Mathematical Model and Simulation for a Helicopter with Tail Rotor

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Abstract: - The helicopters have different aerodynamic characteristics according to their type; therefore different mathematical models can be developed to represent their flying dynamics, which is very complex. In this paper we develop a mathematical model for a helicopter with a principal rotor hub and a tail rotor configuration, which is used to simulate the flying behavior of the helicopter and can be used to design a flight control system. The mathematical model is a nonlinear six-equation system, in which for principal rotor motion and tail rotor motion a decoupled system is established in order to obtain a complete helicopter-flying attitude. Some simulations are made using the mathematical model under a specific flying condition.

Key-words: Helicopter; Flying Dynamic; Mathematical Model, Simulation.

1 Introduction

The most important theories that describe the helicopters flying behavior are basically the momentum theory and blade-isolated theory. Besides, the attitude of a helicopter can be represented by Euler's angles [8]. Using these theories together with the Newton-Euler equations we develop an interesting mathematical model. Certainly helicopters behavior is very complex to be determined, and so the perturbations involve in fly, such as vibrations present in the rotor mechanism, the possible windblast and many others. In this paper we study the flying behavior of a helicopter, and leave the perturbations analysis for future work, although the perturbation must be consider when designing a helicopter flight control system. The mathematical model is developed for a conventional helicopter, which has one principal disk rotor and one tail rotor disk.

2 Mathematical helicopter model concept

We consider the helicopter as a rigid body. A body coordinate system is defined with the origin at the center of gravity of the helicopter. The axis x_B is pointing to the nose of the helicopter and is positive in the forward direction, the axis y_B is positive to the right (this is the usual convention for the analysis of stability and control) [14], the axis z_B coincide with the orientation of the principal rotor axis and is positive downwards.

Helicopter momentum theory in hover, climb and advance flying condition [4, 14], and the theory of isolated blade element [5, 11], has been used to develop the mathematical equations. The movement theory of a rigid body [8] was also used to understand the angular movement quantity components, associated to the inertias and angular velocity.

In fig. 1, it is shown the degrees of freedom of the helicopter. The displacement of the center of gravity, has the components x_B, y_B and z_B; the attitude of the helicopter, respect to the center of gravity, are given by the Euler´s angles roll ϕ_B, pitch θ_B and yaw ψ_B. The main rotor head is to a distance h from the center of gravity, and the tail rotor is to a distance l_{tr} backwards. The linear and angular displacements of the main rotor head are expressed in fig. 1.

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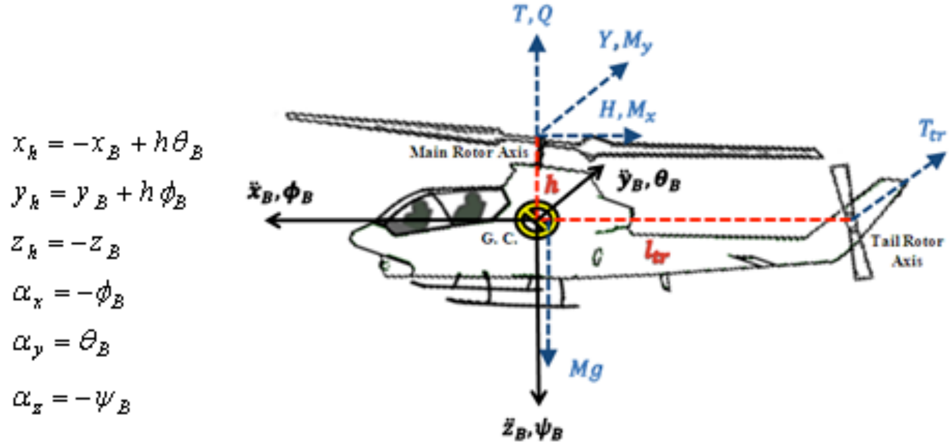

Fig. 1. Definition of the degrees of freedom of the helicopter, and the forces and moments, due to the main rotor, the tail rotor and the gravity.

The subscript h, corresponds to a horizontal virtual axis fixed respect to the center of gravity, while subscript B corresponds to body coordinate of the helicopter. Notice that the body and horizontal axis are equal in hover fly.

By small angles approximation the angular velocity of the helicopter has components $p = \dot{\phi}_B$, $q = \dot{\theta}_B$ and $r = \dot{\psi}_B$. The components of the disturbance linear speed are $u = \dot{x}_B$, $v = \dot{y}_B$ and $w = \dot{z}_B$.

The helicopter has mass M and moments of inertia I_x, I_y, I_z and I_{xz}, on the roll, pitch, and yaw axis respectively. The mass of the rotor is including in the inertia of the helicopter. The fig. 1 also shows the forces and moments acting upon the helicopter, the reactions in the main rotor head, the thrust of tail rotor, and the force of gravity. The effects of the forces and the moments originated by the fuselage, the horizontal and vertical stabilizer are neglected.

The differential equations for the degrees of freedom of a rigid body are defined according to the developed equations of forces and moments:

$$
\begin{aligned}
&M\ddot{x}_B = -H - Mg\theta_B \\
&M\ddot{y}_B = Y + T_{tr} + Mg\phi_B \\
&M\ddot{z}_B = -T + Mg \\
&I_x\ddot{\phi}_B - I_{xz}\ddot{\psi}_B - \left(I_y - I_z\right)\dot{\theta}_B\dot{\psi}_B - I_{zx}\dot{\phi}_B\dot{\theta}_B = -M_x + hY \\
&I_y\ddot{\theta}_B - \left(I_z - I_x\right)\dot{\phi}_B\dot{\psi}_B + I_{zx}\dot{\phi}_B{}^2 - I_{xz}\dot{\psi}_B{}^2 = M_y + hH \\
&I_z\ddot{\psi}_B - I_{zx}\ddot{\phi}_B - \left(I_x - I_y\right)\dot{\phi}_B\dot{\theta}_B + I_{xz}\dot{\psi}_B\dot{\theta}_B = Q - l_{tr}T_{tr}
\end{aligned} \tag{1}
$$

Where:

H	Longitudinal force.
Y	Lateral force.
T	Traction force for the main rotor.
T_{tr}	Traction force for the tail rotor.
Q	Yaw moment or motor pair moment.
M_x	Roll moment.
M_y	Pitch moment.
l_{tr}	Distance between helicopter´s center gravity and tail rotor principal rotation axis.
g	Gravity force.
$\ddot{x}_B$	Helicopter longitudinal acceleration over it body axis.
$\ddot{y}_B$	Helicopter lateral acceleration over it body axis.
$\ddot{z}_B$	Helicopter vertical acceleration over it body axis.
$\ddot{\phi}_B, \dot{\phi}_B$	Angular acceleration and velocity for the roll angle.
$\ddot{\theta}_B, \dot{\theta}_B$	Angular acceleration and velocity for the pitch angle.
$\ddot{\psi}_B, \dot{\psi}_B$	Angular acceleration and velocity for the yaw angle.

3 Forces and moments

The main rotor have two mechanical coupled controls, the collective pitch angle θ_o, and the cyclical pitch control which has a longitudinal and lateral control A_1 and B_1. The main thrust force (2), is control by θ_o and B_1, the corresponding longitudinal (4) and lateral (5) forces H and Y, are controlled in combination by collective and cyclical controls θ_o, A_1 and B_1. Also helicopters have a power control, which can reduce or increase the rotor angular velocity Ω.

$$T = \frac{1}{2}b\rho ac\Omega^2R^3\left[\frac{1}{3}\theta_o + \frac{1}{2}\mu^2\theta_o - \frac{1}{2}\mu B_1 + \frac{1}{2}\lambda\right] \tag{2}$$

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The tail rotor, possess only collective pitch angle, for controlling the thrust T_{tr}, which generates a force in lateral direction against the rolling moment. The collective pitch control is a mechanism that changes the collective pitch angle of tail rotor θ_{otr}, which is directly proportional to the tail rotor thrust (3).

$$T_{tr} = \frac{1}{2} b_{tr} \rho a_{tr} c_{tr} \Omega_{tr}{}^2 R_{tr}{}^3 \left[\frac{1}{3}\theta_{otr} + \frac{1}{2}\mu_{tr}{}^2 \theta_{otr} + \frac{1}{2}\lambda_{tr} \right] \quad (3)$$

In the tail rotor thrust (3), the term corresponding to the lateral collective control B_1 does not appear as appear in (2), because the tail rotor does not have cyclical pitch controls.

The longitudinal force H and Lateral force Y are responsible for the correspondent longitudinal and lateral displacement, are determined by equations (4) and (5).

$$H = \frac{1}{2}\rho abc\Omega^2 R^3 \left[\frac{1}{2a}\delta\mu - \frac{1}{6}a_o b_1 + \frac{3}{4}\lambda a_1 + \frac{1}{3}a_1\theta_o - \frac{1}{2}\lambda\mu\theta_o + \frac{1}{4}\mu a_o{}^2 + \frac{1}{4}\mu a_1{}^2 + \frac{1}{6}a_o A_1 - \frac{1}{4}\mu a_1 B_1 + \frac{1}{4}\lambda B_1 \right] \quad (4)$$

$$Y = \frac{1}{2}\rho abc\Omega^2 R^3 \left(\frac{1}{6}a_o a_1 - \mu^2 a_o a_1 + \frac{3}{4}\lambda b_1 - \frac{3}{2}\lambda\mu a_o + \frac{1}{4}\mu a_1 b_1 - \frac{1}{4}\lambda A_1 - \frac{1}{4}\mu a_1 A_1 + \frac{1}{6}a_o B_1 - \frac{1}{2}\mu b_1 B_1 + \frac{1}{2}\mu^2 a_o B_1 + \frac{1}{3}b_1\theta_o - \frac{3}{4}\mu a_o\theta_o + \frac{1}{2}\mu^2 b_1\theta_o \right) \quad (5)$$

The expression for rotor torque Q, which is the moment generated by the tangential force on the blade element upon the rotation axis is defined as:

$$Q = \frac{1}{2}\rho abc\Omega^2 R^4 \left[\frac{\delta}{4a}\left(1+\mu^2\right) - \frac{1}{2}\lambda^2 - \frac{1}{2}\lambda\mu a_1 - \frac{1}{3}\lambda\theta_o{}^2 - \frac{1}{4}\mu^2 a_o - \frac{1}{8}a_1{}^2 - \frac{3}{16}\mu^2 a_1{}^2 - \frac{1}{16}\mu^2 b_1{}^2 - \frac{1}{8}b_1{}^2 + \frac{1}{3}\mu a_o b_1 + \frac{1}{4}\lambda\mu B_1 - \frac{1}{8}a_1 B_1 + \frac{1}{16}\mu^2 a_1 B_1 - \frac{1}{6}\mu a_o A_1 + \frac{1}{8}b_1 A_1 + \frac{1}{16}\mu^2 b_1 A_1 \right] \quad (6)$$

Where:

c	Blade chord.
ρ	Air density.
R , R_{tr}	Main rotor and tail rotor disk ratio or blade length.
a_o	Flapping blade coning angle.
a_1 , b_1	Longitudinal and lateral flapping coefficient.
b	Number of Blades.
θ_o	Collective pitch angle.
A_1	Lateral pitch cyclic control.
B_1	Longitudinal roll cyclic control.
a	Blade section two-dimensional lift-curve slope.
δ	Polar quadratic resistance.
μ	Climb inflow ratio.
λ	Axial inflow parameter.
Ω , Ω_{tr}	Main rotor and tail rotor angular velocity.

The roll moment M_X respect to the longitudinal axis x has been determined by:

$$M_X = \frac{1}{2}\rho abc\Omega^2 R^4 \left[\frac{1}{3}\mu\theta_o - \frac{1}{8}B_1 + \frac{1}{4}\lambda\mu - \frac{1}{8}a_1 - \frac{3}{16}\mu^2 B_1 \right] \quad (7)$$

The pitch moment M_Y respect to the lateral axis y has been determined by:

$$M_Y = \frac{1}{2}\rho abc\Omega^2 R^4 \left[\frac{1}{8}A_1 + \frac{1}{16}\mu^2 A_1 + \frac{1}{6}\mu a_o - \frac{1}{8}b_1 - \frac{1}{16}\mu^2 b_1 \right] \quad (8)$$

The equations (2), (3), (4), (5), (6), (7) and (8) determine the forces and moments of a generic helicopter with tail rotor, for any flight conditions.

4 Mathematical Model

Replacing the equations (2), (3), (4), (5), (6), (7) and (8) into equation (1), and solve it for the acceleration components and angular accelerations components, we obtain the system equation (9), which easily can be used to design a flight control system for a helicopter, or autonomous radio controlled helicopter. This system is a nonlinear mathematical model for a generic helicopter, which tail rotor configuration in flying attitude.

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$$\ddot{x}_B = -\frac{\rho abc\Omega^2 R^3}{2M}\left[\frac{1}{2a}\delta\mu - \frac{1}{6}a_o b_1 + \frac{3}{4}\lambda a_1 + \frac{1}{3}a_1\theta_o - \frac{1}{2}\lambda\mu\theta_o + \frac{1}{4}\mu a_o{}^2 + \frac{1}{4}\mu a_1{}^2 + \frac{1}{6}a_o A_1 - \frac{1}{4}\mu a_1 B_1 + \frac{1}{4}\lambda B_1\right] - g\theta_B$$

$$\ddot{y}_B = \frac{\rho abc\Omega^2 R^3}{2M}\left[\frac{1}{6}a_o a_1 - \mu^2 a_o a_1 + \frac{3}{4}\lambda b_1 - \frac{3}{2}\lambda\mu a_o + \frac{1}{4}\mu a_1 b_1 - \frac{1}{4}\lambda A_1 - \frac{1}{4}\mu a_1 A_1 + \frac{1}{6}a_o B_1 - \frac{1}{2}\mu b_1 B_1 + \frac{1}{2}\mu^2 a_o B_1\right.$$
$$\left. + \frac{1}{3}b_1\theta_o - \frac{3}{4}\mu a_o\theta_o + \frac{1}{2}\mu^2 b_1\theta_o\right] + \frac{\rho a_{tr} b_{tr} c_{tr}\Omega_{tr}{}^2 R_{tr}{}^3}{2M}\left[\frac{1}{3}\theta_{o\,tr} + \frac{1}{2}\mu_{tr}{}^2\theta_{o\,tr} + \frac{1}{2}\lambda_{tr}\right] + g\phi_B$$

$$\ddot{z}_B = -\frac{b\rho ac\Omega^2 R^3}{2M}\left[\frac{1}{3}\theta_o + \frac{1}{2}\mu^2\theta_o - \frac{1}{2}\mu B_1 + \frac{1}{2}\lambda\right] + g$$

$$\ddot{\phi}_B = \frac{I_{xz}}{I_x}\ddot{\psi}_B + \frac{(I_y - I_z)}{I_x}\dot{\theta}_B\dot{\psi}_B + \frac{I_{zx}}{I_x}\dot{\phi}_B\dot{\theta}_B - \frac{\rho abc\Omega^2 R^4}{2I_x}\left[\frac{1}{3}\mu\theta_o - \frac{1}{8}B_1 + \frac{1}{4}\lambda\mu - \frac{1}{8}a_1 - \frac{3}{16}\mu^2 B_1\right] + h\frac{\rho abc\Omega^2 R^3}{2I_x}\left[\right.$$
$$\frac{1}{6}a_o a_1 - \mu^2 a_o a_1 + \frac{3}{4}\lambda b_1 - \frac{3}{2}\lambda\mu a_o + \frac{1}{4}\mu a_1 b_1 - \frac{1}{4}\lambda A_1 - \frac{1}{4}\mu a_1 A_1 + \frac{1}{6}a_o B_1 - \frac{1}{2}\mu b_1 B_1 + \frac{1}{2}\mu^2 a_o B_1 + \frac{1}{3}b_1\theta_o$$
$$\left. - \frac{3}{4}\mu a_o\theta_o + \frac{1}{2}\mu^2 b_1\theta_o\right]$$

$$\ddot{\theta}_B = \frac{(I_z - I_x)}{I_y}\dot{\phi}_B\dot{\psi}_B - \frac{I_{zx}}{I_y}\dot{\phi}_B{}^2 + \frac{I_{xz}}{I_y}\dot{\psi}_B{}^2 + \frac{\rho abc\Omega^2 R^4}{2I_y}\left[\frac{1}{8}A_1 + \frac{1}{16}\mu^2 A_1 + \frac{1}{6}\mu a_o - \frac{1}{8}b_1 - \frac{1}{16}\mu^2 b_1\right] + h\frac{\rho abc\Omega^2 R^3}{2I_y}\left[\right.$$
$$\left.\frac{1}{2a}\delta\mu - \frac{1}{6}a_o b_1 + \frac{3}{4}\lambda a_1 + \frac{1}{3}a_1\theta_o - \frac{1}{2}\lambda\mu\theta_o + \frac{1}{4}\mu a_o{}^2 + \frac{1}{4}\mu a_1{}^2 + \frac{1}{6}a_o A_1 - \frac{1}{4}\mu a_1 B_1 + \frac{1}{4}\lambda B_1\right]$$

$$\ddot{\psi}_B = \frac{I_{zx}}{I_z}\ddot{\phi}_B + \frac{(I_x - I_y)}{I_z}\dot{\phi}_B\dot{\theta}_B - \frac{I_{xz}}{I_z}\dot{\psi}_B\dot{\theta}_B + \frac{\rho abc\Omega^2 R^4}{2I_z}\left[\frac{\delta}{4a}(1+\mu^2) - \frac{1}{2}\lambda^2 - \frac{1}{2}\lambda\mu a_1 - \frac{1}{3}\lambda\theta_o - \frac{1}{4}\mu^2 a_o{}^2 - \frac{1}{8}a_1{}^2\right.$$
$$\left. - \frac{3}{16}\mu^2 a_1{}^2 - \frac{1}{16}\mu^2 b_1{}^2 - \frac{1}{8}b_1{}^2 + \frac{1}{3}\mu a_o b_1 + \frac{1}{4}\lambda\mu B_1 - \frac{1}{8}a_1 B_1 + \frac{1}{16}\mu^2 a_1 B_1 - \frac{1}{6}\mu a_o A_1 + \frac{1}{8}b_1 A_1 + \frac{1}{16}\mu^2 b_1 A_1\right]$$
$$- l_{tr}\frac{\rho a_{tr} b_{tr} c_{tr}\Omega_{tr}{}^2 R_{tr}{}^3}{2I_z}\left[\frac{1}{3}\theta_{o\,tr} + \frac{1}{2}\mu_{tr}{}^2\theta_{o\,tr} + \frac{1}{2}\lambda_{tr}\right]$$

(9

5 Model parameters

The Flapping parameters a_o, a_1 and b_1 are calculated by:

$$a_o = \frac{1}{2}\gamma\left(\frac{1}{4}\theta_o + \frac{1}{3}\lambda + \frac{1}{4}\mu^2\theta_o - \frac{1}{3}\mu B_1\right) - \frac{M_W}{I_b\Omega^2} \quad (10)$$

$$a_1 = \frac{\mu\left(\frac{8}{3}\theta_o + 2\lambda\right) - \left(1 + \frac{3}{2}\mu^2\right)B_1}{\left(1 - \frac{1}{2}\mu^2\right)} \quad (11)$$

$$b_1 = \frac{4\mu a_o + 3\left(1 + \frac{1}{2}\mu^2\right)A_1}{3\left(1 + \frac{1}{2}\mu^2\right)} \quad (12)$$

The Blade Lock Number γ represents the ratio of the aerodynamic and inertial forces on the blade [14]:

$$\gamma = \frac{\rho ac R^4}{I_b} \quad (13)$$

The blade inertia moment I_b (assuming that the blade mass is constant), is calculates using equation (14), and the moment M_W due to the blade weight is shown in equation (15).

$$I_b = \int_0^R m r^2 dr = \frac{mR^3}{3} \quad (14)$$

$$M_W = \int_0^R mgr dr = mg\frac{R^2}{2} \quad (15)$$

The simplifications than can be made to the model are the easy way to modeling and control; lets assume that we have one helicopter with a rigid rotor, in a flying condition, then these assumptions will produce that some of the variables involve in the system model, become zero. If we handle a specific condition of flight then we can simplify the mathematical model, that will allow us to appreciate the helicopter behavior and make the simulation process an easy and manageable one. For example if we consider an ascending flight, then $\dot{x}_B$, and $\dot{y}_B$ will be equal zero, as well as its respective derivatives of acceleration, this permit to reduce to the number of equations of the model and the consequent complexity of it, this is an useful tool for the helicopter simulation and control.

The Climb inflow ratio μ and Axial inflow parameter λ (These also apply for the tail rotor but with their correspondent velocity components, angular velocity and blade radius), are defined in a way that can be used according to the relative body velocities $\dot{x}_B$, $\dot{y}_B$ and $\dot{z}_B$ so that:

$$\mu = \frac{\sqrt{\dot{x}_B^{\,2} + \dot{y}_B^{\,2}}}{\Omega R} \quad (16), \qquad \mu_{tr} = \frac{\sqrt{\dot{x}_B^{\,2} + \dot{z}_B^{\,2}}}{\Omega_{tr} R_{tr}} \quad (17)$$

$$\lambda = \frac{\dot{z}_B - v_i}{\Omega R} \quad (18), \qquad \lambda_{tr} = \frac{\dot{y}_B - v_{itr}}{\Omega_{tr} R_{tr}} \quad (19)$$

Where v_i and v_{itr} are the induced velocity of main rotor and tail rotor.

The induced velocities change according to the flight condition, then to have a correct use of the mathematical model we must change the induced velocities every time the helicopter changes its flying condition. Basically the flying condition are, hover flight, climb flight (or descend), longitudinal flight (forward or backward), lateral flight (right or left advance), also it can be a oblique flight that is a combination of longitudinal, lateral and climb flight, but establish a correct induced velocity for this specific condition is not on study in this paper.

For climb flight according to the blade isolated element theory [5, 11], the induced velocity can be found using the equation (20) and (21):

$$v_i = \left(\frac{\dot{z}_B}{2} + \frac{bac\Omega}{16\pi}\right)\left(-1 + \sqrt{1 + \frac{2\Omega R\left(\theta_o - \dfrac{\dot{z}_B}{\Omega R}\right)}{\dfrac{4\pi \dot{z}_B^{\,2}}{bac\Omega} + \dot{z}_B + \dfrac{bac\Omega}{16\pi}}}\right) \tag{20}$$

$$v_{itr} = \left(\frac{\dot{y}_B}{2} + \frac{b_{tr} a_{tr} c_{tr} \Omega_{tr}}{16\pi}\right)\left(-1 + \sqrt{1 + \frac{2\Omega_{tr} R_{tr}\left(\theta_{otr} - \dfrac{\dot{y}_B}{\Omega_{tr} R_{tr}}\right)}{\dfrac{4\pi \dot{y}_B^{\,2}}{b_{tr} a_{tr} c_{tr} \Omega_{tr}} + \dot{y}_B + \dfrac{b_{tr} a_{tr} c_{tr} \Omega_{tr}}{16\pi}}}\right) \tag{21}$$

In hovering flight the induced velocity can be obtain according to the quantity movement theory [4, 11, 13], in which $v_i = v_{io}$ (22), where v_{io} is the hover induced velocity, which can be consider as a constant; this is possible because in hover flight the traction force T became equal to the disc loading (weight of the helicopter).

$$v_{io} = \sqrt{\frac{T}{2\rho\pi R^2}} \tag{22}$$

In hovering $\dot{x}_B$, $\dot{y}_B$, and $\dot{z}_B$ are zero this causes that μ, μ_{tr}, θ_B, ϕ_B, A_1, B_1, a_1 and b_1 also became zero, allowing only the yaw movement if the collective tail control θ_{otr} is actuated.

For any condition of flight the air density changes with the altitude *H,* for small flying altitudes, changes in the air density can be consider as constant:

$$\rho = 1.225(1 - 0.0002256H)^{4.2561} \quad (Kg/m^3) \tag{23}$$

For a helicopter in a longitudinal pure flight $\dot{y}_B = \dot{z}_B = \phi_B = \psi_B = B_1 = b_1 = 0$, the induced velocity can be simplified as:

$$v_i = v_{io}\sqrt{\frac{1}{2}\left\{\sqrt{4 + \left(\frac{\dot{x}_B}{v_{io}}\right)^4} - \left(\frac{\dot{x}_B}{v_{io}}\right)^2\right\}} \tag{24}$$

For a lateral flight $\dot{x}_B = \dot{z}_B = \theta_B = \psi_B = A_1 = a_1 = 0$, the induced velocity (24) can be used with it correspondent velocity component $\dot{y}_B$ as:

$$v_i = v_{io}\sqrt{\frac{1}{2}\left\{\sqrt{4 + \left(\frac{\dot{y}_B}{v_{io}}\right)^4} - \left(\frac{\dot{y}_B}{v_{io}}\right)^2\right\}} \tag{25}$$

6 Simulations

For a rotation in Hover Flight, we increase or decrease the collective pitch tail rotor angle θ_{otr} . If a control is established then the yaw angle has to become zero when reaches the desirable rotation yaw angle over the time, otherwise will continue rotate as seen in fig 2.

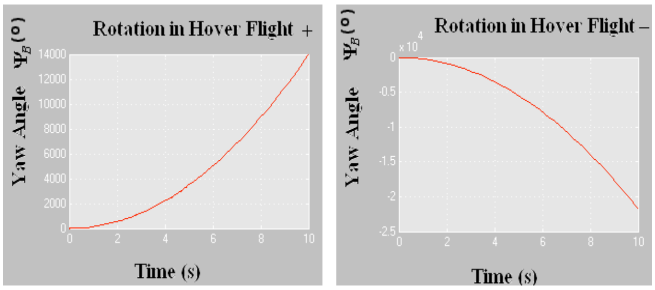

Fig. 2. Rotation under a Hover flight.

For a Hover Flight no rotation can be appearing is θ_{otr} doesn't be activated as shown in fig. 3.

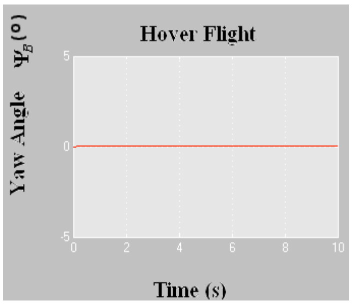

Fig. 3. Pure Hover Flight.

For a longitudinal flight the lateral pitch cyclic control A_1 , is actuated to control the advance, the values of θ_{otr} and θ_o depends on the flying altitude and the desire advance velocity $\dot{x}_B$. As shown if fig. 4, the advance velocity reaches it maximum value over the time, this make the pitch angle became constant that is physically correct.

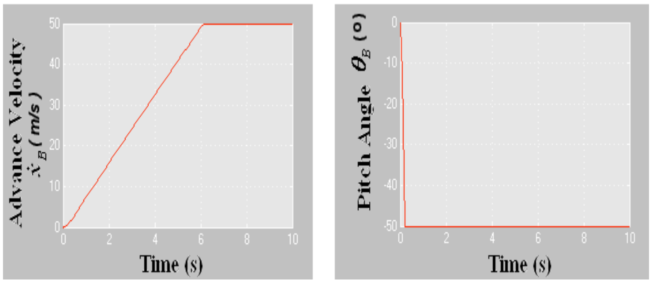

Fig. 4. Longitudinal Flight.

Similar case for a lateral flight but the control actuated is the longitudinal roll cyclic control B_1 and the desire lateral velocity $\dot{y}_B$.

Conclusions

The mathematical model is suitable for simulation and control for a helicopter with tail rotor configuration, many advantages can be obtain if the model is use for the control like an autonomous radio controlled helicopter or for automatic helicopter pilot, also for flying simulation study is suitable for built a flying computer training program.

Many other interesting simulations like for a lateral and ascending flight can be found using this mathematical model.

Perturbations like wind crossing or vibration engine need to be considering if there are interest on built more robust model that can be for simulation or flying control purpose.

The induced velocity change according to the flying condition, there is not a unique formula that can determine this variable for all flying conditions. Each induced velocities must be used according to the flying condition of the helicopter upon the mathematical model.

Assume a specific flight condition can reduce the mathematical model equations, this is because many of the variables involved became zero; this is a practical tool that can be used for simulation and control.

References

[1] Abbott. H. Ira. y Von Doenhoff, Albert E. *Theory Of Wing Sections*. Dover Publications, Inc. New York. 1958.

[2] A.R.S. Bramwell, George Done. *Bramwell's Helicopter Dynamics*. Butterworth-Heinemann Publications. 2001.

[3] Ferdinand P. Beer, y E. Russell Johnston Jr. *Mecánica Vectorial para Ingenieros. Dinámica.* Mc. Graw Hill. 3ra edición. México D.F. 1999.

[4] Gessow Alfred, y Garry C. Myers. *Aerodynamics of the Helicopter.* College Park Press 8th printing. Washington D.C. 1985.

[5] Hennequet A. Gonzalo. *Helicópteros.* UNEFA, Departamento de Ingeniería Aeronáutica. Maracay. 1981.

[6] Hernández S. Roberto, Fernández C. Carlos y Baptista L. Pilar. *Metodología de la Investigación.* Mac. Graw Hill. México D.F.1991.

[7] Ira H. Abbott, Albert E. Von Doenhoff. *Theory of Wing Sections*. Dover Publications, INC. New York, 1959.

[8] J.L. Meriam, y L.G. Kraige. *Mechanic For Engineers Dynamic.* Editorial Reverté, S.A. 3th edition. Barcelona-Spain. 2000.

[9] John H. Matheus., Kurtis D. Fink. *Numeric Methods with Matlab.* Prentice Hall. México D.F. 1999.

[10] Johnson W. *Helicopter Theory.* Princeton University Press. USA. 1980.

[11] López Ruiz J. L. *Helicópteros. Teoría y Diseño conceptual.* ETSI Aeronáuticos de la Universidad Politécnica de Madrid. Gráficas JUMA. Madrid. 1993.

[12] López Ruiz J. L. *Apuntes de Helicópteros.* ETSI Aeronáuticos de la Universidad Politécnica de Madrid. Gráficas JUMA. Madrid. 1980.

[13] McCormick, Barnes W. *Aerodynamics Aeronautics and Flight Mechanics.* John Wiley & Sons Publications. New York. 1979.

[14] Wayne J. *Helicopter Theory*. Dover Publications, INC. New York. 1994.

[15] W.Z. Stepniewski, C.N. Keys. *Rotary Wing Aerodynamics*. Dover Publications, INC. New York. 1981.