Flow and heat transfer characteristics on a moving flat plate in a parallel stream with prescribed surface heat flux

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Abstract: - The boundary layer flow on a moving flat plate in a moving fluid with prescribed surface heat flux is studied. A finite difference scheme is used to solve the system of transformed governing equations. The features of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. It is found that dual solutions exist when the plate and the free stream move in the opposite directions. The rate of heat transfer increases with increasing heat flux exponent parameter.

Key-Words: - Heat transfer, Moving plate, Moving fluid, Boundary layer, Heat flux, Dual solutions

1 Introduction
Different from Blasius [1], Sakiadis [2] considered the boundary-layer flow on a moving plate in a quiescent ambient fluid. He found exactly the same equation as Blasius, but the boundary conditions are different. Ishak et al. [3] extended the classical problems of Blasius [1] and Sakiadis [2], by considering a flat plate moving in the same or opposite directions to a parallel free stream, all with constant velocities. Similar problems with various boundary conditions and in different situations have been considered by Merkin [4], Afzal et al. [5] and Weidman et al. [6]. They also reported the existence of dual solutions.

Continuous surface heat transfer problems have many practical applications in industrial manufacturing processes. Such processes are hot rolling, wire drawing and glass fiber production. Problems with variable surface heat flux has been introduced in many other studies [7-10]. The purpose of this investigation is to study flow and heat transfer characteristics on a moving flat plate in a parallel stream with variable surface heat flux.

2 Problem Formulation
Consider a two-dimensional boundary layer flow on a fixed or continuously moving permeable flat surface immersed in a viscous and incompressible fluid of constant temperature $T_\infty$. It is assumed that the plate is subjected to a variable surface heat flux $q_w(x) = ax^n$, where $a$ and $n$ are constants and $x$ is the distance from the slit where the plate is issued, and moves in the same or opposite direction to the free stream, both with constant velocities $U_w$ and $U_\infty$, respectively. Under these assumptions, the boundary layer equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2},
\]

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2},
\]

subject to the boundary conditions

\[
u = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad \text{at} \quad y = 0,
\]

\[
u \to U_w, \quad T \to T_\infty \quad \text{as} \quad y \to \infty,
\]

where $u$ and $v$ are the velocity components in the $x$- and $y$- directions, respectively, $\nu, T, q_w, k, \alpha$ and $V_w$ are, respectively the kinematic viscosity, temperature of the fluid in the boundary layer, surface heat flux, thermal conductivity, thermal diffusivity and the mass transfer velocity at the surface of the plate.
In order to solve Eqs. (1) – (3) subject to the boundary conditions (4), we introduce the following similarity transformation:

\[ \eta = \left( \frac{U}{v_x} \right)^{1/2} y, \quad f(\eta) = \frac{\psi}{(v_x U)^{1/2}}, \quad \theta(\eta) = \frac{k(T - T_\infty)}{q_w} \left( \frac{U}{v_x} \right)^{1/2}, \]

where \( U \) is the composite velocity defined as \( U = U_w + U_\infty \) (Afzal et al. [5]). Further, \( \psi \) is the stream function defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \), which identically satisfies Eq. (1). By employing the similarity variables (5), Eqs. (2) and (3) reduce to the following ordinary differential equations:

\[ f'' + \frac{1}{2} ff' = 0, \]

\[ \frac{1}{Pr} \theta'' + \frac{1}{2} f \theta' - \frac{1}{2} (2n+1) f' \theta = 0. \]

The boundary conditions (4) now become

\[ f(0) = 0, \quad f'(0) = \lambda, \quad \theta'(0) = -1, \quad f'(\eta) \to 1 - \lambda, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \]

where \( \lambda \) is the velocity ratio parameter defined by

\[ \lambda = \frac{U_w}{U}. \]

The physical quantities of interest are the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_x \), which are defined as

\[ C_f = -\frac{\tau_w}{\rho U^2/2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \]

where the wall shear stress \( \tau_w \) and the wall heat flux \( q_w \) are given by

\[ \tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0}, \quad q_w = -k \frac{\partial T}{\partial y} \bigg|_{y=0}, \]

with \( \mu \) and \( k \) being the dynamic viscosity and thermal conductivity, respectively. Using the similarity variables (5), we obtain

\[ \frac{1}{2} C_f \Re_x^{1/2} = f''(0), \quad Nu_x/\Re_x^{1/2} = \frac{1}{\theta(0)}, \]

where \( \Re_x = Ux/v \) is the local Reynolds number.

3 Results and Discussion

The nonlinear ordinary differential Eqs. (6) and (7) subjected to (8) have been solved numerically by a finite-difference scheme known as the Keller-box method described in [11] for some value of the velocity ratio parameter \( \lambda \) and heat flux exponent \( n \). The variation of the skin friction coefficient \( f''(0) \) with \( \lambda \) is shown in Fig. 1, while the corresponding local Nusselt number \( 1/\theta(0) \) is shown in Fig. 2, for some values of \( n \).

It is seen that the solution is unique when \( \lambda \geq 0 \), while dual solutions are found to exist when \( \lambda < 0 \), i.e. when the plate and the free stream move in the opposite directions. The values of \( f''(0) \) are positive when \( \lambda < 0.5 \), and they become negative when the value of \( \lambda \) exceeds 0.5. The solution can be obtained up to a critical value of \( \lambda \) (say \( \lambda_c \)), beyond which the boundary layer separates from the surface, thus no solution is obtained using the boundary layer equations. Based on our computations, \( \lambda_c = -0.5482 \). This value of \( \lambda_c \) is in agreement with those reported by Merkin [4], Afzal et al. [5], Weidman et al. [6], Ishak et al. [3, 12] and Hussaini et al. [13].

The samples of velocity and temperature profiles for some values of parameters are presented in Figs. 3 and 4, respectively. These profiles satisfy the boundary conditions (8), which support the numerical results, besides supporting the dual nature of the solutions presented in Figs. 1 and 2.

4 Conclusion

The development of the boundary layer on a fixed or moving surface parallel to a uniform free stream with variable surface heat flux has been investigated. The classical Blasius and Sakiadis problems are two particular cases of the present problem. Dual solutions are found to exist when the plate and the free stream move in the opposite directions. The rate of heat transfer increases with increasing heat flux exponent parameter.
Fig. 1 Variation of the skin friction coefficient $f''(0)$ with $\lambda$.

Fig. 2 Variation of the local Nusselt number $1/\theta(0)$ with $\lambda$ for various values of $n$ when $Pr = 1$.

Fig. 3 Velocity profiles $f'(\eta)$ when $\lambda = -0.3$.

Fig. 4 Temperature profiles $\theta(\eta)$ for various of $n$ when $\lambda = -0.3$. 
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References:


