Evaluation of Squeeze-film Damping Effects in MEMS Perforated Plates

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Abstract: In this paper, analytical and numerical methods for simulating squeeze-film air damping affecting suspended and perforated movable micro-surfaces are analyzed. Numerical solutions are obtained by full 3-D Navier-Stokes numerical analysis, carried out by a commercial finite element code, COMSOL Multiphysics, and are compared with those obtained using different analytical models proposed in literature. The numerical and analytical solutions are also compared with published experimental measurements. Investigated cases are experimentally studied rectangular plates, which differ from one another for the size, the number and the position of perforations.

Key-Words: - Squeeze-film damping, MEMS, Navier-Stokes equation, FEA, COMSOL Multiphysics

1 Introduction

The aim of the mechanical design of inertial sensors, like accelerometers, is to get the desired dynamic behavior (i.e. frequency response and quality factor). This achievement strongly depends on damping phenomena, which affect the movement of movable elements, like the seismic mass. At micro-scale, the main damping source is the so-called squeeze-film air damping. It is related to the presence of a thin film of fluid (usually air) confined between two walls in relative normal movement, and becomes significant especially when the thickness of the air layer is at least one third of the wall size. Since such a geometry is very common in MEMS (Micro Electro-Mechanical Systems) devices, for example in plate-shaped or comb-drive accelerometers, this phenomenon has attracted great interest with time. The squeeze-film air damping can be explained by considering what happens into the air gap when one of the plates moves. In particular, when one of the two walls moves normally with respect to the other, one, the fluid is sucked into/pulled out of the gap, and a significant pressure field arises into the channel. This phenomenon causes the generation of a resistive force on the movable plate. When the ambient pressure is high (i.e. atmospheric pressure), the motion of the fluid caused by the relative movement of the two surfaces can be modeled through the Navier-Stokes equation [1]. In most applications, it is possible to further simplify the problem (if non-steady effects, fluid inertia and thermal gradient are negligible) and to use the simpler Reynolds equation [2]. Such an equation can be analytically solved for simple geometries, like rectangular, circular and annular plates [3]. On the other hand, when the movable plate is perforated, the previous equation does not apply. For this reason, during the last two decades new and appropriate models, based on the classical Navier-Stokes and Reynolds approaches, have been introduced. Unfortunately, these analytical models can be adopted for a restricted number of simple regular geometries, while the structures of MEMS devices are generally complex and irregular. In such cases, for accurately modeling, simulating and analyzing squeeze film damping a numerical method, as the finite element method (FEM), should be used. The present work investigates on this perspective. In the paper, solutions, obtained for some referenced cases by a commercial finite element software, COMSOL Multiphysics, have been compared with those obtained using five different analytical models proposed in literature and the experimental results provided by other authors.

2 Analytical modelling of squeeze-film damping effect

Throughout the years, many analytical models have
been proposed to describe the squeeze-film air damping affecting two surfaces in reciprocal movement. Herein, the main expressions for calculating the damping coefficient of both perforated and non-perforated plates are presented.

In the case of simple non-perforated rectangular plates, from analytically solving the Reynolds equation, the damping coefficient can be determined as [3]:

$$C_{\text{rect}} = \frac{\mu Lw^3}{h^3} \beta(\eta) \tag{1},$$

where $\mu$ is the air viscosity, $L$ and $w$ are, respectively, the larger and the smaller side lengths of the moving plate, $h$ is the thickness of the air gap between the two plates and $\beta$ is a correction factor depending on the $w/L$ ratio ($\eta$), which is possible to find in [3].

Similarly, by analytically solving the Reynolds equation, the damping coefficient for a circular plate is [3]:

$$C_{\text{circ}} = \frac{3}{2\pi h^3} \mu A^2 \tag{2},$$

where $A$ is the plate area.

One important assumption for deriving expressions (1) and (2) is that pressure is considered zero at the plate boundaries. So, border effects are neglected. As will be shown in the next section, these effects can be taken into account if effective sizes of the plate instead of their nominal values are introduced into the above reported analytical equations [4].

Usually, the movable plate (for example, the plate of a plate-shaped accelerometers) has perforations in order to improve the sacrificial layer removal during the fabrication process. The presence of holes significantly affects the pressure distribution inside the air gap and the Reynolds equation does not apply. For this reason, different analytical models have been proposed throughout the years to include also the effect of holes [5-9]. Herein, special attention will be given to five models, proposed by Skvor [5] in 1967, Bao et al. [6] in 2003, Kwok et al. [7] in 2005, Veijola [8] in 2006 and Pandey & Pratap [9] in 2007, respectively. Usually, to properly derive suitable analytical expressions, the whole perforated plate is divided into square or circular cells, each including one hole.

According to Skvor’s model, Reynolds equation is solved by considering zero pressure on the inner cell boundary and zero flux on the external cell boundary. So, the damping coefficient can be defined as [5]:

$$C_{\text{Skvor}} = 3\pi \frac{\mu}{Q_{ch}} \frac{a^4 N}{2h^4} K(\lambda) \tag{3},$$

where $a$ is the cell radius, $N$ is the number of perforations, $Q_{ch}$ is function of the Knudsen number [5], and $K$ is function of the ratio $\lambda$ of the hole to cell radius.

In their model, Bao et al. [6] started from solving a modified Reynolds equation for incompressible flow and found that the damping coefficient can be expressed as:

$$C_{\text{Bao}} = \gamma \frac{\mu Lw^3}{h^3} \tag{4},$$

where $\gamma$ is defined as:

$$\gamma = 3\alpha^2 - 6\alpha^3 \sinh\left(\frac{1}{\alpha}\right) - 24\alpha^3 \eta \frac{\sinh(2/\alpha)}{\pi^2} \times$$

$$\times \sum_{n=1,3,5, \ldots}^{\infty} \frac{\tan\left(\sqrt{1+(n\pi\alpha/2)}/\alpha \eta\right)}{n^2 \left[1+(n\pi\alpha/2)^2\right]^{3/2}}. \tag{4'}$$

Here, $\alpha = 1/a$, where $l$ is the attenuation length [6], depending on the ratio $\lambda$ of the hole to cell radius, the effective thickness of the perforated plate, the hole radius and the thickness of the air gap. This equation can be properly modified to take into account rarefaction effects [9].

In Kwok’s model, Reynolds’ equation is solved by considering non-trivial pressure condition on the inner cell boundary and trivial pressure condition on the outer cell boundary. In this case, the damping coefficient is [7]:

$$C_{\text{Kwok}} = \frac{3\mu q^4 N}{8Q_{ch} h} K(\lambda) + 8T_p N \frac{\mu (q^2-s^2)^2}{Q_{ch} s^4} \tag{5},$$

where $q$ is the square cell side, $Q_{ch}$ and $Q_{th}$ are function of the Knudsen number and $s$ is the hole side.

In 2006, Veijola proposed a very complete model, which takes into account the air rarefaction and compressibility, the air flow through the gap between plates and that one through perforations.
The final expression of his damping coefficient is [8]:

\[ C_{\text{Veijola}} = \sum_{m=1,3,5,...}^{\infty} \sum_{n=1,3,5,...}^{\infty} \frac{1}{G_m,n(L_{\text{eff}}, w_{\text{eff}})} + \frac{1}{R_{m,n}} \]

where \( w_{\text{eff}} \) and \( L_{\text{eff}} \) are the plate sides appropriately corrected to take into account rarefaction effects; \( G_{m,n} \) is function of the air viscosity, the thickness of the air gap and the Kundschen number; finally, \( R_{m,n} \) quantifies all the losses due to the air flow through both the holes and the gap [8].

In 2007, Pandey further improves modeling of the squeeze film damping of perforated plates. In fact, he distinguishes the damping contribution of plate portions at different location. In particular, the pressure distribution inside the air gap is different if internal or boundary holes are considered (figure 1).

The final expression he proposed is:

\[ C_{\text{Pandey}} = C_{\text{squeeze}} + C_{\text{hole}}, \]

where the two terms are defined as [9]:

\[ C_{\text{squeeze}} = \frac{16 \pi}{\pi^6} \left( \frac{p_k L^2}{\omega h} \right) \times \sum_{m,n=\text{odd}} \left( \frac{\Gamma^2}{\pi^2 + m^2 k^2 + n^2} \right) \left( 4 - f_{\text{perf}} \right) \]

and

\[ C_{\text{hole}} = 8 \pi \mu \left( \frac{T_r}{Q_{\text{th}}} + \Delta E b \right) \times N. \]

In the previous equations, \( \eta \) is the plate aspect ratio as before, \( p_k \) is the ambient pressure, \( \omega \) is the angular frequency, \( \sigma \) is the squeeze number, \( \Gamma \) includes perforation effects of both internal and external holes, \( f_{\text{perf}} \) takes into account losses due to holes, \( \Delta E \) is the relative elongation of the hole side [9] and \( N \) is the number of perforations.

### 3 Numerical modelling of squeeze-film damping effect

Numerical damping analyses were carried out by the means of COMSOL Multiphysics, a finite element code, which allows to solve problems contemporarily defined in different physical domains. For this reason, such a solver is particularly suited to perform squeeze-film damping analysis, where both fluid dynamics and structural mechanics are involved. Here, COMSOL has been used to perform full 3D Navier-Stokes numerical analysis of perforated and not perforated microsurfaces. As known, the numerical FEM analysis process is very time-consuming, when the geometries of perforated devices are complex, due to the high dimensionality of the finite element model. In order to minimize the computational time a high performing computer should be used. Then, the present simulations have been carried out on a workstation with the following technical features: RAM 16 GB, Intel(R) Core(TM) i7 CPU 860 @ 2,80 GHz. To further reduce computational time the symmetries of the studied components have been exploited. By doing so, the time for analyzing the more complex case study here was not more than fifteen minutes.

### 4 Non-perforated plates: comparison of analytical and numerical results

Before considering the perforated plates (whose results are reported in the next section), simple squeeze-film damping problems for rectangular and circular plates without perforations have been studied. As it has been previously shown, for such geometries, analytical solutions are available for a comparison. In particular, it is interesting to notice that in the numerical simulation it is not needed to assume that pressure is zero at the plate boundaries. So, it is possible to quantify the border effects.

![Figure 2: Comparison of damping coefficient calculated analytically (dotted line) and numerically (solid line) for a square plate.](image-url)
The graph of figure 2 shows the difference between the values of the damping coefficient determined by equation (1) and that obtained by finite element analysis carried out by COMSOL, as the ratio of the side length of the plate $L$ to the thickness of the air gap $h$ varies at atmospheric pressure. Starting value of $L$ was 200 µm.

Exploiting the symmetries of the plates examined, numerical analyses have been carried out using only a quarter of plate instead of the whole extension. Finite element 3D mesh is automatically generated for both the working volume and the two plates by the software and automatically updated during time. Tetrahedral elements have been used and the number of elements was chosen by a convergence analysis. In simulation, the upper plate moves while the lower plate is fixed. Figure 3 shows a typical mesh generation and some results, in terms of air flow and pressure distributions.

![Figure 3: Square plate- (a) finite element model; (b) air flow distribution; (c) pressure distribution.](image)

To introduce border effects also in the analytical model, it is possible to appropriately increase the value of the geometrical dimensions of the plate to be put inside the analytical expressions. The correction factor, $\Delta L/h$, for the plate sides depends on the difference in size between the moving and the fixed plate, as shown in figure 4.

![Figure 4: Variation of the correction factor with the ratio between the length of the fixed and movable plate.](image)

In fact, as the ratio $L_f/L_m$, between the fixed and the movable plate varies, the air flow changes (figure 5).

![Figure 5: Air flow distribution when movable and fixed plates have the same (a), comparable (b) or different (c) size.](image)

From the analyses carried out by the authors, the increment of the plate length to be put in equation (1) is 1.288$h$ when the fixed plate is much larger than the moving plate; while it is 0.8483$h$, when the fixed and the moving plate have similar size. In both cases, the determined values are in good agreement with those reported in [4], which have been numerically obtained, too. Similar analyses were also carried out on circular plates (figure 6).

