Performance analysis of multi-hop system in Nakagami-*m* environment

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Abstract**—**In this paper, performance analysis of multi-hop transmission system is presented. Transmission channels experience Nakagami-*m* fading. The generalized case in which desired signal and co-channel interference (CCI) experience mutually different amount of fading is considered. Furthermore, the analysis includes the case in which fading severity between relays is unequal which is real case scenario in practice. Outage probability, as an important performance measure, is examined. A generalized expression for *N* relays is derived and in order to illustrate the results, cases with 2, 3 and 4 relays are considered and the results are compared.

Keywords**—**Gamma distribution, multi-hop transmission, Nakagami-*m* distribution, outage probability

I. Introduction

MULTI-HOP systems are used to solve the problem of major attenuation in signal transmission by dividing transmitter-receiver channel into shorter channels connected by relays [1]. This principle found broad application now days in cooperative wireless communication systems. Interference and fading influence both the input of receivers of all relay stations and the final receiver station causing variations of signal amplitude up to 30 dB [2]. The values can vary above and below predetermined value that can sometimes cause complete connection break-up. Fading varies depending on path length, frequency, climate conditions as well as terrain profile. Depending on the fading nature the probability density function (PDF) of the signal amplitude at relay stations and receiver inputs can be determined. Nakagami-m distribution is a statistical distribution which can accurately model different types of environments and is used for modelling of fading that occurs in mobile communications. Although empirical and therefore susceptible to different approximations and settings, Nakagami-m distribution proved to be very useful due to easy manipulation and wide range of applicability [3]. The Nakagami model represents a wide variety of realistic line-of-sight/none line of sight fading channels encountered in practice [4]. In [5] the performance of dual-hop system has been analyzed. In this paper the results obtained in [5] are generalized for *N* relays and used for system analysis with 2, 3 and 4 relays.

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II. System model

Multi-hop system consists of a transmitter, a receiver and relays between them therefore the signal is transmitted over several sections. In this paper, the communication between transmitting terminal S and receiving terminal T is realized by *N* relay terminals, which is illustrated in Fig.1.

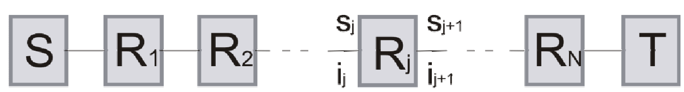

Fig. 1 The model of an *N*-relay station system

If the envelopes of desired signal and interference at the input of *j*-th relay are denoted with s_j and i_j, signal-to-interference-ratio (SIR) at the input of *j*-th terminal can be defined as following:

$$\lambda_j = \frac{s_j^2}{i_j^2} = \frac{z_j}{y_j}, \quad j = \overline{1, N}. \tag{1}$$

Envelopes of desired signal s_j and interference i_j have Nakagami-*m* distribution that can be expressed as following

$$p_{s_j}(s_j) = \frac{2m_{j_1}^{m_{j_1}} \cdot s_j^{2m_{j_1}-1}}{\Gamma(m_{j_1}) \cdot \Omega_{j_1}^{m_{j_1}}} \cdot e^{-\frac{m_{j_1}}{\Omega_{j_1}} s_j^2} \tag{2}$$

$$p_{i_j}(i_j) = \frac{2m_{j_2}^{m_{j_2}} \cdot i_j^{2m_{j_2}-1}}{\Gamma(m_{j_2}) \cdot \Omega_{j_2}^{m_{j_2}}} \cdot e^{-\frac{m_{j_2}}{\Omega_{j_2}} i_j^2}, \tag{3}$$

respectively.

z_j and y_j have Gamma distribution with following form

$$p_{z_j}(z_j) = \frac{1}{\Gamma(m_{j1})} \left(\frac{m_{j1}}{\Omega_{j1}} \right)^{m_{j1}} z_j^{m_{j1}-1} e^{-\frac{m_{j1}}{\Omega_{j1}} z_j}, \quad z_j \geq 0 \tag{4}$$

$$p_{y_j}(y_j) = \frac{1}{\Gamma(m_{j2})}\left(\frac{m_{j2}}{\Omega_{j2}}\right)^{m_{j2}} y_j^{m_{j2}-1} e^{-\frac{m_{j2}}{\Omega_{j2}} y_j}, \; y_j \geq 0 \quad (5)$$

where Ω_{j1} represents average power of desired signal, Ω_{j2} represents average power of interference and m_{j1}, m_{j2} represent Nakagami-*m* parameters ($m \geq 0.5$) which determine fading severity. As parameter *m* increases, the fading severity decreases.

A. PDF

Probability density function (PDF) expression for SIR for *j*-th segment can be calculated using the equation

$$p_{\lambda_j}(\lambda_j) = \int_0^\infty |J_j| \, p_{z_j}(\lambda_j y_j) p_{y_j}(y_j) dy_j \quad (6)$$

where Jacobian function can be calculated by following equation

$$|J_j| = \left|\frac{dz_j}{d\lambda_j}\right| = y_j. \quad (7)$$

Final equation for PDF for *j*-th segment is derived in [5] and has following form:

$$p_{\lambda_j}(\lambda_j) = \frac{\Gamma(m_{j1}+m_{j2})}{\Gamma(m_{j1})\Gamma(m_{j2})} m_{j1}^{m_{j1}} m_{j2}^{m_{j2}} \times \frac{\lambda_j^{m_{j1}-1}}{\left(\frac{1}{\gamma_j} m_{j1}\lambda_j + m_{j2}\right)^{m_{j1}+m_{j2}}} \left(\frac{1}{\gamma_j}\right)^{m_{j1}} \quad (8)$$

where γ_j is ratio of average powers of desired and interference signal, i.e.

$$\gamma_j = \frac{\Omega_{j1}}{\Omega_{j2}} \quad (9)$$

B. CDF

Cumulative distribution function (CDF) for the same segment is:

$$F_{\lambda_j}(\lambda_j) = \int_0^{\lambda_j} p_{\lambda_j}(x) dx \quad (10)$$

Final equation for CDF for *j*-th segment is obtained in [5]

$$F_{\lambda_j}(\lambda_j) = \frac{\Gamma(m_{j1}+m_{j2})}{\Gamma(m_{j1})\Gamma(m_{j2})}\left(\frac{m_{j1}}{m_{j2}}\right)^{m_{j1}} \frac{\lambda_1^{m_{j1}}}{m_{j1}}\left(\frac{1}{\gamma_j}\right)^{m_{j1}} \times {}_2F_1\left(\left(m_{j1}+m_{j2}\right), m_{j1}; 1+m_{j1}; -\frac{\lambda_j}{\gamma_j}\frac{m_{j1}}{m_{j2}}\right) \quad (11)$$

System failure can occur in case of a disconnection in $S-R_1$ channel which happens when the value of λ_1 is below the threshold required for Quality of Service (QoS) to be satisfied. The failure can occur also in following cases: $S-R_2, S-R_3, ..., S-R_N$ where $\lambda_2, \lambda_3, ..., \lambda_N$, respectively are below the threshold required for QoS. In order to analyze the system of *N* relays, it is necessary to find distribution of minimum of all these values $\lambda = \min(\lambda_1, \lambda_2, ..., \lambda_j, ..., \lambda_N)$. PDF of λ can be calculated by substituting (8) and (11) into the equation:

$$\begin{aligned} p_\lambda(\lambda) = {} & p_{\lambda_1}(\lambda)\left(1-F_{\lambda_2}(\lambda)\right)\left(1-F_{\lambda_3}(\lambda)\right)\ldots\left(1-F_{\lambda_N}(\lambda)\right) \\ & + p_{\lambda_2}(\lambda)\left(1-F_{\lambda_1}(\lambda)\right)\left(1-F_{\lambda_3}(\lambda)\right)\ldots\left(1-F_{\lambda_N}(\lambda)\right) \\ & + \ldots + \\ & + p_{\lambda_N}(\lambda)\left(1-F_{\lambda_1}(\lambda)\right)\left(1-F_{\lambda_2}(\lambda)\right)\ldots\left(1-F_{\lambda_{N-1}}(\lambda)\right) \end{aligned} \quad (12)$$

Final expression for PDF of λ is:

$$p_\lambda(\lambda) = \sum_{n=1}^{N} p_{\lambda_n}(\lambda) \prod_{\substack{k=1 \\ k \neq n}}^{N} \left(1 - F_{\lambda_k}(\lambda)\right) \quad (13)$$

Using one of mathematical packages Mathematica or MatLab it can be confirmed that $p_\lambda(\lambda)$ is non-negative function and that $\int_0^\infty p_\lambda(\lambda) = 1$ is true therefore confirming regularity of above mentioned equation.

C. Outage probability

Highly important and widely accepted system performance indicator is outage probability which is defined as probability of having SIR value lower than pre-determined threshold which defines required QoS. Outage probability can be obtained using following formula

$$P_{out} = \int_0^{\lambda_0} p_\lambda(\lambda) d\lambda \quad (14)$$

where λ_0 represents established threshold.

Using the equation different fading environment can be simulated in order to analyze system behaviour.

III. Numerical Results

Mathematica 6.0 and Origin 8 are used for obtaining the figures for system analysis based on above derived equations.

In Fig. 2 PDF of λ for different values of system parameters is shown. It reveals that decreasing severity of fading for desired signal and increasing of γ increases the chance of taking on a larger output SIR, and therefore results in better performance of the system.

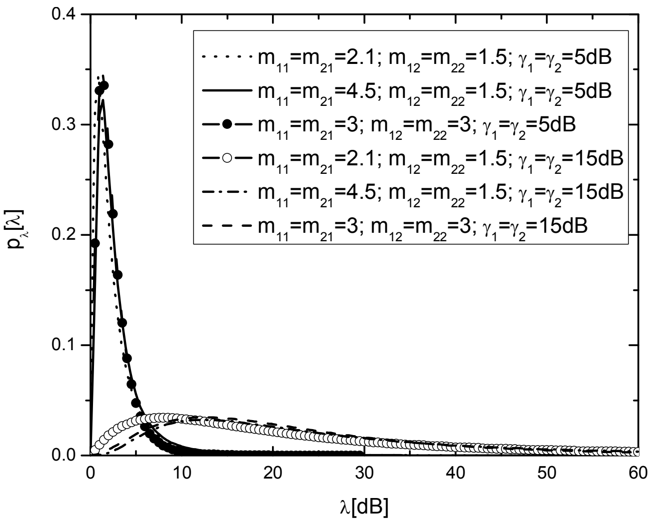

Fig. 2 PDF for dual-hop system for different system parameters

In Fig. 3 and Fig. 4 the results obtained from (14) are used for system analysis. The results will be shown for different number of relays in order to show how the conditions in the system change.

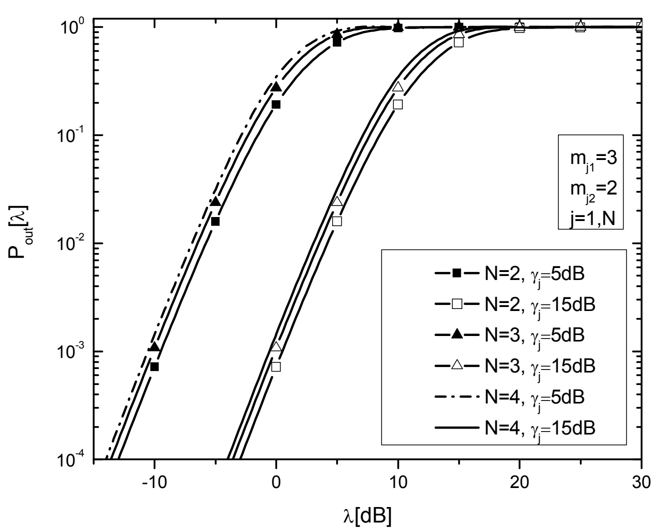

Fig. 3 Outage probability for different value γ_j and for number of branches *N*

First case to be considered is shown in Fig. 3 when fading severity of desired signal and interference is constant. Without loss of generality, we assume that ratios of average powers of desired and interference signal are equal in all relays input. By analyzing Fig. 3, it can be concluded that outage probability is higher for lower values of γ and higher values of *N*. In relay station the signal is amplified so that γ at the input of the next relay station which attenuates, due to propagation crossing the distance between them [7], is equal to γ at the input of previous one implying that the distance between transmitter and receiver is greater therefore the outage probability for the system is higher.

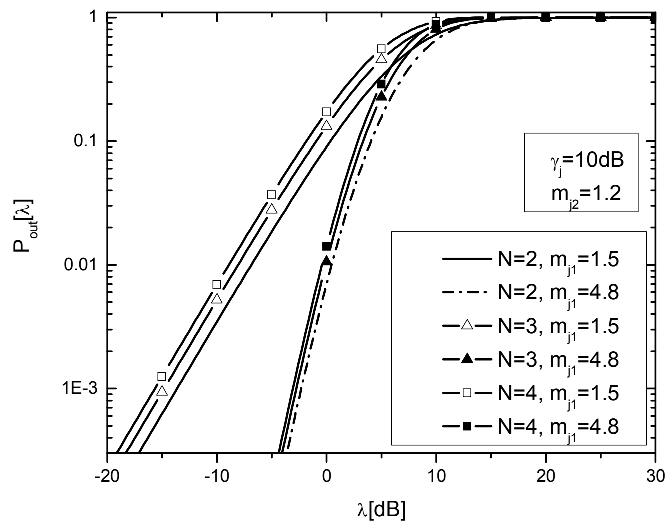

Fig. 4 Outage probability depending on SIR and fading severity of desired signal as well as number of branches *N*

Next case for analysis considers changing fading severity of desired signal along with number of relays while fading severity of interference and γ do not change. In Fig. 4 it is clearly shown that the outage probability of the system is higher for higher fading severity of desired signal, i.e. lower *m*, which is expected. The figure also shows that the outage probability is higher as number of relays i.e. distances between transmitter and receiver increases.

IV. Conclusion

This paper presents the analysis of multi-hop system in Nakagami-*m* environment for different values of system parameters and different number of relays in the system. Realistic cases of the system were considered therefore it refers to different values of parameters in each transmitting segment of the system. The paper includes derivations of equations for PDF of minimum of instantaneous SIR values as well as outage probability used for system analysis. Presented figures show that the outage probability is higher for lower average SIR values as well as for higher fading severity to which the desired signal is exposed. The results also show that the conditions in the system degrade as the number of relays in the system increases, i.e. distance between transmitter and receiver increases.

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