Diversity SSC reception in correlated generalized-*K* (K_G) fading environment

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Abstract**—** In this paper, performance analysis of switch and stay selection combining (SSC) technique in multipath/shadowing fading propagation channel is observed. The short term fading (multipath fading) and the long term fading (shadowing) are both modeled by Generalized-*K* (K_G) distribution. The proposed system is considered in correlated fading environment. The probability density function (pdf) of signal-to-noise ratio (SNR) at the output of SSC receiver is derived in form of MeijerG functions. According to this new closed-form formula, the outage probability is considered and the effects of fading and shadowing parameters and correlation coefficients on the performance gain are analyzed.

Keywords**—**composite fading channel, Generalized-*K* (K_G) distribution, outage probability, SSC diversity system.

I. Introduction

WIRELESS system performance evaluation needs to take into account the presence of channel propagation impairments such as large-scale fading, that affects the link quality by slow variation of the mean level, which arises from shadowing, and small-scale fading because of multipath propagation, introduced due to the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components [1].

This destructive combination of obstacles often occurs simultaneously. The multipath fading can be modeled by Rice, Rayleigh and Nakagami-*m* distributions and shadowing by lognormal distribution. These lognormal based fading models are analyzed in few papers [1]-[3]. However, the analytical analysis of these models is very complicated, so in some other papers the Gamma distribution is employed as an useful solution for describing shadow fading phenomena [4]. Furthermore, based on Gamma distribution, several generalized distributions have been proposed as composite fading channel models, the generalized Gamma, the *G* [5], the *K* [6] and the Generalized-*K* (K_G) [7]-[10] channel models.

There are several techniques that reduce the effects of fading and upgrade transmission reliability. One of the most effective is diversity reception. Space diversity technique improves the transmission performance using more than one independent faded version of the transmitted signal [11]. The most famous space-diversity schemes are: selection combining (SC), switch and stay combining (SSC), equal-gain combining (EGC), maximal-ratio combining (MRC), or a combination of MRC and SC called generalized selection combining (GSC) [11], [12].

SSC diversity technique is the less complex combining technique compared to the other space diversity methods. Because of its low complexity implementation requirements, SSC diversity systems are frequently used. For all other diversity techniques, the receiver processes the obtained diversity signals to maximize the system's power efficiency.

Independently of the channel condition, the transmitted signals are often correlated, due to small distance between the diversity antennas. So, the correlation between branches shoul be taken in consideration.

In [8], the brief performance evaluations of the output signal-to-noise ratio (SNR) for the maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC) and switch and stay combining (SSC) operating over uncorrelated K_G fading channels are presented. The performance analysis of various diversity combining techniques, this time over the correlated K_G fading channels, is considered in [9]. The infinite series expressions for the probability density function (pdf), cumulative distribution function (cdf) and joint moments of two correlated K_G variables are also derived in [9].

In this paper, the performance of SSC diversity receiver is analyzed. The new formulae of output SNR's pdf and cdf are derived. The detailed analysis of outage probability in correlated fading environment is presented. Based on analytical results, some numerical are also presented.

The paper is organized as follows. In the section 2, we give the brief description of channel model and mathematical formulation of proposed problem. The switch and stay combining scheme is presented in section 3. In sections 4 the derived formula for evaluating the outage probability is presented and in section 5 numerical results are given.

II. Channel Characterization

We consider wireless communication system with dual-

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branch SSC receiver operating in K_G fading environment. The K_G distribution accurately approximates a great variety of fading and/or shadowing models. Also, it is convenient and mathematically tractable distribution for evaluating adequate performance criteria.

The probability density function (pdf) of the instantaneous envelope r at the ith input branch (i=1, 2) is [8, eq. 1]

$$p_{r_i}(r)=\frac{4m^{(m+k)/2}}{\Gamma(m)\Gamma(k)\Omega^{(m+k)/2}} \times r^{m+k-1}K_{k-m}\left[2\left(\frac{m}{\Omega}\right)^{1/2} r\right], r\geq 0 \quad (1)$$

where $K_{k-m}(.)$ is the $(k-m)$th order modified Bessel function of the second kind [13, eq. 8.407], $\Gamma(.)$ is the Gamma function [13, eq. 8.31] and Ω=E(r^2)/k with E(.) being expectation operator. The parameter m is a fading severity parameter and k is a shadowing severity parameter. The two shaping parameters, m and k, can take different values between 0 and ∞. Therewith a great variety of short-term and long-term fading (shadowing) conditions can be described.

Since the instantanaous SNR per symbol of ith receiving branch is $\gamma_i = r_i^2 E_s / N_0$, the corresponding average input SNR will be $\bar{\gamma}_i = \Omega_i k E_s / N_0$. Based on this, we have pdf of the instantaneous SNR

$$p_{\gamma_i}(\gamma)=\frac{2}{\Gamma(m)\Gamma(k)}\left(\frac{mk}{\bar{\gamma}}\right)^{(m+k)/2} \times \gamma^{(m+k-2)/2}K_{k-m}\left[2\left(\frac{mk\gamma}{\bar{\gamma}}\right)^{1/2}\right] \quad (2)$$

as well as cdf of the instantaneous SNR

$$F_{\gamma_i}(\gamma)=\frac{1}{\Gamma(m)\Gamma(k)}\left(\frac{mk}{\bar{\gamma}}\right)^{(m+k)/2}\gamma^{(m+k)/2} \times G_{1,3}^{2,1}\left[mk\gamma/\bar{\gamma}\left|\begin{matrix}1-(m+k)/2\\ (k-m)/2,-(k-m)/2,-(m+k)/2\end{matrix}\right.\right] \quad (3)$$

where $G_{p,q}^{m,n}\left[.|.\right]$ is the Meijer's G-function [13, eq. 9.301].

Independent fading assumes antenna elements to be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing when diversity is applied in small terminals.

Assuming the pdf of the correlated K_G-distribution $f_{x_1x_2}(x_1,x_2)$ with identical parameters (m=m_1=m_2 and k=k_1=k_2), [9, eq. 4], with Jacobian of the transformation given by

$$|J|=\begin{vmatrix}\dfrac{\partial x_1}{\partial\gamma_1} & \dfrac{\partial x_1}{\partial\gamma_2}\\ \dfrac{\partial x_2}{\partial\gamma_1} & \dfrac{\partial x_2}{\partial\gamma_2}\end{vmatrix}=\frac{N_0}{4E_s\sqrt{\gamma_1\gamma_2}} \quad (4)$$

we derived the joint pdf of two input SNRs

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=\frac{1}{\Gamma(m)\Gamma(k)}\sum_{a,b=0}^{\infty}\frac{(mk)^{\varepsilon}\rho_n^a\rho_g^b(\gamma_1\gamma_2)^{\varepsilon/2}}{\Gamma(m+a)\Gamma(k+b)\bar{\gamma}^{\varepsilon}} \times\frac{\prod_{i=1}^{2}G_{1,3}^{2,1}\left[mk\gamma_i/\sigma_\gamma\left|\begin{matrix}1-\varepsilon/2\\ \psi/2,-\psi/2,-\varepsilon/2\end{matrix}\right.\right]}{a!b!(1-\rho_n)^{k+a+b}(1-\rho_g)^{m+a+b}} \quad (5)$$

The parameters ε and ψ are defined as $\varepsilon = k+m+a+b$, $\psi = k+b-m-a$; ρ_n is power correlation coefficient between the envelopes and ρ_g is correlation coefficient between the average fading powers of the envelopes.

III. SSC Diversity Reception

The SSC combiner processes one branch unless the instantaneous SNR of that branch falls below previously determined threshold, when the combiner switches the treated branch with the other one (system model is shown in Fig. 1).

The cdf of correlated SNRs is given by

$$F_{SSC}(\gamma)=\Pr(\gamma_T\leq\gamma_1\leq\gamma)+\Pr(\gamma_2<\gamma_T\wedge\gamma_1\leq\gamma) \quad (6)$$

The pdf of instantaneous SNR at the output of SSC, by differentiating equation (6), is given by [14, eq. 9]

$$p_{SSC}(\gamma)=\begin{cases}f_{SSC}(\gamma), \gamma\leq\gamma_T\\ f_{SSC}(\gamma)+p_\gamma(\gamma), \gamma>\gamma_T\end{cases} \quad (7)$$

where $p_\gamma(\gamma)$ is defined as (2) and $f_{SSC}(\gamma)$ is defined as

$$f_{SSC}(\gamma)=\int_0^{\gamma_T}p_{\gamma_1\gamma_2}(\gamma,\gamma_2)d\gamma_2=$$

$$\frac{2}{\Gamma(m)\Gamma(k)}\sum_{a,b=0}^{+\infty}\frac{\gamma^{\varepsilon/2-1}\gamma_T^{\varepsilon/2}(mk)^{\varepsilon}\rho_n{}^a\rho_g{}^b}{\Gamma(m+a)\Gamma(k+b)a!b!\bar{\gamma}^{\varepsilon}(1-\rho_n)^{k+a+b}}$$

$$\times\frac{1}{(1-\rho_g)^{m+a+b}}G_{1,3}^{2,1}\left[mk\gamma_T/\bar{\gamma}\left|\begin{matrix}1-\varepsilon/2\\ \psi/2,-\psi/2,-\varepsilon/2\end{matrix}\right.\right]$$

$$\times K_{\psi}\left[2\left(\frac{mk\gamma}{\sigma_{\gamma}}\right)^{1/2}\right] \quad (8)$$

where $\sigma = (1-\rho_n)(1-\rho_g)\bar{\gamma}$.

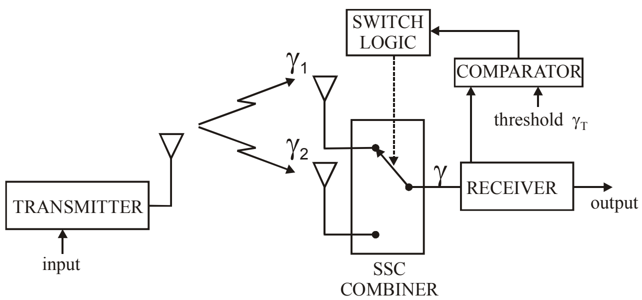

Fig. 1 Block scheme of the system

Moreover, after some manipulations of (6), the cdf of instantaneous output envelope can be expressed as [14, eq. 10]

$$F_{SSC}(\gamma) = \begin{cases} F_{\gamma_1\gamma_2}(\gamma,\gamma_T), \gamma \le \gamma_T \\ F_{\gamma}(\gamma) - F_{\gamma}(\gamma_T) + F_{\gamma_1\gamma_2}(\gamma,\gamma_T), \gamma > \gamma_T \end{cases} \quad (9)$$

where $F_{\gamma}(\gamma)$ is given by (3) and $F_{\gamma_1\gamma_2}(\gamma,\gamma_T)$ is given by

$$F_{\gamma_1\gamma_2}(\gamma,\gamma_T) = \int_0^{\gamma}\int_0^{\gamma_T} p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)d\gamma_1 d\gamma_2 =$$

$$\frac{1}{\Gamma(m)\Gamma(k)}\sum_{a,b=0}^{+\infty}\frac{(mk)^{\varepsilon}\rho_n{}^{a}\rho_g{}^{b}}{\Gamma(m+a)\Gamma(k+b)a!b!(1-\rho_g)^{m+a+b}}$$

$$\times\frac{\gamma^{\varepsilon/2}\gamma_T^{\varepsilon/2}}{\bar{\gamma}^{\varepsilon}(1-\rho_n)^{k+a+b}}$$

$$\times G_{1,3}^{2,1}\left[mk\gamma/\sigma_{\gamma}\left|\begin{matrix}1-\varepsilon/2\\ \psi/2,\,-\psi/2,\,-\varepsilon/2\end{matrix}\right.\right]$$

$$\times G_{1,3}^{2,1}\left[mk\gamma_T/\sigma_{\gamma}\left|\begin{matrix}1-\varepsilon/2\\ \psi/2,\,-\psi/2,\,-\varepsilon/2\end{matrix}\right.\right] \quad .(10)$$

IV. Outage Probability

The outage probability, P_{out}, is a standard measure of the communication system performance and is commonly used to control the noise level in wireless communication systems.

For proposed system model, outage probability is defined as the probability that combined-SNR falls below a given outage threshold q, also known as a protection ratio. The analysis of outage probability is important in design of a wireless communication systems in order to achieve reasonable influence of noise on system performance.

The outage probability can be evaluated as

$$P_{out} == \int_0^{q} p_{SSC}(\gamma)d\gamma = F_{SSC}(\gamma) \quad (11)$$

V. Numerical Results

Fig. 2 shows the outage probability, P_{out}, versus the normalized switching threshold, $\gamma_T/\bar{\gamma}$, for various correlation coefficient values. It is obvious that there is an optimal threshold that minimizes outage probability. For this threshold value, SSC combining can be observed as SC. This figure also shows that higher value of existed correlation degrades the performance gain.

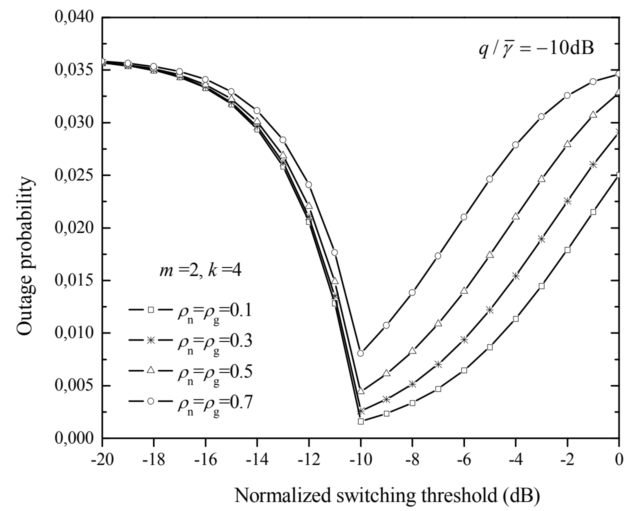

Fig. 2 Outage probability for various correlation coefficient values

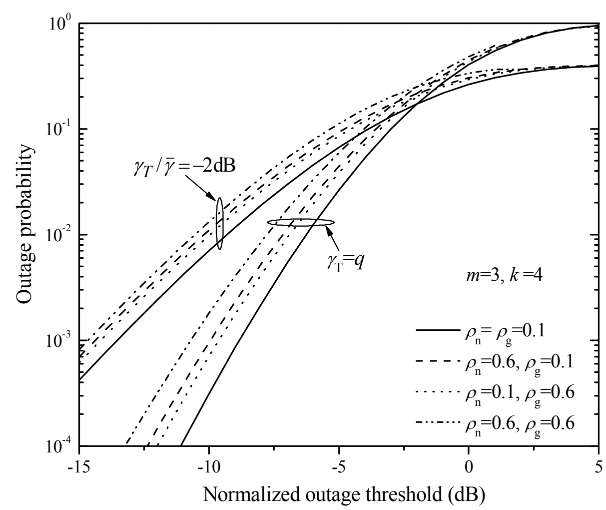

Fig. 3 Outage probability for different values of normalized switching threshold

The outage probability, as a function of normalized outage threshold, $q/\bar{\gamma}$, for different values of normalized switching threshold is shown in Fig. 3. For the special case of γ_T=q, (11) gives the same results as [9, eq. 16]. The presented four dependences (curves) for different values of correlation coefficients and γ_T=q are the same as corresponding four curves in [9, Fig. 1]. It is interesting to note here that system performance is better when $\gamma_T=q$, compared to the randomly picked threshold value ($\gamma_T / \bar{\gamma} = -2\,\mathrm{dB}$). The influence of correlation between branches is much more significant in the case of lower fading and shadowing severity. For m=1.1, the system performance is not acceptable, even for light shadowing condition k=5.1. The performance improves significantly when small scale fading effect decreases m=3.4 (fading parameter increases).

Fig. 4 shows the outage probability as a function of normalized outage threshold for different fading and shadowing severity in correlated and uncorrelated system environment. It is obvious that for uncorrelated case system performance improves (outage probability decreases). We can also observe the same influence of fading and shadowing parameters for correlated and uncorrelated case (when shaping parameters decrease/increase, outage probability increases/decreases).

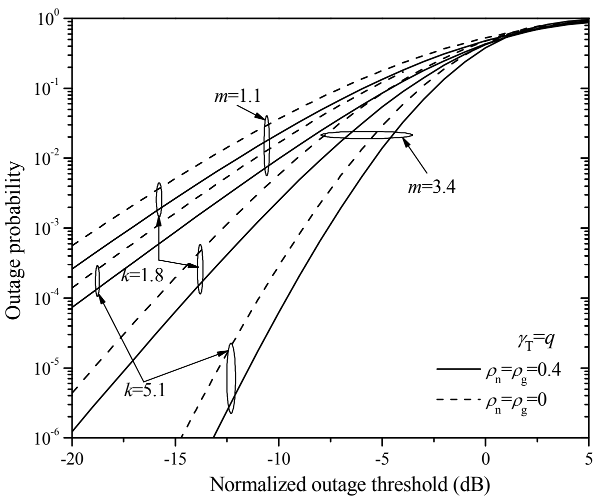

Fig. 4 Outage probability of correlated and uncorrelated system for different values of shaping parameters

VI. Conclusion

In this paper, the performance analysis of system with SSC receiver, over correlated composite fading channels, was carried out. Generalized-K (K_G) distribution was observed for evaluating performances of proposed diversity system. The new closed-form expressions for pdf and cdf of output SSC SNR were derived. Using these new formulae, the outage probability was efficiently evaluated. The effects of various parameters, such as the fading and shadowing severity parameters and level of correlation to the system's performance were also examined.

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