Differential Doppler target position fix
computing methods

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Abstract—The Differential Doppler is one of the target position fix method in passive target location systems. This method has number of configurations. In this paper is analyzed the simplest configuration with one stationary target and three in-line receivers with parallel velocity vector. There are presented four methods for computing target coordination from measured Doppler difference. For better comprehension of presented principles the same example is used for all methods.

Keywords—Target location, Differential Doppler, FDOA, target position.

I. INTRODUCTION

The using a Doppler effect for finding the target position is one of the amounts of target position estimation techniques which use a Differential Doppler effect on different receiving positions and their relative velocity to the target. This method is not so widespread than other methods for its Differential Doppler measuring difficulties and technological complications. Using of modern digital receivers and techniques in signal processing leads to possibility to compute the Cross ambiguity function of received signals where Differential Doppler is extracted [1].

The Differential Doppler (Doppler Difference or Frequency Difference of Arrival – FDOA) may be used in a lot of varieties (except stationary target and stationary receivers) and because of its dependency on receivers and target position coordinates and velocity vectors, the universal position fix algorithms are not published.

For showing of the possibilities to solve this task the simplest configuration was analyzed. The most simple configuration (from the computation point of view) is 2-D task 3-positional system, where the target is stationary and three receiving stations are in line on x-axis and they are moving with the same velocity vector which is parallel to x-axis (Fig.1). For better understanding the situation will be presented in example.

II. BASIC SCENARIO AND COMPUTATIONAL METHODS

Let the stationary target has coordinates:
\[ x=20000 \text{m}, \ y=50000 \text{m} \]

and transmits harmonic signal with frequency:
\( f_0=10 \text{GHz} \) (speed of light is \( c=3\times10^8 \text{m/s} \))

Let there are three receiving station with coordinations:
Receiver C: \([0,0]\)
Receiver L: \([a,0]\), where: \( a=-25000 \text{m} \)
Receiver R: \([b,0]\), where: \( b=30000 \text{m} \)

And all three receivers is moving with velocity vector \([v,0]\), where: \( v=100 \text{m/s} \)

Due to Doppler effect the frequencies of signal transmitted by target received by receivers are
Receiver C:
\[ f_{OC}=f_0+(f_0*c/v)*\frac{x}{\sqrt{x^2+y^2}} = 10000001237.968 \text{ Hz} \quad (1) \]
Receiver L:
\[ f_{OL}=f_0+(f_0*c/v)*(x-a)/(\sqrt{(x-a)^2+y^2}) = 10000002229.882 \text{ Hz} \quad (2) \]
Receiver R:
\[ f_{OR}=f_0+(f_0*c/v)*(x-b)/(\sqrt{(x-b)^2+y^2}) = 999999346.279 \text{ Hz} \quad (3) \]

So the final measured Doppler Differences are:
Left Doppler Difference:
\[ \text{DDL}=f_{OL}-f_{OC} = 991.913 \text{ Hz} \quad (4) \]
Right Doppler Difference:
\[ \text{DDR}=f_{OR}-f_{OC} = -1891.689 \text{ Hz} \quad (5) \]

Fig. 1 Basic scenario of simplest differential Doppler target position fix method with curves of constant differential Doppler shift
For better computation we can substitute the new variables: 
$L,R:$

where:

$L = \frac{DDLc}{(f_0v)} = 0.297574 \quad (6)\]

$R = \frac{DDRc}{(f_0v)} = -0.567506 \quad (7)$

This variables $L$ and $R$ represents the „cosine” differences of angles between velocity vector and receiver to target vector. The left Doppler difference or $L$ represents the curve their points has constant difference of cosine of angles to the foci (receivers positions). These curve is (even though it is not correct) „isodopplera” and is shown on Figure 2.

The main task is to extract target position $x, y$, if the $a, b, c$ and $v$ is determined and $DDL$ and $DDR$ are measured. There are a number of possibilities.

A. Iteration method

There are created three matrices $X_0, J$ and $F$:

$X_0 = [x_0; y_0]$ where $x_0$ and $y_0$ is beginning estimation of possible target position

$J = \begin{bmatrix} f_x & -1 \frac{f_x(x-a)}{c(x^2-a^2+y^2)} - \frac{f_y}{c(x^2+y^2)} & -1 \frac{f_y}{c(x^2+y^2)} \\ \frac{f_y}{c(x^2+y^2)} & -1 \frac{f_x(x-a)}{c(x^2-a^2+y^2)} + \frac{f_x}{c(x^2+y^2)} & \end{bmatrix}$

$F = \begin{bmatrix} f_x(x-a) & -f_x(x-a) & -f_x(x-a) \\ f_x(x-b) & -f_x(x-b) & -f_x(x-b) \\ \end{bmatrix}$

$X_0 = X_0 + J \ \ (-F)$

that computes new estimation of $x, y$ of target compute $J, F$ and $X_0$ repeatedly until required accuracy. [2],[4]

B. Transformation to roots of polynomial solution method

This method transforms basic equations (1),(2),(6),(7) into polynomial. The transformation is strongly complicated but the basic principle is following:

$L = (x^2+2x^2a+x^2a^2+y^2)/c(x^2+y^2)$

$R = (x^2+2x^2b+x^2b^2+y^2)/c(x^2+y^2)$

Let:

$L = x-K^2$ \quad (8)

Then $K^2 = (x^2+2x^2a+x^2a^2+y^2)/c(x^2+y^2)$

and using of substitution we can write:

$K^2 = (x^2+2x^2a+x^2a^2+y^2)/c(x^2+y^2)$

$K^2 = (x^2+2x^2a+x^2a^2+y^2)/c(x^2+y^2)$

where: $A1 = a^2, A2 = 2a, A3 = 1, A4 = a^2, A5 = a^2, A6 = 2$ \quad (9)

Then $K^2 = (x^2+2x^2a+x^2a^2+y^2)/c(x^2+y^2)$

And his roots are:

$0.00037139067656$

$0.00006186678543 + 0.00028515404024i$

$0.00015682634718 + 0.00001252638341i$

$0.00013459534464 + 0.00015109801191i$

$0.00012956291874 + 0.00011823696296i$

$0.00012095763703 - 0.00004411633861i$

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So the proper value of $M$ is $0.37139067$ (the real and positive root). Then is computed $K$ from the equation (8) or (9) and then:

$x = K^2 M$

$y = \sqrt{K^2 - x^2}$

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C. Graphical method

The principle of this method is to draw both isodopplers into grid and than find their intersection. The base problem is how to draw the isodoppler. We can use the definition of isodoppler as mentioned below:

\[ L = \cos(\alpha) - \cos(\beta) \]  

(10)

At first for \( \beta \in \{ \arccos(1-L), \pi \} \) are computed \( \alpha = \arccos(L + \beta) \) so the \( \tan(\alpha) \) and \( \tan(\beta) \) are the slopes of the lines from foci (receiver positions). The final picture is similar to Figure 1.

![Fig. 2 Construction of isodoppler by intersection of two lines](image)

D. Graphical-computational method (polar coordination)

This method is similar to graphical method but the drawing of isodopplers and finding of interception point is provided numerically.

Let \( f_2 = \beta \) a \( f_1 = \alpha \)

And for computing in interval \( \beta \in \{ \arccos(1-L), \pi \} \)

\[ f_1 = \arccos(L + \cos(f_2)) \];
\[ k_1 = \tan(f_1); \text{ slope of line from } C \text{ - position} \]
\[ k_2 = \tan(f_2); \text{ slope of line from } L \text{ - position} \]

Points of isodoppler are on intersection of lines:

\[ x_1 = (k_2 \times a) / (k_2 - k_1) \]; \[ y_1 = k_1 \times x_1, \text{ [3]} \]

After the transformation into polar coordination the isodopplers have the shape shown on Figure 3.

![Fig. 3 Isodopplers in polar coordination and their intersection](image)

So the intersection may be found by the differences of radii where they are equal. This idea can be transformed into one equation where the radii difference is:

\[ RD = \frac{\left(1 + (\tan(\alpha))^2\right) b^2 (B + \cos(\alpha) + 1) (B + \cos(\alpha) - 1)}{-\sqrt{-\left(\frac{B + \cos(\alpha) + 1}{B + \cos(\alpha) - 1} + \tan(\alpha)B + \tan(\alpha)\cos(\alpha)\right)} + \left(1 + (\tan(\alpha))^2\right) a^2 (A + \cos(\alpha) + 1) (A + \cos(\alpha) - 1)}{-\sqrt{-\left(\frac{A + \cos(\alpha) + 1}{A + \cos(\alpha) - 1} + \tan(\alpha)A + \tan(\alpha)\cos(\alpha)\right)}^2} \]

where: \( A \) - is left cosine difference (L) and \( B \) - is right cosine difference (R)

The \( RD \) is shown in Figure 4 and its zero point indicates the interception of isodopplers.

![Fig. 4 The radius difference RD as function of alpha](image)

The function \( RD \) is equal to 0 for \( \alpha = 1.1902899 \text{ rad} = 68.19859^\circ \) And for this angle the radius \( R \) is:

\[ R = \frac{1 + (\tan(\alpha))^2}{\left(1 + (\tan(\alpha))^2\right) b^2 (B + \cos(\alpha) + 1) (B + \cos(\alpha) - 1)}{-\sqrt{-\left(\frac{B + \cos(\alpha) + 1}{B + \cos(\alpha) - 1} + \tan(\alpha)B + \tan(\alpha)\cos(\alpha)\right)} + \left(1 + (\tan(\alpha))^2\right) a^2 (A + \cos(\alpha) + 1) (A + \cos(\alpha) - 1)}{-\sqrt{-\left(\frac{A + \cos(\alpha) + 1}{A + \cos(\alpha) - 1} + \tan(\alpha)A + \tan(\alpha)\cos(\alpha)\right)}^2} \]

\[ R = 53.85164804 \text{ km} \]

\[ x = R \cos(\alpha), \]
\[ y = R \sin(\alpha) \]

III. CONCLUSION

In this paper was presented four methods for Differential Doppler Target position fix computing. There is no pure analytical solution. The iteration method is very universal and usable for many different Doppler Difference configuration but it is not suitable for understanding to characteristic and ambiguity of method. This method is applicable for Error estimation and covariance matrix computing. The conversion to polynomial method is usable only for this configuration and the computing of polynomial roots needs very high numerical resolution. The graphical method is universal similar to iteration method but it is very good for visualization of different configuration situation and their isodopplers. The polar coordination method is usable only for this method but is simpler for computing the target position than polynomial method. The Differential Doppler method can be use neither for target location target velocity vector estimation nor for narrow frequency band short navigation. [5]
REFERENCES


