ANALYSIS OF STRESS DUE TO CONTACT BETWEEN SPUR GEARS

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Abstract: - In this paper a study of the stresses in the contact zone among a couple of spur gears is realized using the finite elements method. The analysis is done by using a plane model involving the contact between two teeth. The geometry is defined according to the standards of the American Gear Manufacturers Association (AGMA). The results obtained are compared with the value given by two others approaches. The first is the theory of Hertz when it is applied to two curved segments in contact. The second approach is the AGMA procedure for calculating contact stresses in spur gear. The results obtained are very similar either using FEM and Hertz´s theory. The contact pressure obtained by FEM is lower than the one obtained by means of Hertz´s theory, this fact reflects the influence of the geometry profile which allows the occurrence of contact zones with greater area, and therefore lower pressures.

Key words: Spur Gears, Solids Mechanics, Contact Stresses, Hertz theory.

1 Introduction

The performance of many of the elements used in machines, mechanisms or structures, involves interaction with another element through contact. Depending on the type of contact stresses generated can be as significant that they need to be considered in designing the mechanical element. Such is the case of Spur Gears (SG), in which the load is transmitted through contact between the teeth, often requiring special surface treatments to make them ready to withstand the stresses that are generated. In this vein, the stress analysis in the contact area, assessing the most critical conditions, is of particular interest to the appropriate design of the transmission. The present study uses the Finite Element Method (FEM) to analyze the stresses due to contact between a pair of spur gears (SG) and results are evaluated by comparing the Hertz contact theory [1] for curved elements in two dimensions with the theory provided by the AGMA [2] for SG. There are numerous studies of operating conditions and stresses generated by the contact a couple of SG, highlighting (for the purposes of this study) the Filiz and Eyercioglou work [3] which assessed the stress at the base of SG tooth due to a static charge; and varying various parameters of the gear as the module, the contact ratio, the fillet radius, pressure angle and number of teeth, presenting a detailed analysis of the boundary conditions for the model developed. Similar work is done by Sfakiotakis, Anifantis, and Vaitsis [4] in which they simulate the joint action of a SG; making considerations for the dynamic study, describing quite efficiently load conditions and stresses achieved along the tooth flank. These work presents a study using a FEM, of the stresses occurred in the contact zone between a SG pair under the action of static loads by making a detailed study of the most critical operating conditions.

2 GEOMETRY, MATERIALS, AND BORDER CONDITIONS

2.1 Tooth profile.

Gears are wheels with teeth used to transmit power (torque and angular velocity) between two rotating shafts. Power is transmitted from one gear to another by the contact between the teeth. Gears discussed in this work are spur, where the teeth are parallel to the axis of the gears. Spur Gears are standardized in regard to the shape and size of their teeth through AGMA standards [2] (Gear American Manufacturers Association), which serves of support in the research on gear design, used materials and manufacturing processes.

The standard tooth profile gear most used is the involute profile. The involute curve is the one that can be defined

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as the stroke of the end of a rope unwound from the base circle (r_b). The involute profile has the characteristics of conjugated profiles, allowing the fundamental law of gears, been of vital importance for the purpose of developing a model that emulates the gear working conditions, generate accurate and precise geometry of the model. For this purpose a programming code in Autoslip have been developed which can generate gears with involute profile in the English system of complete tooth height. To generate the profile is required to provide the diametral pitch, pitch diameter, pressure angle and number of divisions that are desired in the profile. This allows convenient control of the tooth geometry.

2.2 Material.

The material used in the present investigation it is comercial steel SAE 8620. The study it is done inside the elastic range with the English measurement system. The yield stress is σ_Y = 86x10^3 lbf/pulg^2, modulus of elasticity E= 30 x10^6 lbf/pulg^2 and the poisson ratio υ = 0,3.

2.3 Boundary conditions.

The restrictions and boundary conditions are critical for the model to successfully simulate continuous the part under actual operating conditions. In this regard is of interest to determine the accuracy and appropriate simplifications for a proper balance between results convergence and computing time. In the developed finite element model, using the symmetry of the gear geometry, a twentieth of the upper and lower gear subject to restrictions is used, as shown in Fig. 1, following the approach of Filiz and Eyeriogloy [3].

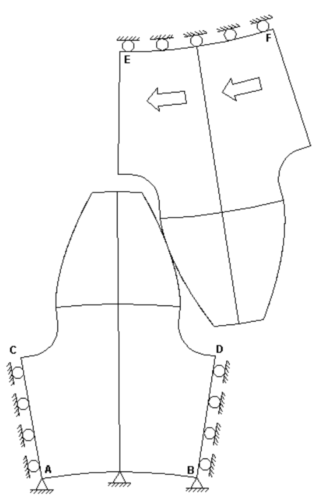

Fig. 1 FEM Boundary conditions

2.4 Highest contact point of a pair of gears teeth.

For purposes of a realistic analysis of the performance characteristics of a pair of gears, it should be noted that the most severe load conditions is found when a single pair of teeth assume the full burden. If no inaccuracies or defects in workmanship, load variations, misalignment and other variants of operation are considered, the condition in which the contact starts, ergo where the tip of a gear is attached to another, is not the most critical, since in this position a couple of other teeth takes some of the burden. Dean [5] developed a procedure for determining the point where one tooth assumes the entire load.

3DISCRETIZATION METHODOLOGY

The discretization procedure from which the convergence was achieved in the simulation results, was based on the generation of four areas for each tooth of the gear pair (see Figure 1). Free meshing tools offered by the simulation program used was were used, with size and shape control of the elements used in the model, allowing to achieve a controlled refinement to suit the number of points generated on the involute profile. Fig. 2 shows the mesh developed.

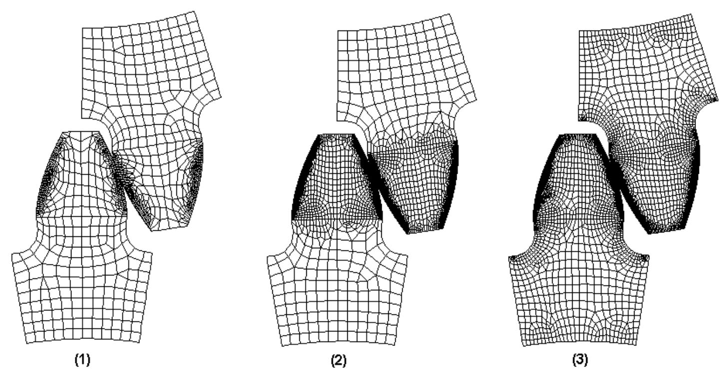

Fig. 2 Discretization in the FEM

4 CONTACT MECHANICS (Hertz cylinders contact in two dimensions)

One of the contact problems raised by Hertz is the determination of the contact stress for frictionless contact of two long cylinders on a line parallel to its axis. When two cylinders are placed in contact, its footprint is rectangular and generates a pressure distribution that consists of a semi-elliptical prism of half width "a". The expressions obtained by Hertz [12] for calculating the contact length and the maximum contact pressure (Po) are:

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$$a = 2\sqrt{\frac{PR}{\pi E^*}} \qquad (1)$$

$$P_O = \sqrt{\frac{PE^*}{\pi R}} \qquad (2)$$

Were P is the normal load transmitted and E^* is a constant that takes into account the elasticity modulus and the Poisson ratio of the gears pair. The pressure distribution in the contact zone it is obtained by the expression:

$$p(x) = \frac{E^*}{2R}\sqrt{a^2 - x^2} \qquad (3)$$

AGMA fundamental equation [2] for contact stress in teeth is:

$$\sigma_C = Cp.\left(\frac{W_t}{F.D_P I}\frac{C_a.C_m}{C_V}C_S.C_f\right)^{1/2} \qquad (4)$$

Were "σc" is the superficial contact stress, "Wt" is the tangential component of the real transmitted load, and "I" the geometric factor. It is also known as pitting resistance geometric factor by AGMA and, takes into account the curvature radio and pressure angle of gear teeth. The constants "C" consider the load application factor (Ca), load distribution (Cm), dynamic factors (Cv), size factor (Cs), and surface conditions factor (Cf). In this study are taken equal to one. C_P elastic coefficient and takes into account the different materials of the pinion and the wheel. According to 2001- B88 standard, for steel with Poisson ratio υ= 0,3 and elasticity modulus E= 30x106MPa, a CP= 2300 it is obtained.

5 ANALYSIS AND RESULTS CONCLUSIONS

For numerical calculations purposes the following values were used: A pair of spur gears with pitch diameters equal d_1= d_2= 5,0 inches. Teeth number N= 20, diametral pitch P_d= 4, width b= 1,0 inch, and hole shaft diameter D_{eje}=3,5 inches. The load transmitted by the gear consist of a torque of T= 10000 Lbf.inch.

5.1 Results.

Fig. 3 presents a comparison of the σ_Z stress along involute profile, obtained from the las mesh studied and the AGMA and Hertz stresses. Can be verified that the models show a similar trend to theoretical curves, with a maximum of stress in the lowest point were a single pair of teeth assume the full burden.

A comparison in the pressure distribution in the contact line (z=0) for the three models with the theory is presented in Fig. 4. The theoretical curve is obtained using equation 3 with the geometric parameters of the studied point. The distribution of model (3) is the one that greatest similarity represents to the Hertz curve, with a maximum of 5 per cent lower than the theoretical.

The values of half width of contact for the three models are shown in Table 1, and a comparison of the variation of results with the theoretical value. The model (3) has a value of "a" 16.6 percent higher than the theoretical.

Table 1 Contact half width "a"

Model	a (inches)	%Variation of "a"
1	0.02088986	79.5%
2	0.013927753	19.7%
3	0.013573617	16.6%
Teorethical	0.011637404	0.0%

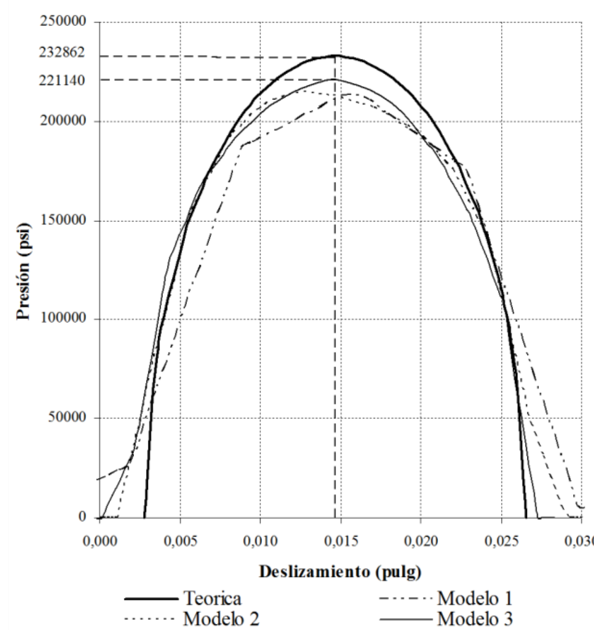

Fig. 3 σ_Z stress along involute profile.

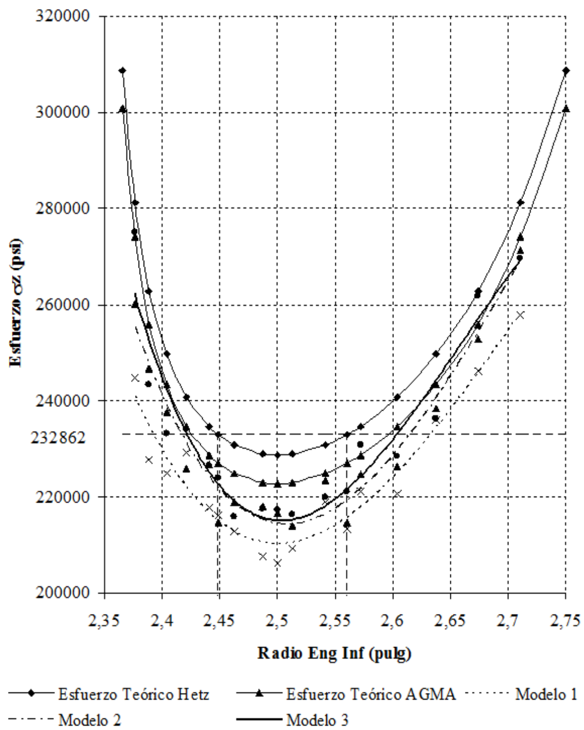

Fig. 4 Hertz pressure contact

Importantly, is seen a strong decline in the value of "a" for the different discretized model (1) to (2), but not as marked for the model (3) and no significant decrease for more refined models.

In Table 2 are shown the results of the maximum shear stress that reach the contact zone, verifying that τ_{MAX} is 3.6 per cent inferior to Hertz theory, and it is reached under the surface at a distance of z ≅ 0.60a (Fig. 5).

Table 2 Maximum shear stresses

	a (inches)	z (inches)	τ_{MAX} (psi)	z/a	τ_{MAX} /Po
Model 3	0.0136	0.00814	67428.0	0.60	0.29
Teorethical	0.0116	0.00908	69922.6	0.78	0.30
%Variation	16.6%	10.3%	3.6%	23.1%	3.6%

Figure 5 shows the detail of the shear stress contours with standard limits respect to contact half width "a". The results are verified graphically, were the maximum shear stress occurs within the surface at a distance of z ≅ 0.60a and it is displaced a distance x ≅ -0.49a respect to the symmetry axis.

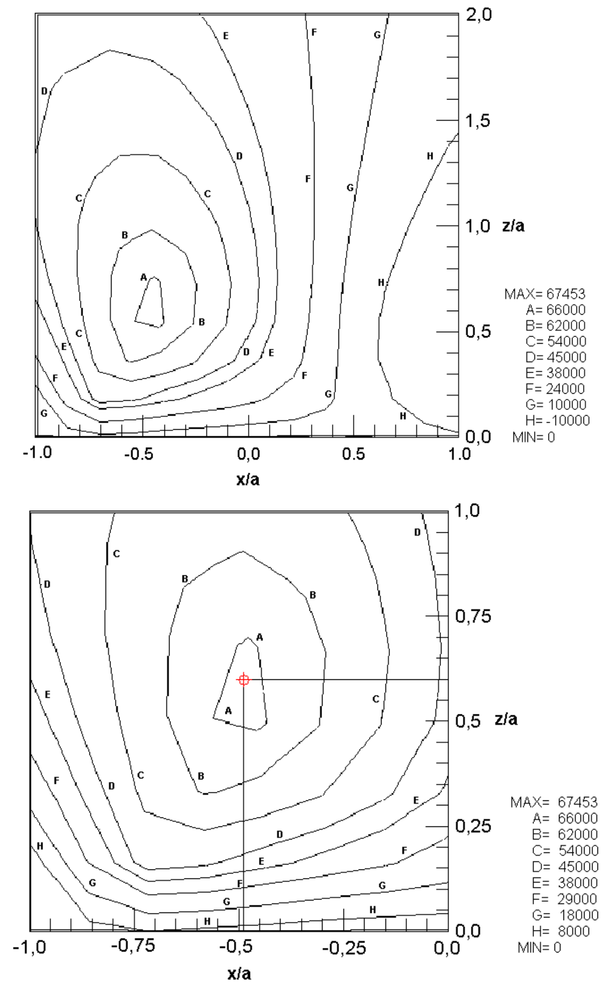

Fig. 5 Maximum Shear Stress Contour τ_{MAX}

5.2 Discussion.

The contact stresses obtained by FEM and the theory of Hertz and AGMA are very similar, showing a tendency to increase exponentially towards the end of the involute profile (Fig. 3) and with a minimum value at the pitch point. It is to be noticed that, in the range in which a single pair of teeth assume the full burden efforts are truly critical. Stresses according to AGMA are lower than those obtained from the Hertz theory, but if different correction factors are used, stresses are higher and more attached to the real working conditions of gears. Despite the FEM developed (with the simplifications that were made) shows the equivalence of trends and results. In this vein, they found differences of about 5.0% compared to the theory of Hertz and 2.6% from the values of AGMA.

The pressure distribution on the surface (Fig. 4) resembles the theoretical semi-elliptical shape, with some asymmetry characteristic of the geometry of involute profile, taking values of zero at the edges and maximum $x \cong 1.08a$. The value of "a" is 16.6% higher than the theory of Hertz for two long cylinders in contact, with a tendency to maintain this ratio. Can be inferred that lower stresses obtained from the FEM are due to increased contact length in the model, since these are inversely proportional to that length.

The principal stresses σ_2 y σ_3 are maximum for (z=0) and decrease very fast with the deepness inside the material and away from the central line of symmetry. The maximum shear stress is reached below the surface for $z \cong 0.6a$, which differs significantly from the Hertz theory which states that is reached for $z \cong 0.78a$. The results obtained suggest that the geometrical factors of the involute profile (variable radius of curvature) and the slippery conditions resulting from operations of the gears, represent an important considerations in the determination of the occurrence of such shear forces. However, the value of maximum shear stress is approximately $\tau_{MAX} \cong 0.3Po$ consistent with the Hertz theory, allowing the same criteria of yield point for contact.

6 CONCLUSIONS

The ability of the FEM for the simulation of mechanical spur gears contact have been proved, presenting estimates of contact pressure and stress states with similar results and tendencies to those obtained by the contact theory of plane models of Hertz and AGMA. The contact pressure and stress state are highest for higher points on the involute and lower were a single pair of teeth assumes the full load transmitted, and minimal for the contact at the pitch point. The maximum shear occurs below the contact surface to a depth of approximately $z \approx 0.6a$ with a value of about $\tau_{XY} \approx 0.3Po$.

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