Concept Diagram on the Cognition Diagnosis of Statistics Learning for University Students

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Abstract: Statistics is an important course for university students because it is the foundation of quantitative research. The purpose of this study is to analyze the concept diagram of statistical concepts for university students. Methodology in this study is CAISM (concept advanced interpretive structural modeling). This method can not only present the personal concept structure by hierarchical diagram, but also calculate the magnitude of mastery on each concept. Empirical data comes form paper-and-pencil assessment of statistics course. The results also show characteristics of concept diagram for task-takers of different total score. It shows that task-takers of different total score have their own specific features of concept diagrams. Moreover, task-takers of same total score with different response pattern have distinct concept diagram. According to the results, it shows CAISM can provide useful information for cognition diagnosis. Finally, some suggestions and recommendations for future investigation are discussed.

Key-Words: concept diagram, knowledge structure, cognition diagnosis, statistics concepts.

1 Introduction
Teaching and learning is an interactive process between teachers and students. Assessment plays an important role for teachers to understand achievement of students. As to this point, concept diagram is one method of assessment and it will improve remedial instruction [2].

There are many approaches to do analysis after assessment is finished. One is to analyze total score and the other is to analyze concept diagram. Total score of test is adopted by classical assessment to indicate achievement of students. However, this method has some shortcomings. For example, total score is hard to provide information of cognitive diagnosis and total score can not distinguish features of concepts involved in the items. Therefore, analysis of total score exist limitations.

It will bring great benefits if analysis of concept diagram is adopted. How to analyze concept diagram is a prospective methodology. Approach of graphic representation for personal assessment should be essential and helpful. It is because diagram could be easily understood and individualized concept diagram can improve realization for characteristics of concepts for each task-taker.

Many statisticians are involved in teaching statistic either formally in a college classroom or informally in an industrial setting [17]. Regardless of the setting, a major concern of those who teach statistics in how to ensure that the student understand statistical ideas and are able to apply what they learn to real-world situations [16]. Although teachers of statistics often express
frustration about difficulties students have learning and applying course material, many may be unaware of the growing body of research related to teaching and learning statistics [15]. It is needed to design a systematic way to realize student's understanding about their concept diagrams.

In this study, foundations of $\alpha$-cut operation from fuzzy theory will be used [4]. The empirical data is statistics test. Although CAISM (concept advanced interpretive structural modeling) can reveal individualized concept diagram, only brief discussion is considered in this study. Concept diagrams for students of different total score will be selected and compared. Students of the same total score with different response pattern will also be chose and discussed. Results of this research are prospective on the cognition diagnosis for statistics learning and help teachers design remedial instruction.

2 Literature Review

The algorithm of interpretive structural modeling is the foundation of this study. They will be discussed as follows. Moreover, issue on the statistics assessment will be also discussed.

2.1 Interpretive Structural Modeling

The theory of interpretive structural modeling (ISM) is based upon discrete mathematics and graph theory [5] [11]. J. N. Warfield [8] provided ISM and it aims to arrange elements in a hierarchical relation. For any set that contains $K$ elements, we can make a hierarchical graph of all elements if the binary relationship among elements is known [9]. Namely, the relationship of $A_i$ and $A_j$ must be acquired in advance. The relationship could be expressed in the form of matrix $A=(a_{ij})_{K\times K}$. If $a_{ij}=1$ exists, $A_i$ is the precondition of $A_j$. On the other hand, if it is $a_{ij}=0$, $A_i$ is not the precondition of $A_j$. The analytical procedure of ISM is as follows [6].

The ISM adopts Boolean operation. The transitive closure is $\bar{A}=A \oplus A^2 \oplus A^3 \oplus \cdots A^K$ and reachability matrix is $R=\bar{A} \oplus I = (A \oplus I)^K$. With transitive closure $\bar{A}$ and reachability matrix $R$, the hierarchical graph of elements in matrix $A=(a_{ij})_{K\times K}$ could be plotted [5]. For example, let

$$A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}_{10 \times 10}
$$

The relationship and hierarchies of elements is depicted in Fig. 1.

Fig. 1. The Linkage of Elements in Hierarchies
2.2 Statistics Education and Assessment

Students learn by constructing knowledge and active involvement in learning activities. It should not underestimate the difficulty students have in understanding basic concepts of statistics.

It is often neglected how students understand basic concepts. Students will learn better if they receive consistent and helpful feedback on their performance. Therefore, it is important to evaluate what students know. Learning will be enhanced by investigating concept diagrams so that misconception and mastery of concepts will be understood [15].

A better understanding of students’ learning not only can be important references in determining success in statistic, but also be beneficial to both teachers and students. Assessment will help teachers understand the learning styles of students and adjust their own instructions accordingly. Furthermore, adaptively remedial instruction could depend on the information of students’ shortcomings [13]. In addition, students will also benefit by managing their learning process more efficiently.

However, most researches related to statistics learning and measurement adopt descriptive methods. Little is known about the analysis of knowledge structures. Therefore, the methodology to analyze concept diagrams about university students should be prospective. [14].

3 Method and Algorithm

CAISM is developed to integrate method of fuzzy logic model of perception (FLMP) and interpretive structural modeling (ISM) [3] [7] [10] [12]. The method analyzes concept diagram based on response data matrix and item attribute matrix. It will reveal the individualized concept diagram and relationship.

The process of CAISM algorithm is depicted in Fig. 2.

4 Data Resource

The statistics assessment is designed by the author. There are 51 university students who are sophomores and take elementary statistics course. The test includes 25 items and each item contains one concept attribute. The item concept matrix and the correct ration of each concept are depicted in Table 1. For example, from item 1 to item 4, these items measure concept 1. The correct ratio of concept 1 is .72.

All the concepts are the basic statistics. From concept 1 to concept 5, they are random sampling, sampling distribution, sampling distribution of mean, central limit theorem and sampling distribution of
ratio. All these items are dichotomous and $\alpha = .55$ is decided in the CAISM algorithm [1].

Table 1. Item Concept Matrix and Correct Ratio

<table>
<thead>
<tr>
<th>Item</th>
<th>Concept 1</th>
<th>Concept 2</th>
<th>Concept 3</th>
<th>Concept 4</th>
<th>Concept 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>item1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>item2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>item3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>item4</td>
<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>item5</td>
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</tr>
<tr>
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<td>1</td>
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<tr>
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<td>1</td>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
<th>Sum</th>
<th>4</th>
<th>9</th>
<th>13</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct ratio</td>
<td>.72</td>
<td>.88</td>
<td>.83</td>
<td>.79</td>
<td>.44</td>
</tr>
</tbody>
</table>

5 Results

The correct ratio of concepts varies. It implies that difficulties of these concepts are quite different.

Although CAISM could provide the concept diagram of each task-taker, it is unfeasible to display the concept diagrams of all task-takers respectively. Thus, the following two subsections will display concept diagrams by giving example from several task-takers.

In the subsection 5.1., we will provide two task-takers of different total score so that we could realize the characteristics of concept diagrams. In the subsection 5.2., we will provide one pair of task-takers who have the same total score with different response pattern. It is predicted that task-takers have the same total score with different response pattern will reveal varied concept diagrams.

5.1 Concept Diagrams of Different Total Score

Two task-takers are selected and discussed in this section. The task-taker with ID 27 has total score which is below the mean. The other task-taker with ID 23 has total score which is above the mean. These two task-takers have total score of 16, 21 respectively.

In Fig. 3 and Fig. 4, the symbol of each concept and magnitude of mastery on concept $d_{nu}$ is shown. For example, in Fig. 3, the mastery of concept 3 is .59 and this concept is the precondition of concept 1, concept 4 and concept 5. These two task-takers own different concept diagrams.

Fig. 3. Concept Diagram of Task-Taker ID 27 (Total Score= 16)
5.2 Concept Diagram of the Same Total Score with Different Response Pattern

As shown in Table 2, one pair of task-takers with the same total score is randomly selected. Both task-takers have total score 19 with different response pattern.

Table 2. Two Pairs of Task-Takers with Response Pattern and Total Score

<table>
<thead>
<tr>
<th>ID</th>
<th>Response Pattern</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>10011111111111111110</td>
<td>19</td>
</tr>
<tr>
<td>12</td>
<td>111111111111110111000</td>
<td>19</td>
</tr>
</tbody>
</table>

As shown in Fig. 5 and Fig. 6, these two task-takers have varied concept diagrams. In addition to the distinction on mastery of concepts, the linkages and prerequisite relationship among concepts also differ. This phenomenon supports viewpoints of cognitive psychology and psychometrics that response pattern could distinguish characteristics of concept diagram, but not total score.

6 Conclusions

This study investigates the concept diagrams of university students. One is concluded that the study help understand personal concepts diagram. Thus, teachers could design the adaptively remedial instruction. Another finding is that concept diagram reveal personal misconceptions or erroneous linkage among concepts.
Future research could apply CAISM to another fields or subjects [1]. Furthermore, it is prospective to integrate CAISM with instructional technology so that e-learning approach of remedial instruction becomes feasible.

References: