AN APPLICATION OF THE INFORMATIC AND COMMUNICATION SYSTEMS TO DEVELOP STUDENTS’ ABILITIES TO SOLVE PATTERN PROBLEMS

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ABSTRACT
The purpose of this paper was to explore the development of students’ abilities to solve linear and quadratic pattern problems with pictorial contexts via an internet discussion board. It will be a pedagogy innovation that mathematical activities accompanied with the teacher’s guidance is shifted to Internet to carry out. The participants in this study were forty 10-to-11-year-old students from Taiwan. They were free to develop their abilities under a non-threatening environment of Internet. An Internet discussion board can be viewed as another channel for supporting students to work on mathematics. This activity lasted for a semester. The conclusions drawn from this study were: The students’ patterning abilities consists of the levels: 0, 1, 2, 3a, 3b, 4a, 4b, 4c. Students are from level 0, 1 (seeing of sensory) and level 2, 3a (pattern seeking), arithmetic thinking, to level 3b (pattern recognition), arithmetic-algebraic thinking transition with understanding how to approach generality, and finally to level 4 (generalization), algebraic thinking with ability in making a general rule. In addition the students at level 4 can achieve far generalizations, while those at level 3b can only achieve near generalizations. An ability model of solving linear and quadratic pattern problems with pictorial contents was established based on the relationships above.

KEY WORDS
Ability model, Internet discussion board, Pattern, Thinking, Web-based education

1. Introduction
Patterns are greatly important for establishing a coherent research base in early algebra [1]. Several researches [2–3] denote that patterning activities directly develop a sense of pattern and regularity and they provide students with practice in the skills of searching for patterns, extending patterns, and making generalizations. These processes will involve in variable and the concept of function. Thus, algebra is viewed as the language of patterns [4]. Expressing generality is described as one of four roots of algebra [5]. Compared to arithmetic thinking which is from known to unknown thinking, algebraic thinking is from unknown to known thinking through expressing generality [6].

The school practice involving generalization in algebra often starts from pictorial and numerical patterns. The activity of pictorial patterns is the best way to connect early algebra to concrete context [2]. There are three purposes for setting pattern tasks within pictorial contexts [7]. The first one is for those students who think from a more geometrical approach. The second one is that pictorial content might be more elementary than purely symbolic content. The third one is to vary the format to create more problems to be solved. Four fifths 10-11-year-old students in Taiwan prefer pictorial patterns because of easiness, novelty and interest, or more hints [8].

It will be a pedagogy innovation that mathematical activities are shifted to Internet to carry out. Several pedagogical strategies may be used, including giving students sufficient time to generate and explore their own ideas. Students would be interested in learning under the internet environment [9–10]. Working on mathematics via the Internet can motivate students’ interests and gain low-achievers’ confidence in grades five and six content [11–12]. It can be viewed as another channel for supporting the fifth and the sixth graders to work on mathematics [12].

The 10-11-year-old students’ patterning abilities includes the levels of 0, 1, 2, 3, 4 [13–14]. The investigative process to solve the patterns consists of four phases: 1. seeing of sensory, 2. pattern seeking, 3. pattern recognition: mathematical analysis, 4. generalization [15]. There are two types of patterns used often in textbooks. They are linear and quadratic patterns. Quadratic patterns are those where the difference of the differences (i.e., the second difference) is constant, while linear patterns are those in which the difference between successive terms is constant. This paper would explore the development of 10-to-11-year-old students’ abilities to solve linear and quadratic pattern problems with pictorial contexts via an internet discussion board. The relationships between students’ patterning abilities, the investigative process, generality, and algebraic thinking would be investigated simultaneously. Thus, the purposes were as follows:

1) To explore the relationships between the level of patterning abilities, the phase of investigative process, generality and algebraic thinking while students solving pattern problems;
2) To develop an ability model of students solving linear and quadratic pattern problems with pictorial contents.
2. Theoretical Framework

The investigative process to solve the patterns in numbers and shapes consists of three phases: 1. pattern seeking: extracting information, 2. pattern recognition: mathematical analysis, 3. generalization: interpreting and applying what was learned [16]. Stacey [17] refers to two kinds of generalizations, that is, near generalizations (e.g., term 10 or 20) and far generalizations (e.g., term 100). The generalization problems require students to see, say, record, and test a pattern. Thus, the first stage of it for the learner is always ‘seeing’ which refers to grasping mentally a pattern or relationship (Mason et al., 1985). Thus, the key to success seemed to be at the first stage of pattern perception [18]. According to the perspective three phases of processes above is extended to a lower phase of seeing of sensory [15]. The way of students to see for generalizing problem gave rise to a way of counting. The phase involves whether students are aware of the ignoring and stressing in the midst of seeing.

Ma [14] combines and revises the stage and level of children’s patterning abilities, suggested by Orton and Orton [14]. She establishes the levels about fifth and sixth graders solving linear patterns with pictorial contents. They are 0, 1, 2, 3a, 3b, 4a, 4b, 4c. The procedure is adopted in which students’ responses to what is noticed are placed at one of the levels. Level 0: no progress at all. Level 1: student notices some properties of the numbers, with perhaps partial patterns described. Level 2: student notices but not to describe a pattern, so the next number might not be described. Level 3: student knows how to obtain the next number using patterns extrapolated from the differences. 3a: recognize a relationship between successive terms, but only notice the answer; 3b: recognize a relationship between successive terms and the structure in relation to the shape. Level 4: student shows clear evidence of understanding the relationship, though an algebraic formula might not be expressed. 4a: a correct verbal statement; 4b: a worthy attempt at an algebraic expression; 4c: a correct algebraic representation.

3. Methods and Procedures

3.1 Design of the Study

The participants in this study carried out the patterning activities via an Internet discussion board. An Internet discussion board which function is as Bulletin Board Systems is easily learned and used, even the fifth and sixth graders are familiar with the usage of it. Each participant had a specific account and password to enter the board. S/he was free to develop their abilities to solve pattern problems under a non-threatening environment of Internet. Besides word typing and recording of an Internet discussion board, new functions such as basic summation and figures and tables pasting, enable students to express themselves more effectively. At the same time, students’ data would be conveniently collected according to the functions of the board. Figure 1, for example, shows a screenshot on the Internet discussion board.

3.2 Participants

The subjects were forty upper-grade students in Taiwan. They were twenty-eight 11-year-old pupils of grade six and twelve 10-year-old pupils of grade five. There were 16 boys and 12 girls in grade six and 6 boys and 6 girls in grade five. They had basic computer skills and used the Internet regularly. Each participant was anonymous but had fixed codes, such as bl3, gh4, mbm1, and mgh2.

Four researchers were involved in this study. Two elementary school teachers have been teaching in the participants’ schools more than five years. They were responsible for posing problems on the Internet and executing the patterning activities. The other two on the research team are mathematical educators who have been teaching at the university level more than ten years. One of them was the author who designed the study and analyzed the participants’ data from the Internet work and the interviews. The other one was responsible for follow-up discussions and crosschecking of data.

3.3 Instrument

Eight problems involving pictorial and numerical patterns that were to be generalized were given to the students. These problems were revised by the researchers, who considered reference material such as numerical patterns of the Assessment of Performance Unit [19], Orton and Orton [13], and Hargreaves et al. [20], as well as pictorial patterns of Orton, Orton, and Roper [7]. Among these eight problems, five problems are quadratic patterns, and the rest are a linear pattern (problem 5), a geometric pattern (problem 2) and the Fibonacci pattern (problem 4). Problems 1, 3, 5, and 7 were presented with pictorial contents, while problems 2, 4, 6, and 8 were presented with numerical contents.

3.4 Procedures

In the patterning activities, the students were asked to search for a pattern, extend the pattern, and develop a generalization for the pattern. They were advised to solve the problems by themselves, because what they did had nothing to do with their academic achievement; their own methods were the best. The
teachers posed a problem on the Internet once two to three weeks. The students worked on the problems at lunch time or after school in the computer room of their schools. They were allowed to solve a problem based on their own perception over and over again. Many students expected to work on new problems combined with enjoying the pleasure surfing the net. This activity lasted six months.

After having completed the posing of the problems on the Internet, students who had demonstrated unclear ideas or had much better performance on this activity were subsequently interviewed individually by their teacher. This methodology enabled the researchers to match the methods and processes students used when responding to the problems. For example, student mgm2 wrote down ‘Shape 1: 1x2+2x1=4…Shape 4: 4x2+5x4=28?’ To ensure that the expressions mgm2 generated were from her perception, her teacher asked her: What is ‘4x2+5x4=28?’ She then needed to explain her ideas. For example, mgm2 expressed that the rectangular are composed of an upward and a downward rectangular. Thus, data relating to students’ understanding of patterns was collected in two forms: one was the written form to which students responded to students’ understanding of patterns was collected in two forms: one was the written form to which students responded on the Internet discussion board, and the other was in oral form by interviewing students on a one-to-one basis.

4. Results

This paper will take problem 5, a linear pattern, and problem 7, a quadratic pattern, as examples. They will be shown as Figure 2. The problems for the students to solve were: How many dots will be used to make the 5th and the 10th (20th) shapes? How many in general? Eighteen protocols which direct quote from the Internet posting will be analyzed here. Focusing on them is justified for three reasons. First, students needed to leave a rich trace of what they were thinking as they progressed. The trace included the written form and the interviews. Second, responses generated by these students needed to depend on their own perception. Third, students needed to find a rule associated with the pattern at least. These protocols will be taken account of four phases of investigative processes (i.e., seeing of sensory, pattern seeking, pattern recognition, generalization) and generalization along with the level of patterning abilities (i.e., 0, 1, 2, 3a, 3b, 4a, 4b, 4c).

For convenience, the author will use the following method to code the responses. The first code is L or Q, representing the linear pattern or the quadratic pattern respectively. The last three or four codes identifies a specific student (e.g., bl3 or mgm1). For example, “Q-bl3” represents bl3 responds to the quadratic pattern (Q). Within protocols, the author used the sign ‘...’ to symbolize tediously or similar responses of students.

Figure 2 Two examples

【Level 0】

L-gl1: There are 5 dots in shape 1, 8 dots in shape 2, and 11 dots in shape 3. 5 times 8 is 40, and 40 times 11 is 440.

Q-b13: There are 4 dots in shape 1, 10 dots in shape 2, and 18 dots in shape 3. There should be 32 dots in shape 5 and 128 dots in shape 20. The rule is ‘+ 6, + 6, + 8’. For example, 4+6=10 , 10+8=18 , 18+6=24 , 24+8=32...".

Students gl1 and bl3 adopted idiosyncratic methods unpredictably (i.e., L: x 8, x 11; Q: + 6, + 8) to extend the pattern such as 5x8=40, 40x11=440; 4+6=10, 10+8=18. They were unaware of the incorrect content of seeing the picture, so their process only showed the phase of seeing of sensory. Their abilities were at level 0 because they did not notice any properties of the pictures and made no progress at all.

【Level 1】

L-mgm1: There are 6 dots minus 1 dot in shape 1. There are 8 dots minus 1 dot in shape 2. There are 10 dots minus 1 dot in shape 3. …Keep going and you would get shape 100. …There would be “x n” in the nth shape.

Q- mgh2: The first concave consists of 4 circles. The second concave consists of 10 circles. 3 dots are added to the row of the bottom and 4 dots are added to the column of the left-hand side of the shape 1. Form the shape 2 keep going these steps and you will get the answers…..keep adding the row and the column to the shapes and you will get the hundredth. The nth shape should be n²...

The individual interviews revealed that mgm1 viewed all T shapes as the structure of two lines with equal length. It is an incorrect structure. Student mgh2 viewed all shapes as the structure of concave. She recognized a relationship between shape 1 and shape 2 such as ‘3 dots are added to the row…4 dots … to the column’. She ignored the row and the column which overlaps in a common left dot, and she did not continue in making attempts at successive terms. Thus, both of them were unaware of the incorrect content of seeing the picture. Their process only showed the phase of seeing of sensory. Their abilities were at level 1 because they only noticed some properties of the pictures with partial patterns described (i.e., L: two lines; Q: concave).
L-bm5: The first is 5, the second is 8, and the third is 11. 5+3=8. 8+3=11. Thus, the 20th will be 20+3=23. The 100th will be 100+3=103. The nth would be “n+3”.  
Q-bh2: There are 4 dots in shape 1. Add six dots to shape 2. Add eight dots to shape 3. Add ten dots to shape 4. Add twelve dots to shape 5 and you will have the answer.  
Students bm5 and bl2 extracted information from the pattern (i.e., L: +3; Q: +6, +8, +10), so their processes showed the phase of pattern seeking. They noticed a pattern (i.e., 3 or six, ten, twelve more dots are required in each new shape) but did not yet derive the next number (i.e., L: 11+3=14; Q: 18+10=28). Thus, their abilities were at level 2. Student bm5 made a wrong description of the 20th, 100th, nth (e.g., 20+3=23), and bl2 did not made attempts at these terms. They did not achieve near and far generalizing tasks.  

**Level 3a**  
L-mbl2: 5 dots, 8 dots, 11 dots. 5+3=8. 8+3=11. 11+3=14. 14+3=17. There are 17 dots in the fifth T. 17+3=20. 20+3=23. There would be 68 dots in the twentieth T.  
L-gm3: Add a dot to the three vertexes of the T shape respectively. Keep going and you would get the answers. 5+3=8. 8+3=11. …14+3=17.61+3=64. The twentieth would be 64.  
Q-bh5: There are 4 dots in shape 1, 10 dots in shape 2, and 18 dots in shape 3. You get shape 2 if you add 6 on shape 1; you get shape 3 if you add 8 on shape 3. 6+2=8. 8+2=10. 10+2=12. You get shape 4 if you add 10 on shape 3; you get shape 5 if you add 12 on shape 4.  
Students mbl2, gm3, and bh5 extracted information from the pattern (i.e., L: +3; Q: +6, +8, +10), so their processes showed the phase of pattern seeking. They recognized relationships between successive terms, but only noticed answers (e.g., L: 11+3=14, 14+3=17; Q: shape 4 if …add 10 on shape 3. …shape 5 if you add 12 on shape 4). Their abilities were at level 3a. Students mbl2 adopted a short-cut method (i.e., A_{n+2}=A_n+3+A_{n-2}; here A_n expresses the number of the nth), and bh5 did not made attempts at 20th term. Thus, they both did not achieve near generalization. However, gm3 achieved a near generalizing task based on a recursive approach.  

**Level 3b**  
L-bh3: The rule is “8-5=3”, each T shape adds 3 dots.” The first shape is 5. The second shape is 5+3. The third shape is 5+3+3. The fourth shape is 5+3+3+3. …The tenth shape is 5+3+3+3+3+3+3+3+3+3.  
Q-bm4: There are 4 dots in shape 1 …Add 6 to shape 1 and you will have the second. Add 8 to shape 2 and you will have the third. Add 6+8+10+12 to the first 4 and you will have 40 dots in shape 5. Add 6+8+…+22 to the first 4 and you will have 130 dots in shape 10. 4+6+8+…+48+50 and keep adding to 100; you will get shape 100. The number of dots of the nth is counted by this way.  

Q-mgh1: The first is 4, the second is 10, and the third is 18. Add (4 + 2 x ‘the number of its position -1’) to the previous number and you will have the subsequent number. Thus, the fourth will be 28; the fifth will be 40; the twentieth will be 460. The 100th: 4+[(4+2)+[(4+2)+2]+…+[(4+2)+2]+2]+…]. There are (100-1) times in the [ ]. The nth: 4+[(4+2)+[(4+2)+2]+…+[(4+2)+2]+2]+…]. There are (n-1) times in the [ ]  
Student bh3, bm4, and mgh1 conceived of the adding processes they were using relative to the method of showing all the addends (e.g., L: 5+3+3+3; Q: 6+8+10+12 to the first 4, and 4+[(4+2)+[(4+2)+2]+…+[(4+2)+2]+2]+…), not just the last result plus the new amount. Thus, they recognized the structure in relation to the shape, and their abilities at level 3b. Also they made mathematical analysis (e.g., 5+3+…+3), so their processes showed the phase of pattern recognition. Such analyses reflected they could understand how to approach generality and achieve near generalizing tasks (e.g., L: The tenth shape is 5+3+…+3; Q: 130 dots in shape 10 and the twentieth … 460). Among them only mgh1 could make a general rule and achieved a near generalizing task (i.e., The nth).  

**Level 4a**  
L-bh5: Totally add on three dots each time, because add a dot to the right, left, and under sides of T shape respectively. The fifth will be 11 plus 3 plus 3. The fifth will be 5 plus 3 plus 3 plus 3, because 11 are 5 plus 3 plus 3. There would be 302 dots in the 100th because 3 times 99 is 297 and 297 plus 5 is 302.  
Q-gb4: There are 4 circles in shape 1 because of 3 and 1 circle in the first and second column respectively. There are 10 circles in shape 2 because of 4, 4, and 2 circles in the first, second, and third columns respectively. There are 18 circles in shape 3 because of 5, 5, and 3 circles in the first, second, and third columns respectively. One more circle is added to each column of the next shape. …Thus, there are 40 circles in shape 5 because of…There are 130 circles in shape 10 because of ten columns and 12 circles in each column besides other 10 circles in the 11th column. I think there will be 100 columns and 102 circles in each column besides other 100 circles in shape 100 and n columns and other n circles in shape n.  
Student bh5 and gb4 focused on component parts of the shapes. They made a correct verbal statement (e.g., L: 3 times 99 is 297 and 297 plus 5 is 302; Q: 100 columns and 102 circles in each column besides other 100 circles) to show clear evidence of understanding the relationship. They achieved not only near generalizing tasks but also far ones (i.e., shape 100). An algebraic formula might not be expressed (e.g., L: 3 x 99 + 5; Q: 102 x 100+100), so their abilities was at level 4a. Their processes showed generalization phase.
The individual interviews revealed that mgm2 viewed the shapes of the quadratic pattern as the rectangular arrays of dots. The rectangular are composed of an upward and a downward rectangular. It is always 2 in height of the upward rectangular. The numbers of base of the upward rectangular and length of the downward rectangular are related to the position of the term in the pattern; the number of base of the downward rectangular is one more related to the position of the term. Similarly mgh2 denoted that the first, second, and third number (e.g., 4.4.6) is the length of the left, right, and down of T shape respectively. Thus, mgh2 and mgm2 focused on component parts of the shapes. They showed clear evidence of understanding the relationship and made worthy attempts at an algebraic thinking could achieve near and far generalization. Students with algebraic thinking from unknown to known through expressing understanding how to approach generality, because their thinking about the methods of arithmetic will be a possible precursor to approach generality. They might easily make progress from an additive approach to a multiplicative approach (e.g., \(3+3=3\times2\)). These multiplicative numerical expressions will be the foundation for algebraic expressions. Students at level 3b, who could (e.g., L-mb2, Q-bh5) achieve the tenth (or twentieth), did not guarantee to do near generalizing tasks. Students only with arithmetic thinking could not achieve both near and far generalizations. 2. Students at level 3b might involve arithmetic-algebraic thinking transition with understanding how to approach generality, because their thinking from known to unknown is related to arithmetic thinking. Students at level 3a, who could (e.g., L-gm3) or could not (e.g., L-mb2, Q-bh5) achieve the tenth (or twentieth), did not guarantee to do near generalizing tasks. Students with arithmetic-algebraic thinking transition could not achieve far generalization. (3) Students at level 4 engaged in algebraic thinking. They could interpret and apply what was learned.

5. Conclusions

The conclusions relative to the linear patterns were presented in Ma’s study [15].

5.1 Relationships between students’ abilities and the processes

1. Students’ abilities at level 0 (L-gl1, Q-bh3) and level 1 (L-gm1, Q-mgh2), are corresponding to the phase of seeing of sensory. They might be unaware of the incorrect content of seeing the picture. 2. Students’ abilities at level 2 (L-bm5, Q-bh2) and level 3a (L-mbl2, L-gm3, Q-bh5) are corresponding to the phase of pattern seeking. They could extract information from the patterns. 3. Students’ abilities at level 3b (L-bh3, Q-bm4, Q-mgh1) are corresponding to the phase of pattern recognition. They could make mathematical analysis. 4. Students’ abilities at level 4a (L-bh5, Q-gh4), level 4b (L-mgh2, Q-mgm2), and level 4c (L-mbl1, Q-mbm1) are corresponding to generalization phase. They could interpret and apply what was learned.
[13–14]) and the findings above. The model will be shown as Figure 3. It denotes that there are levels of 0, 1, 2, 3a, 3b, 4a, 4b, 4c. (1) Students with level 0, 1 of abilities only showed the phase of seeing of sensory. Students with level 2, 3a of abilities showed the phase of pattern seeking. Both of the phases above possessed arithmetic thinking. (2) Students with level 3b of abilities showed the phase of pattern recognition and prepared arithmetic-algebraic thinking transition with understanding how to approach generality. They can achieve near generalization (3) Students with level 4a, 4b, and 4c of abilities showed the phase of generalization and possessed algebraic thinking with ability in making a general rule. They can achieve near and far generalizations.

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References

[8] H. L. Ma, A study of promoting mathematical reasoning abilities of students via communication in computer network, (Final report of the project, National Science Council in Taiwan, 2002).