds.

(a) (b)

Figure 2: Edge detection result with a discretely spaced system of the FitzHugh-Nagumo reaction-diffusion equations [14, 16]. (a) Original image with 524×684 (pixels). (b) Distribution of $u(x, y, t = 50)$. The parameter settings were $D_u = 1.0, D_v = 5.0, a = 0.2, b = 1.0, \varepsilon = 1.0 \times 10^{-3}$; a finite difference in space was $\delta h = 0.5$ and that in time was $\delta t = 1.0 \times 10^{-3}$.

emulates an active pulse transmission process along a nerve axon. The equations have activator and inhibitor variables, which diffuse and also trace the nonlinearity of the cubic function, as follows:

$$f(u,v) = [u(u-a)(1-u) - v]/\varepsilon, \qquad (3)$$

$$g(u,v) = u - bv, \qquad (4)$$

in which a and b are constants and ε is a positive small constant. Figure 1 shows a trajectory of the ordinary differential equations $\mathrm{d}u/\mathrm{d}t = f(u,v)$ and $\mathrm{d}v/\mathrm{d}t = g(u,v)$ with Eqs. (3) and (4). The trajectory depends on the parameter settings of a, b and ε. A bi-stable system refers to that having two stable steady states and a uni-stable system does to that having one stable steady state.

The important point of the reaction terms of Eqs. (3) and (4) is that the parameter a works as a threshold value for an initial condition. When an initial condition of the solution (u,v) is $(u,v) = (a + \delta, 0)$, if $\delta > 0$, the solution (u,v) becomes an excited state; if $\delta < 0$, it becomes a resting state (see Fig. 1). Thus, the system of the ordinary differential equations $\mathrm{d}u/\mathrm{d}t = f(u,v)$ and $\mathrm{d}v/\mathrm{d}t = g(u,v)$ has the function of thresholding for its initial condition. This brings the basic idea of detecting edges and segments with thresholding for image intensity distribution. By performing numerical computation for the FitzHugh-Nagumo reaction-diffusion equations, we have proposed to realize computer algorithms of edge detection and segmentation from image intensity distribution given as its initial condition [14].

For a stable stationary solution of edges and segments, an additional important point is that the system is discretely spaced under rapid inhibitory diffusion [16]. Although the earliest work done by Kuhnert et al. show the impressive results of edge detection and segmentation with a real chemical reaction system [9], it does not provide stable results, which are necessary for a realistic algorithm of image processing. In comparison to that, Kurata et al. have found that the discretely spaced version of the FitzHugh-Nagumo reaction-diffusion system under the rapid inhibitory diffusion brings stable stationary results of edges and segments. This important point is similar to the Turing's scenario [3, 4] and also to the long-range inhibition causing the Mach bands effect [8]. These similarities inspire us to develop reaction-diffusion algorithms. It would be interesting, if reaction-diffusion systems developed for modeling pattern formation processes are helpful in modeling visual functions required for pattern recognition processes and realizing their algorithms. Figure 2 shows an example of an edge detection result for a binary image; pulses or areas being an excited state are edges. We utilized the finite difference method for discretization of the FitzHugh-Nagumo reaction-diffusion equations of Eqs. (2) and (2) with Eqs. (3) and (4).

3 Reaction-Diffusion Algorithm Designed for Edge Detection

As shown in the previous section, a single pair of reaction-diffusion equations has a function of detecting edges for binary image. By utilizing two pairs of reaction-diffusion equations, we show that it becomes possible to realize an edge detection algorithm applicable to gray level image [15, 20].

Let us consider a one-dimensional distribution having a step-wise edge. If smoothing the distribution with a Gaussian filter or a simple diffusion equation, we can observe that the smoothed and original distributions intersect at the inflection point of the step-wise distribution. Thus, if we can divide the original distribution into a higher level or a lower level than the smoothed distribution, we can realize an edge detection algorithm that is applicable to gray level image. Let us recall that the FitzHugh-Nagumo reaction-diffusion equations have the constant parameter a which works as a threshold value for an initial condition. Thus, we consider the parameter a as a distribution and substitute the smoothed distribution for the distribution a [15].

A reaction-diffusion system having a distribution a of a threshold level self-organizes false pulses at false edge positions as well as true pulses at around true edge positions. This is because the step-wise edge and its smoothed distribution intersect at the inflection point and also converges at inside of a region having a higher intensity level. Thus, the false edges are organized inside of the region. We need to eliminate the false edges from outputs of the reaction-diffusion equations.

By considering the parameter a as a threshold level distribution and for eliminating false edges, we presents an edge detection algorithm [20]. The algorithm consists of two pairs of the FitzHugh-Nagumo reaction-diffusion equations coupled with a simple diffusion equation, as follows:

$$\partial_t u_0 = D_u \nabla^2 u_0 + f(u_0, v_0, a_0) + \Theta(\partial_t u_1), \quad (5)$$

$$\partial_t u_1 = D_u \nabla^2 u_0 + f(u_1, v_1, a_1), \quad (6)$$

$$\partial_t v_i = D_v \nabla^2 v_i + g(u_i, v_i), \quad (7)$$

$$\partial_t a_i = D_a \nabla^2 a_i, \quad (8)$$

in which the variables u_i, v_i, a_i are also defined in space and time, and $i = 0, 1$ is the index number of the two pairs. The function $\Theta(s)$ gives s, if $s < 0$, and otherwise 0. The algorithm provides an original image for initial conditions of u_i and smoothed images for initial conditions of a_i, in which a_1 is smoothed more than a_0.

The term $\Theta(\partial_t u_1)$ in Eq. (6) eliminates false pulses organized in u_0. The temporal derivative $\partial_t u_1$ becomes negative, when a solution (u_1, v_1) moves from an excited state to a resting state along the solution trajectory shown in Fig. (1). Thus, temporal changes of the other distribution u_1 eliminates the false pulses locating between the true edge position and the false pulse in u_1, when the distribution u_1 goes back to the resting state from the excited state. After a finite duration of time, we obtain an edge detection result by searching pulses from u_0.

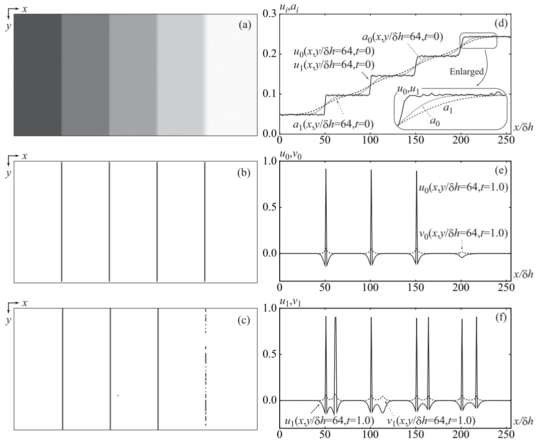

Figure 3: Edge detection for a step-wise image intensity distribution with zero-mean Gaussian noise. (a) Original image with 256×128 (pixels). (b) Edge map obtained by the Canny's algorithm [21] with $\sigma = 3.50$ and threshold levels of 0.70 and 0.90. (c) Edge map obtained by the reaction-diffusion algorithm. (d) Initial conditions of $u_0 = u_1$,a_0 and a_1. (e) Spatial distribution of u_0 obtained at $t = 1.0$. (f) Spatial distribution of u_1 obtained at $t = 1.0$. (d), (e) and (f) show spatial distributions at the vertical position $y/\delta h = 64$. Refer to Section 4 for the parameter settings of the reaction-diffusion algorithm.

Figure 3 shows a simple result of edge detection, in which an image intensity distributes as a multi-level step-wise function and has four edges. The Canny's edge detection algorithm provides a complete result of edge detection, as shown in Fig. 3(b). In comparison, the reaction-diffusion algorithm consisting of Eqs. (6)-(8) provides an almost correctly detected result except for the fourth edge, as shown in Fig. 3(c). These situations show that the algorithm works in the simple situation.

4 Experimental Results

This section presents experimental results of edge detection done by the Canny's algorithm [21] and the

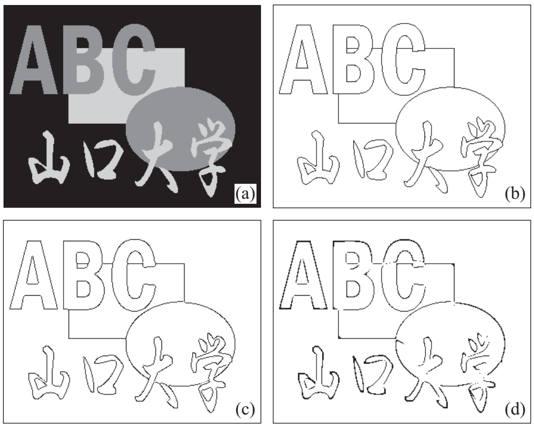

Figure 4: Results of edge detection for an artificial image. (a) Original image with 341×269 (pixels). (b) Ground-truth data of an edge map. (c) Edge map obtained by the Canny's algorithm [21] with $\sigma = 0.20$ and threshold levels of 0.01 and 0.10. (d) Edge map obtained by the reaction-diffusion algorithm.

reaction-diffusion algorithm. In the reaction-diffusion algorithm, we fixed its parameter settings at $D_u = 1.0, D_v = 5.0, D_a = 10.0, b = 1.0, \varepsilon = 1.0 \times 10^{-3}$ and the finite differences δh and δt on space and time at $\delta h = 0.5$ and $\delta t = 1.0 \times 10^{-4}$.

Figure 4 shows the results of edge detection for an artificial image having three intensity levels. The Canny's algorithm almost completely detected edges contained in the ground-truth data of its edge map and also did not detect any false edges. In comparison, the reaction-diffusion algorithm failed to detect edges and detected false edges in several areas, in particular, in areas facing three different intensity levels.

Figure 5 shows results of edge detection for a real image. As shown in Fig. 5(a), the real image contains defocused edges as well as focused edges. Thus, we tested an algorithm detecting edge positions and simultaneously evaluating edge strength. The algorithm consists of multiple reaction-diffusion systems, each of which consists of Eqs. (6)–(8). By providing images smoothed with different diffusion coefficients for each of the systems, we tried to evaluate edge strength [20]. Figure 5(c) shows the obtained edge strength distribution and Fig. 5(d) shows the strong edge extracted from the distribution. From these results, we can confirm that the edge strength evaluation algorithm utilizing multiple reaction-diffusion system works. Since we do not have the ground-truth data, we can not confirm the performance of the algorithm at this moment. Since defocusing cause weak edges, the edge strength information may provide additional information for detecting a depth map [24].

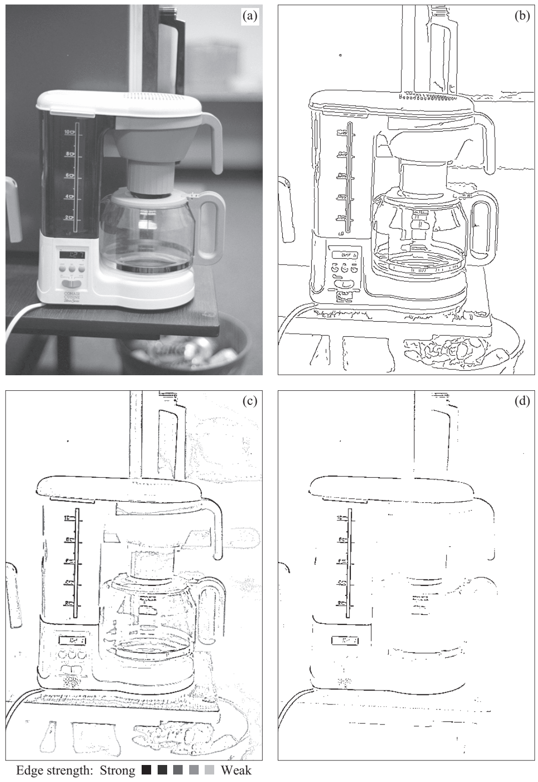

Figure 5: Results of edge detection for a real image. (a) Original real image provided on the website [22, 23]; a size of the image is 461×665 (pixels). (b) Edges extracted by the Canny's algorithm with $\sigma = 1.20$ and threshold levels of 0.20 and 0.80. (c) Edge strength distribution obtained by the reaction-diffusion algorithm. (d) Strong edges extracted from the result (c).

5 Conclusion

In this paper, we presented a quick review of reaction-diffusion systems observed in pattern formation processes and also presented a basic idea for realizing visual functions of edge detection and segmentation required in pattern recognition processes. Furthermore, we presented an edge detection algorithm applicable to gray level image. We imposed the rapid inhibitory diffusion on the algorithms. In addition, by utilizing multiple reaction-diffusion systems, we tested to evaluate edge strength as well as edge position for a real image. These results show that the reaction-diffusion algorithms work for grey level image and also evaluate edge strength. However, their performance is not satisfactory in comparison to a standard algorithm of edge detection. Future research work required for the reaction-diffusion algorithms is the performance improvement and further development of an edge strength evaluation algorithm.

Acknowledgements: The present study was supported in part by the Grant-in-Aid for Scientific Research (C) (No. 20500206) from the Japan Society for the Promotion of Science.

References:

[1] J. D. Murray, *Mathematical Biology*, Springer-Verlag, Berlin, 1989.

[2] J. P. Keener and J. J. Tyson, Spiral waves in the Belousov-Zhabotinskii reaction, *Physica D* 21, 1986, pp. 307–324.

[3] A. M. Turing, The chemical basis of morphogenesis, *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences* 237, 1952, pp. 37–72.

[4] A. Gierer and H. Meinhardt, A theory of biological pattern formation, *Kybernetik* 12, 1972, pp. 30–39.

[5] S. Kondo and R. Asai, A reaction-diffusion wave on the skin of the marine angelfish Pomacanthus, *Nature* 376, 1995, pp. 765–768.

[6] S. Sick, S. Reinker, J. Timmer and T. Schlake, WNT and DKK determine hair follicle spacing through a reaction-diffusion mechanism, *Science* 314, 2006, pp. 1447–1450.

[7] H. K. Hartline, H. G. Wagner and F. Ratliff, Inhibition in the eye of *Limulus*. *Journal of General Physiology* 39, 1956, pp. 651–673.

[8] R. B. Barlow and D. A. Quarles, Mach bands in the lateral eye of *Limulus*, *The Journal of General Physiology* 65, 1975, pp. 709–730.

[9] L. Kuhnert, K. I. Agladze and V. I. Krinsky, Image processing using light-sensitive chemical waves. *Nature* 337, 1989, pp. 244–247.

[10] T. Sakurai, E. Mihaliuk, F. Chirila and K. Showalter, Design and control of wave propagation patterns in excitable media. *Science* 296, 2002, pp. 2009–2012.

[11] A. Adamatzky, B. D. L. Costello and T. Asai, *Reaction-Diffusion Computers*, Elsevier, Amsterdam 2005.

[12] T. Matsumoto, A chaotic attractor from chua's circuit, *IEEE Transactions on Circuits and Systems* 31, 1984, pp. 1055–1058.

[13] M. Gómez-Gesteira, V. P. Pérez-Muñuzuri, L. O. Chua, and V. Pérez-Villar, Coexistence of excitability, hopf and turing modes in a one-dimensional array of nonlinear circuits. *IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications* 42, 1995, pp. 672–677.

[14] A. Nomura, M. Ichikawa, H. Miike, M. Ebihara, H. Mahara and T. Sakurai, Realizing visual functions with the reaction-diffusion mechanism, *Journal of the Physical Society of Japan* 72, 2003, pp. 2385–2395.

[15] A. Nomura, M. Ichikawa, R. H. Sianipar and H. Miike, Edge detection with reaction-diffusion equations having a local average threshold, *Pattern Recognition and Image Analysis* 18, 2008, pp. 289–299.

[16] N. Kurata, H. Kitahara, H. Mahara, A. Nomura, H. Miike and T. Sakurai, Stationary pattern formation in a discrete excitable system with strong inhibitory coupling, *Physical Review E* 79, 2009, 056203 [5 pages].

[17] A. Nomura, M. Ichikawa and H. Miike, Reaction-diffusion algorithm for stereo disparity detection, *Machine Vision and Applications* 20, 2009, pp. 175–187.

[18] R. FitzHugh, Impulses and physiological states in theoretical models of nerve membrane, *Biophycal Journal* 1, 1961, pp. 445–466.

[19] J. Nagumo, S. Arimoto and S. Yoshizawa, An active pulse transmission line simulating nerve axon, *Proceedings of the IRE* 50, 1962, pp. 2061–2070.

[20] A. Nomura, M. Ichikawa, K. Okada and H. Miike, Edge strength evaluation with reaction-diffusion systems, *Proceedings of the 24th International Image and Vision Computing New Zealand Conference*, pp. 442–447.

[21] J. Canny, A computational approach to edge detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 8, 1986, pp. 679–698.

[22] K. Bowyer, C. Kranenburg and S. Dougherty, Edge detector general evaluation, http://marathon.csee.usf.edu/edge/

[23] K. Bowyer, C. Kranenburg and S. Dougherty, Edge detector evaluation using empirical ROC curves, *Proceedings of Computer Vision and Pattern Recognition* 1, 1999, pp. 354-359.

[24] J. H. Elder and S. W. Zucker, Local scale control for edge detection and blur estimation, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 20, 1998, pp. 699–716.