![Figure 6: Circular plate- (a) finite element model; (b) air flow distribution; (c) pressure distribution.](image)

Figure 7 shows the variation of the increment ($\Delta L$ for rectangular plates and $\Delta r$ for circular plates) of the plate length/radius to be put in equation (1)/(2) as the air gap $h$ varies for cases where the movable and fixed plates have different or the same size. The correction factor is the slope of the appropriate curve. For circular plates, $\Delta r/h$ was found to be 0.610 and 0.437 when the movable and fixed plates have different or the same size, respectively. To the authors’ knowledge, such correct factors have not been reported yet.

![Figure 7: Variation of the increment ($\Delta L$ for rectangular plates and $\Delta r$ for circular plates) of the](image)
size of a rectangular plate when the movable and fixed plates have different (dotted line) and the same size (solid line) and of a circular plate when the movable and fixed plates have different (dotted line with circles) and the same size (solid line with circles) with respect to the thickness of the air gap, h.

5 Perforated plates: comparison of analytical and numerical results with experimental data

COMSOL software and the analytical equations previously described were used to determine the damping coefficient characterizing some geometries reported in [9] and [10] for which experimental data are given. In particular, the geometries reported in [10] are 6.3 µm thick gold perforated square plates. They differ from one another for the dimensions of the square holes and the side of the plate.

In order to make a comparison, the three plates of side 55 µm, 96 µm and 137 µm, respectively, were considered. For each plate four identical patterns of different sized holes (7.2 µm, 9.2 µm, 10.7 µm, 12.6 µm, respectively) were taken into account.

Figure 8 shows the finite element model and the results in terms of air flow and pressure distributions for a quarter of plate with 4x4 holes.

![Figure 8: 4x4 holes square plate](image)

(a) finite element model; (b) flow distribution; (c) pressure distribution.

Figure 9 compares the results obtained by Skvor, Kwok, Bao, Veijola and Pandey’s analytical models and those given by numerical analyses in terms of percentage errors with respect to the experimental data. From figure 9, it is possible to notice that results obtained by COMSOL numerical analyses are in good agreement with the experimental and analytical values.

Errors carried out by numerical analyses are always lower than 30%, except one case.

![Figure 9: Percentage errors of damping coefficient](image)

Figure 9: Percentage errors of damping coefficient (with respect to experimental value [10]) determined by numerical analysis (solid line) and different analytical models- Pandey (dotted line with triangles), Bao (dotted line), Kwok (solid line with triangles), Skvor (dotted line with squares) and Veijola’s (solid line with circles)- for square gold plates with 2x2 holes (a), 4x4 holes (b) and 6x6 holes (c).
Figure 10: Damping coefficients (black bars) and their corresponding percentage errors (white bars) with respect to the experimental value [10] determined by numerical analysis and different analytical models.

As regards the case reported in [9], it concerns a silicon square perforated plate with thickness of 3.5 µm, side of 500 µm, 16x16 holes and side of 11.25 µm, air gap thickness of 6 µm. Figure 10 compares the values for the damping coefficient determined by the different analytical models and that one obtained by COMSOL. Also in this case, COMSOL estimates quite accurately the damping coefficient, with an error with respect to the experimental value lower than 30%.

6 Conclusion
In this work, the feasibility of using a commercial multiphysics solver, COMSOL, to simulate damping effects was investigated. To this aim the software was applied for determining the damping coefficient characterizing some perforated and not perforated regular geometries for which the experimentally measured values are reported in literature. The values of the damping coefficient numerically determined by such a solver were compared with those determined by some analytical models and with the experimental values. It has been shown that the numerical solutions obtained using COMSOL appear to reflect experimental values with equal or sometimes better accuracy than the analytical solutions. So, the accuracy of the software was verified and, even if, as it could be expected, the numerical FEM analysis process is more time-consuming than analytical methods, it represents a not contrasted method for analyzing complex perforated micro-devices.

References: