Probability density function of M-ary FSK signal in the presence of Gaussian noise, intersymbol interference and *G* fading

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Abstract**—**A performance of the receiver for the demodulation of M-FSK signals in the presence of Gaussian noise, intersymbol interference and composite fading that consists of a multipath Nakagami-*m* fading and a shadowing is considered in this paper. The composite fading is modeled by the G distribution. The probability density function and the outage probability of M-ary FSK signal in the presence of noise, interference and fading is derived. Various numerical results are graphically presented for dual branch FSK receivers to illustrate the obtained mathematical analysis and to show the influence of system parameters to the system performance.

Keywords**—** intersymbol interference, Gaussian noise, G fading, M-ary frequency shift keying, probability density function, outage probability.

I. Introduction

THE performances of wireless communications systems can be seriously degraded by the influence of the Gaussian noise, intersymbol interference, multipath fading and shadowing [1]-[4]. In order to view the influence of all these effects on the performances of an M-ary FSK system, the probability density function (PDF) and outage probability of an M-ary FSK receiver output signal will be derived.

The performance evaluation of several types of FSK and CPFSK receivers was investigated in detail in the paper [5]. Also, the error probability of the cross-correlator receiver for binary digital frequency modulation detection is studied using theoretical analysis and computer simulations [6]. In [7] average bit-error probability performance for optimum diversity combining of noncoherent FSK over Rayleigh channels is determined.

Fading, which appears as the result of spreading multichannel signals in the wireless telecommunication systems, is the main cause of performance degradation of digital wireless communication systems. When a received signal experiences fading during transmission, its envelope and phase both fluctuate over time. The overall fading process for land mobile satelite systems is a complex combination of multipath fading and a log-normal shadowing. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal is considered in [1], [8-9].

The use of lognormal distribution to model the average power which is random variable due to shadowing doesn't lead to a closed form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) at the receiver. A compound K and Generalized-K fading models use a gamma distribution to account for shadowing instead of the lognormal distribution [10-12]. The Generalized-K distribution has two shaping parameters, and as a consequence includes the K distribution as a special case.

The Inverse-Gaussian pdf was proposed recently as a substitute to the log-normal [13]. Using Inverse-Gausssian distribution in compound model, it was introduced the new distribution for multipath/shadowed fading, called the G distribution [14]. Detailed performance analysis for the SNR statistics of a receiver with different type of combining techniques operating over Generalized-K fading channel is published in [15]. It was made a performance evaluation of single-user communication systems operating in a composite channel. Analytical expressions for the outage probability and the moment generating function of the G-distribution, were derived, facilitating the calculation of average bit error probabilities. The correlated bivariate G distribution with arbitrary and not necessarily identical parameters is analyzed in [16].

In order to view the influence of the Gaussian noise, intersymbol interference and G fading to the M-ary FSK system, we are derived the probability density function and the outage probability of the M-ary FSK receiver output signal. The bit error probability and the signal error probability can be determined by the probability density function of the output signal. Also, the moment generating function, the moment and variance of the output signals can be derived by the probability density function of output signals.

The study starts by defining the model of the M-ary FSK system. The expressions for the probability density function of

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the output signal at one time instant are obtained in the next section. Fourth section consists of the numerical results in the case $M=2$. The last section is the conclusion. Numerical results are graphically presented to show the effects of the influence of the Gaussian noise, intersymbol interference, multipath fading and shadowing on the probability density function and the outage probability.

II. System Model

The model of an M-ary FSK system, which is considered in this paper, is shown in Fig. 1. This system has M branches. Each branch consists of the bandpass filter and correlator. The correlator is consisting of multiplier and lowpass filter.

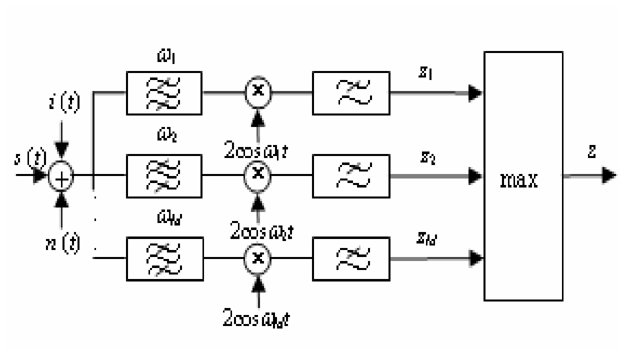

Fig. 1 Block diagram of the system for coherent demodulation of M-ary FSK signal

Transmitted signal for the hypothesis H_i is:

$$s(t) = A\cos\omega_i t \tag{1}$$

where A denotes the amplitude of the modulated signal and has G distribution [15]:

$$p_A(A) = \left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}} \sqrt{\frac{\lambda}{2\pi}} \frac{4m^m \exp\left(\frac{\lambda}{\theta}\right) A^{2m-1}}{\Gamma(m)\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)^{m+\frac{1}{2}}} \cdot K_{m+\frac{1}{2}}\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right), \quad A \geq 0 \tag{2}$$

In the above equation m is the Nakagami-m shaping parameter, λ *and* θ are shadowing shaping parameters given in Inverse-Gaussian (IG) distribution [15], $K_{m+1/2}(\bullet)$ is the $(m+1/2)$th order modified Bessel function of the second kind, $\Gamma(\bullet)$ is the gamma function.

Gaussian noise at the input of the receiver is given by:

$$n(t) = \sum_{i=1}^{M} x_i \cos\omega_i t + y_i \sin\omega_i t, \quad i=1, 2, \ldots M \tag{3}$$

x_i and y_i represent the components of Gaussian noise, with zero means and variances σ^2.

The interference $i(t)$ can be obtained as:

$$i(t) = \sum_{i=1}^{M} A_i \cos(\omega_i t + \theta_i) \tag{4}$$

where phases θ_i, have uniform probability density function.

These signals pass first through bandpass filters whose central frequencies $\omega_1, \omega_2, \ldots, \omega_M$ correspond to hypotheses $H_1, H_2, \ldots, H_M$.

If $z_1, z_2, \ldots z_M$ are the output signals of the receiver branches, then the M-FSK receiver output signal is:

$$z = \max\{z_1, z_2 \ldots z_M\} \tag{5}$$

III. Probability Density Function

In the case of the hypothesis H_1, transmitted signal is:

$$s(t) = A\cos\omega_1 t \tag{6}$$

while the output branch signals of the receiver are:

$$z_1 = A + x_1 + A_1\cos\theta_1 \tag{7}$$

$$z_k = x_k + A_k\cos\theta_k, \quad k=2,3,\ldots,M \tag{8}$$

Define the probability density functions on the output of branches and the cumulative density of these signals is necessary to obtain output probability density function of M-ary FSK receiver.

The conditional probability density functions for the signals $z_1, z_2, \ldots, z_M$ are:

$$p_{z_1/A,\theta_1}(z_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1\cos\theta_1)^2}{2\sigma^2}} \tag{9}$$

$$p_{z_k/A,\theta_k}(z_k) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k\cos\theta_k)^2}{2\sigma^2}}, \quad k=2,3,\ldots,M \tag{10}$$

By averaging (9) and (10) we obtain the probability density functions of the branches output signals:

$$p_{z_1}(z_1) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_1 - A - A_1\cos\theta_1)^2}{2\sigma^2}} \cdot \left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}} \sqrt{\frac{\lambda}{2\pi}} \frac{4m^m \exp\left(\frac{\lambda}{\theta}\right) A^{2m-1}}{\Gamma(m)\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)^{m+\frac{1}{2}}} \cdot K_{m+\frac{1}{2}}\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right) dA \frac{1}{2\pi} d\theta_1 \tag{11}$$

$$p_{z_k}(z_k) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_k - A_k\cos\theta_k)^2}{2\sigma^2}} \frac{4\left(\frac{m}{\Omega}\right)^{(k+m)/2}}{\Gamma(m)\Gamma(k)} \cdot$$

$$\cdot\left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}}\sqrt{\frac{\lambda}{2\pi}}\frac{4m^m\exp\left(\frac{\lambda}{\theta}\right)A^{2m-1}}{\Gamma(m)\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)^{m+\frac{1}{2}}}\cdot$$

$$\cdot K_{m+\frac{1}{2}}\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)dA\frac{1}{2\pi}d\theta_k \quad (12)$$

The cumulative distributions of the signals $z_1, z_2, ..., z_M$ are:

$$F_{z_1}(z_1)=\int_0^{z_1}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z_1-A-A_1\cos\theta_1)^2}{2\sigma^2}}\cdot$$

$$\cdot\left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}}\sqrt{\frac{\lambda}{2\pi}}\frac{4m^m\exp\left(\frac{\lambda}{\theta}\right)A^{2m-1}}{\Gamma(m)\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)^{m+\frac{1}{2}}}\cdot$$

$$\cdot K_{m+\frac{1}{2}}\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)dA\frac{1}{2\pi}d\theta_1 dz_1 \quad (13)$$

$$F_{z_k}(z_k)=\int_0^{z_k}\int_0^{\infty}\int_{-\pi}^{\pi}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(z_k-A_k\cos\theta_k)^2}{2\sigma^2}}\cdot$$

$$\cdot\left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}}\sqrt{\frac{\lambda}{2\pi}}\frac{4m^m\exp\left(\frac{\lambda}{\theta}\right)A^{2m-1}}{\Gamma(m)\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)^{m+\frac{1}{2}}}\cdot$$

$$\cdot K_{m+\frac{1}{2}}\left(\sqrt{\frac{2\lambda}{\theta^2}\left(mA^2+\frac{\lambda}{2}\right)}\right)dA\frac{1}{2\pi}d\theta_k dz_k \quad (14)$$

The probability density function of the M-ary FSK receiver output signal in the case of the hypothesis H_1 can be obtained from:

$$p(z)=\sum_{i=1}^{M}p_{z_i}(z)\cdot\prod_{\substack{j=1\\ j\neq i}}^{M}F_{z_j}(z) \quad (15)$$

The outage probability is standard performance criterion of communication systems operating over fading channels and it is defined as the probability that the instantaneous value of signal exceeds a specified value:

$$P_{out}(z_{th})=\int_0^{\gamma_{th}}p(z)dz \quad (16)$$

In the same way the probability density function and the outage probability of the M-aryl FSK receiver output signal can be derived for hypothesis H_i, $I=2, M$ and the results are identical.

IV. NUMERICAL RESULTS

Dual branch FSK receiver is used in many practical telecommunication systems. We now consider this system because of its easy implementation and very good performances.

The probability density function of dual branch FSK receiver output signal can be derived from (15) for $M=2$ and expressed as:

$$p(z)=p_{z_1}(z)\cdot F_{z_2}(z)+p_{z_2}(z)\cdot F_{z_1}(z) \quad (17)$$

The probability density functions $p(z)$, for various values of the system parameters are given in Figs. 2. to 4.

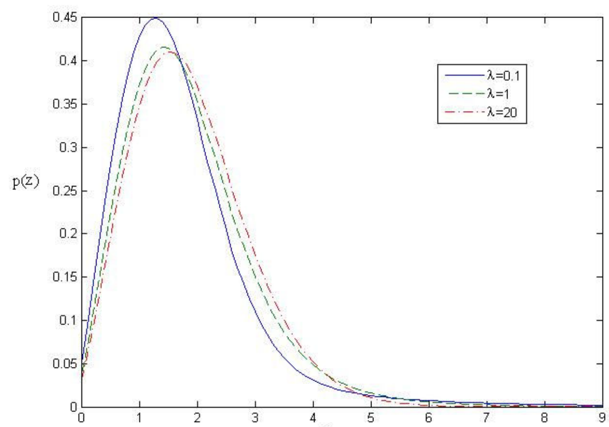

Fig.2 - The probability density functions $p(z)$ for the parameters $A_i=1$, $\sigma=1$, $m=1$, $\lambda=0.1,1,20$ and $\theta=2$

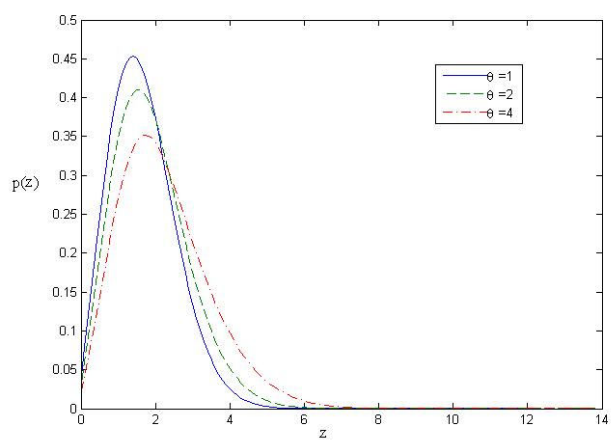

Fig.3 - The probability density functions $p(z)$ for the parameters $A_i=1$, $\sigma=1$, $m=1$, $\lambda=30$ and $\theta=1,2,4$

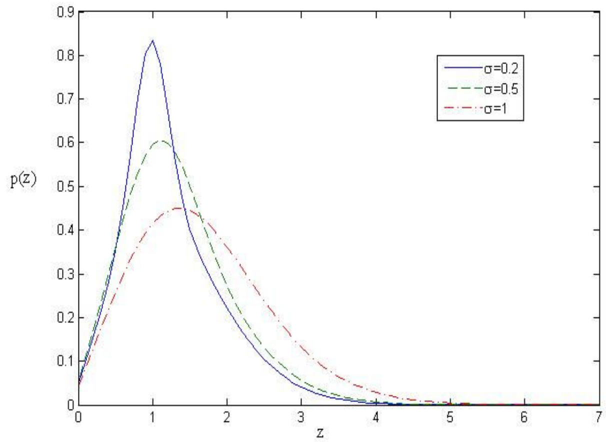

Fig.4 - The probability density functions $p(z)$ for the parameters $A_i=1$, $\sigma=0.2,0.5.1$, $m=1$, $\lambda=30$ and $\theta=1$

The outage probability $P_{out}(z)$, for the same values as in Figs 2.to 4. are given in Figs. 5. to 7.

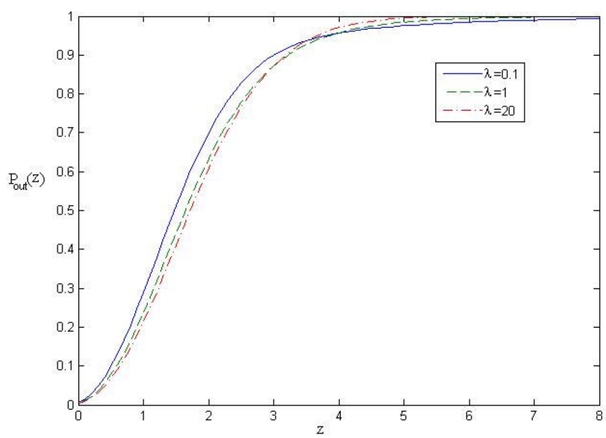

Fig.5 - The outage probability $P_{out}(z)$ for the parameters $A_i=1$, $\sigma=1$, $m=1$, $\lambda=0.1,1,20$ and $\theta=2$

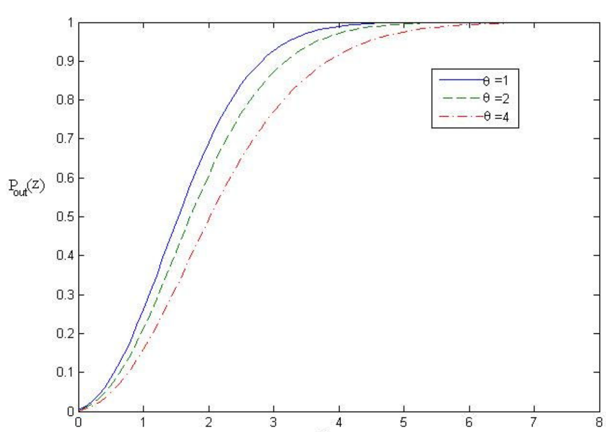

Fig.6 - The outage probability $P_{out}(z)$ for the parameters $A_i=1$, $\sigma=1$, $m=1$, $\lambda=30$ and $\theta=1,2,4$

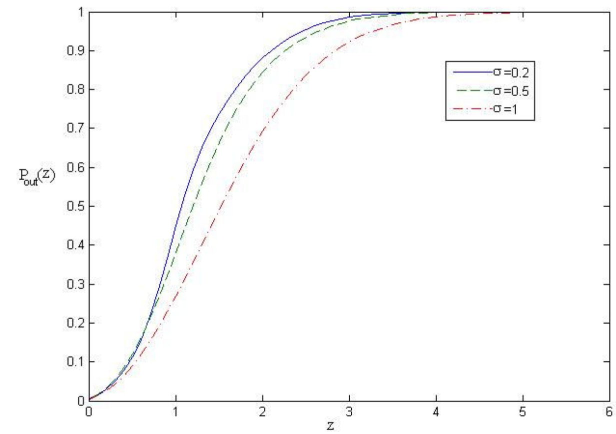

Fig.7 - The outage probability $P_{out}(z)$ for the parameters $A_i=1$, $\sigma=0.2,0.5.1$, $m=1$, $\lambda=30$ and $\theta=1$

It can be seen from the figures in this chapter that receiver has better characteristics for bigger values of parameter σ and for smaller values of parameters λ and θ.

V. Conclusion

In this paper, the statistical characteristics of the signal at the receiver output for coherent M-FSK demodulation are determined. The input signal of the receiver is digital frequently modulated signal corrupted by additive Gaussian noise, intersymbol interference and G fading.

In order to view the influence of the Gaussian noise, intersymbol interference and G fading to the M-FSK system, we are derived the probability density function of the M-FSK receiver output signal and the outage probability. Also, the moment generating function, the cumulative distribution of the output signals and the moment and variance of the output signals can be derived by probability density function of output signals. Based on this, the results obtained in this paper have a great significance and they are good base for determination of other system performances.

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