Change of Water Droplets Thermal State in Evaporation Chamber under Different Heating Modes

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Abstract: - Thermal state of water droplet and intensity of phase transformation on its surface is modeled in the evaporation chamber. Due to radiation unsteady temperature field with negative gradient is formed in water droplet and essentially alters the intensity of phase transformation on the droplet surface.

Keywords: - water droplet, heat and mass transfer, radiant heat source

1 Introduction
Understanding of heat and mass transfer in the systems of sprayed liquid droplets in the gas is essential for thermal technologies using this process, like combustion, solutions evaporation, recovery of heat from flue gas, gas cooling, etc.

In the evaporation chamber there is a small energy potential difference between air and water injected into it. Therefore, the process of phase transformation is inefficient. It is possible to intensify heat and mass transfer by delivery of additional heat into water droplets. This requires one to understand the influence of radiation to heat and mass transfer process of sprayed water droplets. It is necessary to take into account the intensity of heat and mass transfer in the air flow and inside the evaporating droplets. Processes of dispersion and two-phase flow are closely related [1, 2].

Research of heat and mass transfer of the condensed particles is known as “droplet problem”. It covers research of heat and mass transfer phenomena inside the droplet and in its ambience. Last decade achievements in droplet problem are described in [3-5].

2 Analytical-Numerical Method of Research
The combined analytical-numerical method to solve droplet problem is applicable when it is necessary to consider the interaction of unsteady heat and mass transfer process with the peculiarities of spectral radiation [2-4]. The advantages of this method include prevention of timing and space coordinate indeterminacies, as well as, reliable control of convergence. The method of droplet surface temperature calculation is based on the principle of energy flow balance at the droplet surface:

\[ q_x^-(x) + q_z^-(x) + q_f^-(x) = 0. \] (1)

Droplet surface temperature determines the mode of phase transformation. The convective and radiant components \( q_x^-(x) + q_z^-(x) \) describe the total heat flow. In spectrum of infrared radiation the complex refractive index for water alters significantly, though it is finite [6]. When the droplet surface does not absorb spectral radiation \( q_x^-(x) = q_x^+ \). Then the condition (1) for the condensing mode of the droplet phase transformation corresponds to expression \( q_x^+ + q_f^+ = q_c^- \), and in the case of evaporative mode the expression is \( q_c^- - q_c^+ = q_f^- \).

The density of heat flow \( q_f^+ = m_v^- \cdot L \) is determined by the density of water vapor flow on the droplet surface, which is described by the Shorin-Kuzikovsky model:

\[ m_v^+ = \frac{D_w \mu_v}{T_{w} R \mu \rho \ln \frac{p - p_{v,w}}{p - p_{v,g}}}, \] (2)

which evaluates the influence of Stefan flow to mass transfer.

The external convective flow between air and the droplet is

\[ q_c^- = \frac{Nu \lambda_w}{2R} \left( f_B \right) \left( f_R - T_R \right) \] (3)
The function of Spalding transfer number evaluates the influence of hydrodynamic Stephan flow. Nusselt number for the droplet is described by Abramzon-Sirignano model [7], and Spalding transfer number is determined based on recommendations presented in [2]. The internal convective flow in a droplet is described by the modified Fourier law

$$q_c = \lambda_{ef} \frac{\partial T(r, \tau)}{\partial r} \bigg|_{r=R} ,$$  

(4)
as the effective coefficient of thermal conductivity, which takes into account water circulation inside the droplet according to the recommendations in [7].

Unsteady temperature field in water droplet is described by model of combined heat transfer in a droplet by conduction and radiation [4].

$$T(r,t) = T_0(t) + \frac{2}{\pi} \sum_{n=1}^{\infty} \sin(n\pi\eta) \times \int_0^t f_n(\tau) \exp \left[ -a \left( \frac{n\pi}{R} \right)^2 (t - \tau) \right] d\tau.$$  

(5)

Function $f_n$ takes into account the influence of radiation absorption in the droplet and the rate of change of surface temperature to the temperature field in the droplet. To determine the volume variation of a spherically symmetrical droplet the intensity of phase transformation on droplet surface and expansion of warming water are evaluated:

$$\frac{\partial (\rho R^3)}{\partial \tau} = -3R^2m_i^e.$$  

(6)

The system of equations (2)–(6) is solved numerically using an iterative method. Number $J$ of a control droplet cross-section is freely selected. The position of cross-section is defined by dimensionless droplet coordinate $\eta_j$ ($\eta_j = 0$ when $j = 1$; $\eta_j = 1$ when $j = J$). Control time $t$ is selected and number $I$ of time coordinate steps is provided ($\tau = 0$ when $i = 1$; $\tau = t$ when $i = I$).

The thermal state of water droplet convectively heated by air and influenced by radiation is calculated numerically solving the equation (5). The warming droplet surface temperature is calculated using the method of steepest descent down to the accuracy of balance (1) being no less than one hundredth of the percent.

The local radiant flux density in the water droplet is calculated by technique described in [3] based on the temperature field calculated in previous iteration. The technique takes into account the radiation absorption inside the droplet and spectral effects on the surface of the contact between air and water.

3 Results and Discussion

Water droplet warming and phase transformation at different heating conditions with initial temperature $T_{r,0}$ in air of temperature $T_{g,0}$ and relative humidity $\varphi_{g,0}$ is modeled numerically. Droplet convective heating is modeled assuming droplet-to-air phase slip $\Delta\varphi_0$. In the case of complex heating source of radiation is assumed of temperature $T_{w}$ with black body radiation source properties.

Model is for a single lonely droplet $g_0 = G_1 / G_g \to 0$ and assumption that heat and mass transfer process doesn’t influence the bulk air temperature and humidity in the evaporation chamber is made, i.e. $T_g(\tau) = T_{g,0}$ and $\varphi_g(\tau) = \varphi_{g,0}$. The research revealed influence of droplet heating mode to their thermal state. The characteristic lifetime of the conductively heated droplet in the scale, expressed by Fourier number:

$$0 \to F_{co,k} + F_{e,k} + F_{f,k},$$  

(7)
independent of sprayed water dispersity [8]. The characteristic lifetime (7) of droplet combines three modes of droplet phase transformation, in which energetical interpretation of droplets is distinctive (Fig.1).

During the first - condensation mode of phase transformation $0 \to F_{co,k}$ the droplet is heated by condensing water vapor together with conductive heat from air. The time instant $F_0 = F_{co,k}$ reflects the characteristic duration of the mode of condensation phase transformations, which is determined by temperature and humidity of air and temperature of sprayed water into the evaporation chamber. During the condensation mode $q_j^e (F_0 \to F_{co,k}) \to 0$ the droplet warms up till reaches dew point (Fig.2). Time instance, when nature of phase transformation at droplet surface changes is $q_k^e (F_{co,k}) = q_k^e (F_{co,k})$. During the condensation mode it is possible to dehumidify an air. During the second - unsteady evaporation mode $F_{co,k} \to F_{e,k}$ heat supplied to the droplet by conduction heats them and causes its evaporation. As warm-up of the droplet declines and evaporation increases, the process gradually reaches equilibrium evaporation mode, which by...
At time instance $Fo = Fo_{co,e}$ droplets warm-up till temperature $T_{e,k}$ (Fig.2), which identifies the mode of equilibrium evaporation (Fig.2). During the third mode of phase transformation the temperature $T_{e,k}$ remains constant. The characteristic curve (Fig.2b) of droplet surface temperature shows that temperature of conductively heated water droplet is determined by temperature and humidity of air, as well as, by the temperature of sprayed water into the evaporation chamber.

Humidification of air can be performed during the second and third modes of phase transformations. The universal curve describes the dynamics of conductively heated droplet surface temperature and is normalized in regards to sprayed water temperature and equilibrium evaporation temperature in the space, based on the ratio of Fourier numbers $Fo/Fo_{co,e}$. Its existence is verified in [2]. The characteristic curve for thermal state of the warming droplet depends on ambient air temperature and humidity. The sprayed water droplet temperature in the evaporation chamber can be presented by the universal curve in evaporation mode, as well as, in the condensing mode of phase transformations, when appropriate normalized parameters are used. In the mode of condensation it is necessary to normalize the droplet temperature in regards to temperature of sprayed water and dew point temperature, but the duration of the warming process has to be expressed in the scale, based on the ratio of Fourier numbers $Fo/Fo_{co,k}$. In [7] the universality of variation of water and liquid hydrocarbon droplets mass average temperatures in the above-mentioned time scale is presented. Non-isothermality of the droplet heated by conduction in the evaporator chamber of air conditioning system is not significant. Therefore, the universal curve of droplet mass average temperature dynamics and droplets surface temperature dynamics will be similar.

Variation of phase transformations parameters in the air humidifying chamber in real time scale is distinctive and very dependent on droplets dispersity (Fig.3a). Phase transformation at droplet surface and expansion of warming water determine the dynamics of droplet size. Water vapor condensates on the surface of droplet sprayed into humid air and its size rapidly increases until $R(\tau) = R_{co}(\tau = \tau_{co})$, when the intensity of phase transformation on the droplet surface becomes equal to zero $m(\tau = \tau_{co}) = 0$. The expansion of droplet will be more significant when air is more humid and sprayed water is cooler. After change in the mode of phase

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transformation, the droplet size will grow slightly due to expansion of warming water.

Fig.3. Variation of normalized droplet volume (1-3) and normalized vapor flow on the water droplet surface conductively heated in air for real time scale (a) and for Fourier number based time scale (b). 

\( R = 10^6, m = (1) 50, (2) 75, (3) 100; \quad R_{co} = 10^6, m = (1) 50.08, (2) 75.12, (3) 100.17; \quad m_0 = 10^3, \quad kg/m^2\cdot s: (1) 2.78, (2) 1.85, (3) 1.39; \quad T_g = 300 K; \quad \phi = 50 \%

However, the expansion of warming water is compensated by rapidly increasing evaporation of water. When both effects counterbalance each other a point of extremity (Fig.3a 1-3 curves) is observed in the curve showing variation of \( R(\tau) \). Numerical simulation confirms that evolution of normalized phase transformations parameters for conductively heated droplets can be described by characteristic curves (Fig.3 b). The characteristic curves for the warming droplet constructed for the cases defined by temperature and humidity of air and temperature of sprayed water in the evaporation chamber can be presented using the universal curves for evaporative and condensing modes based on the universal time scales.

Fig.4. Influence of sprayed water temperature (a) and air humidity (b) on variation of water droplets thermal state in evaporation chamber. \( \phi \%: (1) 0, (2) 25, (3) 50, (4) 75, (5) 95.

The temperature of sprayed water and humidity of air (Fig.4) significantly influence the characteristic curves of water droplet thermal state. For qualitative evaluation of the droplet thermal state in the evaporation chamber the parameter \( T_{0,e} = T_0/T_e, \) expressed by the ratio of initial temperature of sprayed water and equilibrium evaporation temperature of droplets is important. When \( T_{0,e} < 1 \), water droplets in the mode of unsteady phase transformation will warm-up till temperature of equilibrium evaporation is reached. When \( T_{0,e} > 1 \), water droplets in the mode of unsteady evaporation will cool down until temperature of equilibrium evaporation (Fig.4a). Temperature of equilibrium evaporation for water droplets is closer to air temperature if humidity of air is higher (Fig.4b).

The radiation absorbed by water droplets substantially changes the thermal mode in droplets (Fig.5). Initially, similarly to the case of conductive
heating, as surface layers of the droplets warm intensively the temperature field of positive gradient (Fig.6) is being formed. This temperature field is typical for the first period of the mode of unsteady phase transformation.

Fig.5. Influence of an external radiant source on the thermal state of water droplets in the evaporation chamber. $Fo$: (1) 0.153, (2) 0.61, (3) 0.915, (4) 1.22, (5) 1.68, (6) 2.14, (7) 2.6, (8) 3.36, (9) 5.8; $T_{sr} = 1000$K; $T_g = 300$ K; $\varphi = 75\%$; $R = 0.0001$m.

The condensation mode of phase transformation is possible also in the first period. The second period begins when the internal layers of a droplet get warmer than its surface. During this period the maximum of the unsteady temperature field in droplets gradually shifts from the droplet surface into its center. In the second period, the curve 5 represents the peculiarities of the unsteady temperature field (Fig.5).

During this period the peculiarities of the temperature field gradients (Fig.6, curve 5) determine distribution of absorbed radiation in the droplet by conduction and ensure that part of this energy already contributes to the process of water evaporation. The third period of the mode of unsteady phase transformation onsets after formation of negative temperature gradient field in the droplets and ends, as certain assumptions occur, according to which radiation energy, absorbed in droplets completely takes place in the process of water equilibrium evaporation. Hence, energy of the additional radiation source appreciably influences the thermal state of water droplets (Fig.7). In the initial stage of the mode of condensation phase transformations non-isothermality of the droplets reaches the first peak due to intensive heating of their surface layers, as $T_R > T_C$. Later non-isothermality of the conductively heated droplets gradually declines, as the equilibrium evaporation mode approaches (Fig.7, curve 1).

Fig.6. Influence of the external radiation source on the gradient of unsteady field in water droplets. Nomenclatures similar to Fig.5.

Fig.7. Influence of the external radiation source temperature on non-isothermality of water droplets. $T_{sr}$, K: (1) 0, (2) 500, (3) 600, (4) 700, (5) 800, (6) 900; $T_g = 300$ K; $\varphi = 50\%$; $T_0 = 280$K; $R = 0.0001$m.
As the internal layers of the droplets are intensively heated under the influence of the external radiation source, non-isothermality of the droplets decreases significantly. Later on non-isothermality begins to increase again, as a negative gradient temperature field is formed in the droplets and at the beginning of equilibrium evaporation it reaches its second peak, as $T_C > T_R$.

4 Conclusion
The research results show that irrespective of droplet dispersivity a peculiar change of sprayed liquid thermal state exists in the time scale expressed by Fourier number. The above mentioned change can be conveniently defined by the characteristic curves representing the change of a droplet surface, centre and bulk temperatures, which are sensibly influenced by temperature of gas mixture and partial pressure of liquid vapour in it.

As droplets are heated by conduction, the characteristic curves exist representing the change of a normalized droplet volume and normalized vapour flow on the droplet surface. Under the influence of radiation source temperature field with negative gradient forms in water droplets. In such case the change of droplet thermal state is described by distinctive curves, the deviation of which from the characteristic curves for droplets will be peculiar and dependent on the intensity of radiant flow absorption in the droplets.

5 Nomenclature

\[ a \quad \text{— Thermal diffusivity (m}^2/\text{s)}; \quad B_t \quad \text{— Spalding transfer number}; \quad D \quad \text{— Mass diffusivity (m}^2/\text{s)}; \quad F_o \quad \text{— Fourier number}; \quad g \quad \text{— Ratio of the initial flow rates}; \quad G \quad \text{— Flow rate (kg/s)}; \quad I \quad \text{— Index of control time}; \quad L \quad \text{— Latent heat of evaporation (J/kg)}; \quad m_v \quad \text{— Vapour mass flux density (kg/ m}^2 \text{s)}; \quad n \quad \text{— Number of the term in infinite sum}; \quad N_u \quad \text{— Nusselt number}; \quad p \quad \text{— Pressure (Pa)}; \quad q \quad \text{— Heat flux density (W/m}^2); \quad r \quad \text{— Coordinate of a droplet (m)}; \quad R \quad \text{— Radius of a droplet (m)}; \quad R_v \quad \text{— Universal gas constant (J/kmol K)}; \quad t \quad \text{— Control time (s)}; \quad T \quad \text{— Temperature (K)}; \]

Greek letters:

\[ \Delta_0 \quad \text{— Droplets slip velocity in the air (m/s)}; \quad \eta \quad \text{— Non-dimensional droplet coordinate}; \quad \lambda \quad \text{— Thermal conductivity (W/m K)}; \quad \mu \quad \text{— Molecular mass (kg/kmol)}; \quad \rho \quad \text{— Density (kg/m}^3); \quad \tau \quad \text{— Time (s)}; \quad \varphi \quad \text{— Relative humidity (%)}; \]

Subscript:

\[ c \quad \text{— Convective}; \quad co \quad \text{— Condensation regime}; \quad e \quad \text{— Equilibrium evaporation regime}; \quad ef \quad \text{— Effective}; \quad f \quad \text{— Evaporation regime}; \quad g \quad \text{— Gas}; \quad vg \quad \text{— Vapour-gas mixture}; \quad i \quad \text{— Time index in a digital scheme}; \quad I \quad \text{— Index of control time}; \quad J \quad \text{— Index of droplet cross-section}; \quad k \quad \text{— Conductive}; \quad l \quad \text{— Liquid}; \quad r \quad \text{— Radiative}; \quad rt \quad \text{— Dew point}; \quad R \quad \text{— Droplet surface}; \quad sr \quad \text{— Radiative source}; \quad \text{Droplet surface}; \quad \nu \quad \text{— Vapour}; \quad 0 \quad \text{— Initial state}; \quad \Sigma \quad \text{— Total}; \quad \infty \quad \text{— Far from a droplet}. \]

References: