Application of LPV Type Force Model in Vehicle Crash Dynamics

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Abstract: In the research field of vehicle crash mechanics one of the most important task is finding a model for the deformational force, which gives acceptable approximation, but doesn't need perfect knowledge about the parameters of the vehicle. The FEM based methods, which are applied in the engineering practice give good approximations, but they have extremely large computational complexity and need a large amount of data. On the other hand there exist simple force models, but they not approximate well in details the force-deflection curve. In this paper we introduce a force model for vehicle deformation based on the LPV-HOSVD paradigm, which well approximates the force during the deformational process and has acceptable complexity.

Key–Words: Vehicle crash, LPV model, HOSVD.

1 Introduction

For all of the car factories one of the most important task is developing better and better passive and active vehicle safety systems. This project require a lot of vehicle crash tests and computer simulations, and based on the results of these, different kind of models are developed for passanger safety, vehicle stiffness, etc. In the field of vehicle crash mechanics, accident analysis, accident reconstruction and crash analysis one of the most important task is finding a model for the deformational force, which gives acceptable approximation, but doesn't need perfect knowledge about the parameters of the vehicle.

In the engineering practice in general a kind of finite element method (in usually a commercial software) is applied to simulate the deformational process. The main disadvantages of these approaches that require a lot of detailed data the vehicle, for example elasticity, stiffness, etc. These parameters are usually unkown, but if we know all of the necessery parameters, then we have to deal with a very complex system of nonlinear partial differential equations. There are commercial softwares which are able to handle this kind of problems, but they have extremely large computational complexity.

In real life we don't know all of the detailed parameters of the vehicle, we just have partial informations about the circumstances of the vehicle crash and about the vehicle itself (and of course there are no two identical vehicles). This fact inspires us researching on modeling techniques which not require the detaild knowledge of physical and mechanical parameters of the examined vehicle, but deal with just a few data, which are relatively easy to measure (quantities which are measurable without "looking inside" the vehicle, for example velocity, deformation, acceleration, time etc.).

In this paper we introduce a force model for vehicle deformation based on the LPV-HOSVD paradigm, which well approximates the force during the deformational process and has acceptable complexity.

2 Previous Force Models and Real Crash Tests

2.1 Previous Simple Force Models

Several force models were developed for describing the force during the deformational process (see for example [1], [2], [3]). These not require too many knowledge about the parameters of the vehicle, but not gives good approximations. The main difference

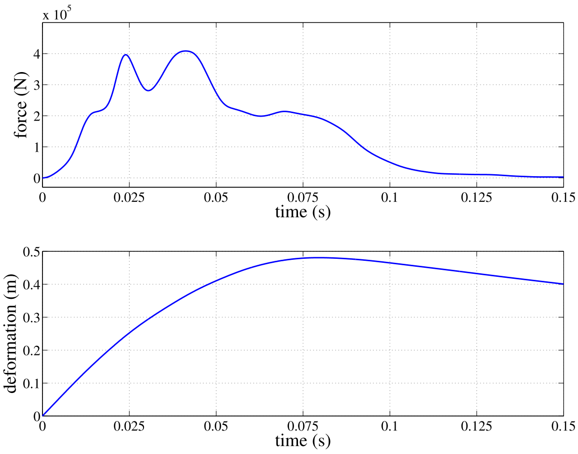

Figure 1: Force and deformation vs. time from a real crash test (source: NHTSA).

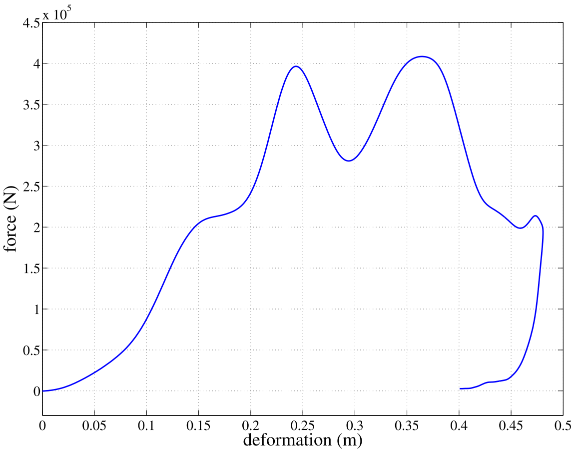

Figure 2: Force-deflection curve derived from the force vs. time and deformation vs. time curves introduced above.

between these models is the concept of stiffness of the vehicle. All of the models are generalizations of the simple linear model, and achieve better approximation applying a more and more complex stiffness concept.

2.2 The 'Stiffness' of a Vehicle

The stiffness of a vehicle is a widely used quasi-heuristic notion in the field of crash and accident analysis, accident reconstruction and vehicle safety research ([4], [5], [6] [7], [8]). The stiffness, as a numerical value, is unambiguous in case of the linear model, but not for vehicle crash process [9]. Generalizations of the stiffness of the linear modell ('k') yield stiffness which depends on the depth of the deformation ([2]).

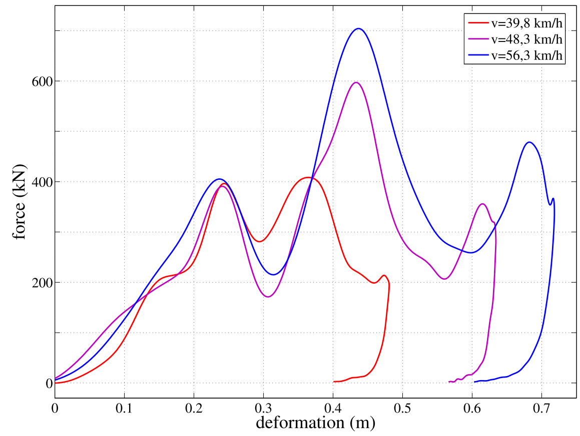

Figure 3: Force-deflection curves for the same type of vehicles for different velocities.

2.3 LCB Crash Test Data

In certain vehicle crash tests the examined vehicle is driven into a rigid wall, which is equipped with special instruments able to register the force during the very short time of the defromational process. This is the so-called lad cell barrier (LCB) test. Before the test a large number of sensors, accelerometers, forcemeters etc. attached to the analyzed vehicle. From the point of view of our task the most important that there are accelerometeres on the vehicle, which are not belonging to the zone of deformation. From data of this type of accelerometer and from the impact velocity, one can determine the dynamic deformation in time.

We deal with data which are available from the free database of NHTSA (National Highway Traffic Safety Administration, USA): *www.nhtsa.dot.gov*. These data are available 'as measured' form, so we have to filter them before of further computing. There are rigorous prescriptions for filtering the crash test data [10]. According to SAE J211 the force and acceleration data were filtered with CFC60 filter. So after filtering we have two data sets: force vs. time and deformation vs. time (See Fig. 1). From these one can easily produce the force–deflection curve (See Fig. 2).

We can state that the previous force models are too simple to be able to describe the peaks and the elastic recovery (turning back) in the force–deflection curve. Moreover, from the concepts of stiffness it is clear, that this cannot be the same numerical value during the deformation process for the whole car body. Based on these experiences we are searching for a model, which approximates the measured data more better and which is a kind of generalization of the linear ($k \cdot x$) model, but deals with a non-constant stiffness.

3 Mathematical Background of Model Reduction

In this section we shortly introduce the mathematical tools which will be applied for reduction of the LPV system. For more detail see for example [11], [12], [13].

3.1 HOSVD based Canonical form

Consider such LPV state-space model

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \mathbf{S}(\mathbf{p}(t))\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix} \tag{1}$$

where $\mathbf{p}(t) = (p_1(t), ..., p_N(t)) \in \Omega$ and which can be given in the form of

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \left(\mathcal{S} \boxtimes_{n=1}^{N} \mathbf{w}_n^T(p_n))\right)\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t),\end{pmatrix} \tag{2}$$

where column vector $\mathbf{w}_n(p_n) \in \mathbb{R}^{I_n}$ $n = 1, \ldots, N$ contains one variable bounded and continuous weighting functions $w_{n,i_n}(p_n)$, $(i_n = 1..I_n)$. The $(N+2)$-dimensional coefficient (system) tensor $\mathcal{S} \in \mathbb{R}^{I_1\times\cdots\times I_{N+2}}$ is constructed from linear time invariant (LTI) vertex systems

$$\mathbf{S}_{i_1\ldots i_N} = \{S_{i_1\ldots i_N,\alpha,\beta}, 1 \le \alpha \le I_{N+1}, 1 \le \beta \le I_{N+2}\}$$

$$\mathbf{S}_{i_1\ldots i_N} \in \mathbb{R}^{I_{N+1}\times I_{N+2}}.$$

Symbol $\boxtimes_n$ represents the n-mode tensor-matrix product.

For this model, we can assume that the functions $w_{n,i_n}(p_n)$,$i_n = 1, \ldots, I_n$, $n = 1, \ldots, .N$, are linearly independent over the intervals $[a_n, b_n]$, respectively.

The linearly independent functions $w_{n,i_n}(p_n)$ are determinable by the linear combinations of orthonormal functions (for instance by Gram–Schmidt-type orthogonalization method): thus, one can determine such a system of orthonormal functions for all n as $\varphi_{n,i_n}(p_n)$, $1 \le i_n \le I_n$, respectively defined over the intervals $[a_n, b_n]$, where all $\varphi_{n,k_j}(p_n)$, $1 \le j \le I_n$ are the linear combination of w_{n,i_j}, where i_j is not larger than k_j for all j. The functions w_{n,i_j} can respectively be determined in the same way by functions φ_{n,k_j}. Thus, if the form (2) of (1) exists then we can determine it in equivalent form as follows:

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \left(\mathcal{C} \boxtimes_{n=1}^{N} \varphi_n^T(p_n(t))\right)\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix}, \tag{3}$$

where tensor $\mathcal{C}$ has constant elements, and column vectors $\varphi_n(p_n(t))$ consists of elements $\varphi_{n,k_n}(p_n(t))$.

Corollary 1 *We can assume, without the loss of generality, that the functions w_{n,i_n} in the tensor-product representation of $\mathbf{S}(\mathbf{p})$ are given in orthonormal system:*

$$\forall n: \int_{a_n}^{b_n} w_{n,i}(p_n) w_{n,j}(p_n) dp_n = \delta_{i,j}, \quad 1 \le i, j \le I_n,$$

where $\delta_{i,j}$ is the Kronecker-function ($\delta_{ij} = 1$, if $i = j$ and $\delta_{ij} = 0$, if $i \ne j$).

Theorem 2 (HOSVD) *Every tensor $\mathcal{S} \in \mathbb{R}^{I_1\times\cdots\times I_L}$ can be written as the product*

$$\mathcal{S} = \mathcal{D} \boxtimes_{l=1}^{L} \mathbf{U}_l \tag{4}$$

in which

1. $\mathbf{U}_l = \begin{bmatrix}\mathbf{u}_{1,l} & \mathbf{u}_{2,l} & \ldots & \mathbf{u}_{I_l,l}\end{bmatrix}$ is an orthogonal $(I_l \times I_l)$-matrix called l-mode singular matrix.

2. tensor $\mathcal{D} \in \mathbb{R}^{I_1\times\ldots\times I_L}$ whose subtensors $\mathcal{D}_{i_l=\alpha}$ have the properties of

(i) all-orthogonality: two subtensors $\mathcal{D}_{i_l=\alpha}$ and $\mathcal{D}_{i_l=\beta}$ are orthogonal for all possible values of l, α and $\beta : \langle\mathcal{D}_{i_l=\alpha}, \mathcal{D}_{i_l=\beta}\rangle = 0$ when $\alpha \ne \beta$,

(ii) ordering: $\|\mathcal{D}_{i_l=1}\| \ge \|\mathcal{D}_{i_l=2}\| \ge \cdots \ge \|\mathcal{D}_{i_l=I_l}\| \ge 0$ for all possible values of l.

The Frobenius-norm $\|\mathcal{D}_{i_l=i}\|$, symbolized by $\sigma_i^{(l)}$, are l-mode singular values of $\mathcal{D}$ and the vector $\mathbf{u}_{i,l}$ is an i-th singular vector. $\mathcal{D}$ is termed core tensor.

Theorem 3 (Compact HOSVD) *For every tensor $\mathcal{S} \in \mathbb{R}^{I_1\times\cdots\times I_L}$ the HOSVD is computed via executing SVD on each dimension of $\mathcal{S}$. If we discard the zero singular values and the related singular vectors $\mathbf{u}_{r_l+1}, \ldots, \mathbf{u}_{I_l}$, where $r_l = rank_l(\mathcal{S})$, during the SVD computation of each dimension then we obtain Compact HOSVD as:*

$$\mathcal{S} = \widetilde{\mathcal{D}} \boxtimes_{l=1}^{L} \tilde{\mathbf{U}}_l, \tag{5}$$

which has all the properties as in the previous theorem except the size of $\mathbf{U}_l$ and $\mathcal{D}$. Here $\tilde{\mathbf{U}}_l$ has the size of $I_l \times r_l$ and $\widetilde{\mathcal{D}}$ has the size of $r_1 \times ... \times r_L$.

Consider (1) which has the form of (2). Then we can determine:

$$\begin{pmatrix}\dot{\mathbf{x}}(t)\\ \mathbf{y}(t)\end{pmatrix} = \left(\mathcal{D}_0 \boxtimes_{n=1}^{N} \mathbf{w}_n(p_n(t))\right)\begin{pmatrix}\mathbf{x}(t)\\ \mathbf{u}(t)\end{pmatrix}, \tag{6}$$

via executing CHOSVD on the first N-dimension of $\mathcal{S}$. The resulting tensor $\mathcal{D}_0 = \tilde{\mathcal{D}} \boxtimes_{n=N+1}^{N+2} \tilde{\mathbf{U}}_n$ has the size of $r_1 \times ... \times r_N \times I_{N+1} \times I_{N+2}$, and the matrices $\tilde{\mathbf{U}}_k \in \mathbb{R}^{I_k\times r_k}$, $k = N+1, N+2$ are orthogonal.

The weighting functions have the property of:

1. The r_n number of weighting functions $w_{n,i_n}(p_n)$ contained in vector $\mathbf{w}_n(p_n)$ form an orthonormal system. The weighting function $\mathbf{w}_{i,n}(p_n)$ is an i-th singular function on dimension $n = 1..N$.

Tensor $\mathcal{D}$ has the properties as:

2. Tensor $\mathcal{D} \in \mathbb{R}^{r_1 \times \ldots \times r_{N+2}}$ whose subtensors $\mathcal{D}_{i_n=i}$ have the properties of

(i) all-orthogonality: two subtensors $\mathcal{D}_{i_n=i}$ and $\mathcal{D}_{i_n=j}$ are orthogonal for all possible values of n, i and j : $\langle \mathcal{D}_{i_n=i}, \mathcal{D}_{i_n=j} \rangle = 0$ when $i \neq j$,

(ii) ordering: $\|\mathcal{D}_{i_n=1}\| \geq \|\mathcal{D}_{i_n=2}\| \geq \ldots \geq \|\mathcal{D}_{i_n=r_n}\| > 0$ for all possible values of $n = 1, \ldots, N+2$.

3. The Frobenius-norm $\|\mathcal{D}_{i_n=i}\|$, symbolized by $\sigma_i^{(n)}$, are n-mode singular values of $\mathcal{D}$.

4. $\mathcal{D}$ is termed core tensor consisting the LTI systems.

3.2 Tensor Product Transformation

Tensor product (TP) transformation is numerical approach, which make a connection between linear parameter varying models and higher order tensors ([12], [13], [14]). The main steps are the followings:

- Discretize the LPV model over a hyper-rectangular grid in the parameter space (dimension is defined by the number of the parameters). If we deal with state space representation, we get matrices $\mathbf{S}^D_{m_1 m_2 \ldots m_N}$.

- Store the matrices into the tensor $\mathcal{S}^D \in \mathbb{R}^{M_1 \times M_2 \times \ldots M_N \times O \times I}$.

- Execute HOSVD on the first N dimension of tensor $\mathcal{S}^D$ and we get the following:

$$\mathcal{S}^D \approx \mathcal{S} \overset{N}{\underset{n=1}{\otimes}} \mathbf{U}_n$$

  Tensor $\mathcal{S} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N \times O \times I}$ contains the LTI (parameter independent) matrices.

- The weighting functions for the LTI matrices are stored in discretized form in the columns of matrices $\mathbf{U}_n$.

4 LPV Type Force Model

The results of crash tests show more difficult force and displacement behaviors than the simple models mentioned above. We are going to develop a model which gives better approximation for the deformational force during the whole deformational process.

4.1 Model Identification

According to the observations, the stiffness parameter of a vehicle (or a part of the vehicle) is not a constant value, but depends on the measure of deformation (x) and on the impact velocity of the vehicle (v). Based on this fact, we assume the force can be approximated well by a nonlinear form, which is a generalization of the linear spring model:

$$F = k(x, v)x. \tag{7}$$

Or, in differential equation form:

$$m\ddot{x} = k(x, v)x. \tag{8}$$

From this, with $k' = k(x, v)/m$, $x_1 = x$ and $x_2 = \dot{x}_1$ we obtain the following matrix form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ k' & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{9}$$

This is a parameter varying matrix and our main assumption is that the behavior of original system (force and displacement) can be described quite well using this kind of nonlinearity. In general state-space model form

$$\begin{aligned} \dot{\mathbf{x}}(t) &= f(\mathbf{x}(t)) \\ y(t) &= c(\mathbf{x}(t)) \end{aligned} \tag{10}$$

where

$$\begin{aligned} f(\mathbf{x}(t)) &= \begin{pmatrix} x_2(t) \\ k\left(x_1(t), v\right) \end{pmatrix} \\ c(\mathbf{x}(t)) &= \begin{pmatrix} x_1(t) & 0 \end{pmatrix} \end{aligned} \tag{11}$$

The next task is to determine the function k'. The approach is similar to the methods introduced in [15] and [16]. Firstly the functional dependence of k' on the variables x (depth of deformation) and v (impact speed) must be specified, for example piecewise linear, polynomial, spline or other linear combinations of given functions of x and v.

The model identification includes two major steps: identification of the local models (LTI models) with the same structure of the LPV model and on the base of these models identification of the final LPV model.

For local model identification we need some data from well-measured crash tests: depth of deformation vs. time, force (at sensors) vs. time. From this data set for a certain deformation x a linear spring model can be identified. Certainly, for other x an other model is valid. The stiffnes k' depends on x, x depends on time (t), so we handle k' as a function of t, which is determined by the measured $F(t)$ and $x(t)$. In this way, for a certain impact speed a set of simple linear models is determined. After that we have to repeat this measuring and identifying process at other impact speeds, but with the same division on the parameter t. Finally we get a large amount of local models in the space of the impact velocity (v) and the time (t), with the same structure of the searched LPV.

A set of linear models means a set of certain values of the parameter varying k' at different parameter values. From these points and using our assumption about the type of the functional dependence, the function k' identified.

Because of the large amount of obtained parameter independent models our system may become very complex. In order to reduce the complexity of the system we apply the introduced tensor product transformation and higher order singular value decomposition below.

4.2 Application on Real Crash Test Data

The method described above is executed on real crash tests data taken from NHTSA. There were three different impact velocity, deformation in time and force in time were measured. From these data sets we obtained by interpolation the $F(t, v)$ and the $x(t, v)$ functions which determined the $k(t, v)$ stiffness.

4.3 HOSVD Based Reduction

The computation was carried out with Matlab TPToolbox ([14]). We applied 108 grid lines in the dimension of the time and 34 grid lines in the dimension of the velocity. Computing HOSVD on each dimension we got 10-10 singular values (which are numerically not zero). So the maximal model was given by keeping all of these singular values. Neglecting singular values step by step we can check the approximation capability of the reduced model (see Fig. 4), and one can see model produced by keeping three of the singular values is practically the same as the measured data.

5 Conclusion

Aplying the LPV-HOSVD paradigm we introduced a novel method based on real crash test data for modeling the force during the vehicle deformational process. This model based on the natural fact that the stiffness of a vehicle depends on the depth of deformation and on the impact velocity. The applied concept of the stiffness is more complex than the others used in this field, but gives more better approximation, and with the HOSVD based reduction the complexity of the model can be reduced significantly, while the approximating capability remains acceptable.

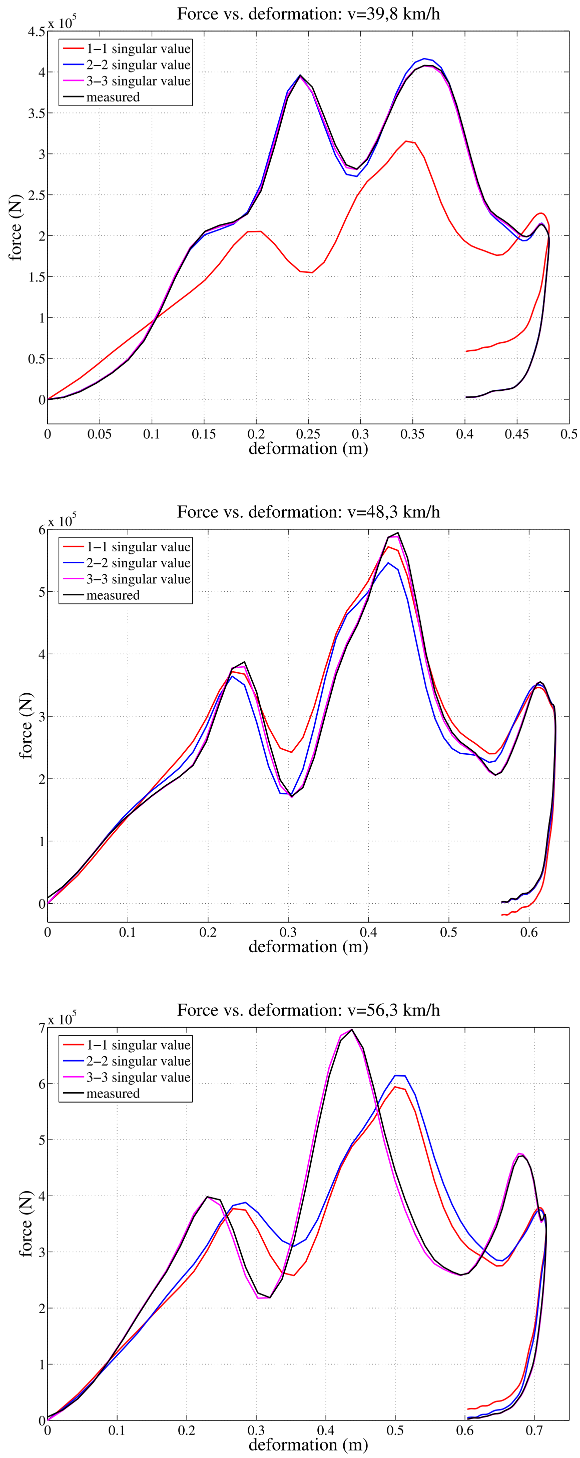

Figure 4: Comparison of the measured data and the reduced models.

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